RELIABILITY ANALYSIS OF SEISMIC SHEAR-TYPE STRUCTURES

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SUMMARY

Statistical analysis and significance test of structural seismic response variable have shown that the story maximum ductility ratio $\mu_{\max}$ and equivalent hysteretic loop numbers $\overline{N}$ all belong to log-normal distribution. Reliability analysis method of structures under different earthquake intensities in structural serviceability limit state is put forward on the basis of results of seismic risk analysis and that of equivalent ductility damage criteria accounting for low-cycle fatigue characteristics of structures. And one-order, second-moment point estimation method is used to focus the reliability analysis on analysis of reliability index β for practical convenience.

INTRODUCTION

The object of structural analysis is to assess the reliability of structures, i.e., to assess quantitatively the structural safety based on probabilistic sense. In recent years, the reliability analysis of seismic structures has attracted some researcher's attention and many interesting results are obtained.

The structural seismic reliability analysis has to solve the following problems: 1.Probabilistic models of earthquake ground motions. 2. Mechanical models of structures. 3. Mathematical methods to solve nonlinear stochastic vibration problems. 4.Failure criteria of seismic structures and assessment standards of structural seismic reliability.

Earthquake ground motion models must de based on surveyed strong ground motion factors. However, earthquake ground motion is a very complex process. Until now, all proposed stochastic ground motion, either often used Kanai Filtered White Noise Model(Kanai,1957) or Markov Colored Spectrum Model(Mstsugashima,1986) and Two-freedom Filtered Noise Model(Tanyi Ye,1993) can only be regarded as approximate math abstract of earthquake ground motion.

Generally speaking, with enriching of surveyed strong ground motion factors, earthquake ground motion models will tend to more reasonable and more accurate to reflect real conditions. But, after all, earthquake is a kind of natural phenomenon which can not be forecasted very correctly. To such a strongly unexpected thing, relying on data possessed now, it is not easy to build a stochastic model that can not only describe earthquake ground motions accurately but also facilitate practical applications.

Structures can not avoid to be degraded into heavy non-linearity under severe earthquake. Though ground motion input may be simplified often as Gauss excitation, the modest even light non-linearity of structures will two ends of the response probability distribution curve deviate from Gauss type obviously. This deviation will greatly influence the system damage probability estimation. Although many methods to solve nonlinear random vibration problems have been put forward, equivalent linearization is still the most efficient and practicable method till now. To evaluate the equivalent parameters in this method, it needs to know the probability density function of the response process and its high order differential process, but this is unknown, so assumed or approximate method has to be adopted. Even equivalent linearization does not ask the structural non-linearity to

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be small, the precision of the approximate method relies on the magnitude of nonlinear item.

The reliability analysis is based on some structural failure criteria. The structural failure criteria reflect our realization on earthquakes and structural properties. The structural cumulative damage caused by the low-cycle fatigue characteristics of structures and the time effect of earthquakes should be considered in the criteria.

Based on the Monte-Carlo numerical analogue rule, the study in this paper gets structural seismic reliability by directly acquiring stochastic characteristics of structure response. The damage criteria used in the paper takes account of the low-cycle fatigue characteristics of structures and integrate first passage failure with cumulative damage. The analysis done by this paper uses amount of several artificial seismic waves from the same ensemble as input, through time history analysis seismic waves often encountered in engineering, gets structural seismic reliability by directly acquiring stochastic characteristics of structural responses. Thus the errors caused by making simplified ground motion models and solving strongly nonlinear stochastic vibration problems may be avoided. The first-order and second-moment point estimation method is used in reliability analysis, and the reliability of seismic structures is analyzed in terms of safety index • to keep in accordance with the Architecture and Structure Design Uniform Standards of China (GBJ68-84) and to facilitate the practical applications.

BASIC FORMULAS FOR STRUCTURAL SEISMIC RELIABILITY

Limit State Equation for Structural Damage

The damage index defined by the author (LIU,1994) considering low-cycle fatigue characteristic of structures is

$$D = \mu_{\max} / \mu^* \tag{!}$$

Where $\mu_{\max}$ ----maximum story ductility response (displacement response), μ^* ----equivalent structural ductility. In the serviceability limit [0,T],

Basic Equations for Structural Seismic Reliability

In the serviceability limit[0,T], the probability of a structure that no damageoccur under different levels of seismic action can be expressed as:

$$P(Y_i, T) = \sum_k P_E(I_k, T) \cdot P_{yi}(D_i < 1 | I_k \;) = \sum P_E(I_k, T) \cdot P_{iy}(\frac{\mu_{\max}}{\mu^*} < 1 | I_k) \tag{2}$$

In which, $P_E(I_k T)$ is the probability in the serviceability limit T (T is equal to 50 years in China) while the intensity of an earthquake is I_k. As intensity is used to measure earthquake severity, I_k is ranging from 5° to 12°. $P_{yi}(D_i < 1 | I_k)$ is the no-damage probability of the ith story of the structure when ground motion of I_k intensity occurs.

In view of the limit-state equation, the conditional no-damage probability of the ith story is:

$$P_{yi}(D_i < 1 | I_k) = P_{yi}(\mu_{\max} \big/ \mu^* < 1 | I_k) = P_{yi}[(\ln \mu_{\max} - \ln \mu^*) < 1 | I_k] \tag{3}$$

According to the experiment result (Liu,1994)

$$\mu^* = \overline{N}^{-\beta} . N_p^{\beta} . \mu_p \tag{4}$$

In which μ_p is the maximum story ductility, $\overline{N}$ is equivalent hysteretic loop number, β is a constant obtained from experiments. It is recommended from LIU(LIU,1994) that β is equal to 0.152.

Then Eq.(3) can be expressed as

$$P_{yi}(D_i < 1|I_k) = P_{yi}[(\ln \mu_{\max} + \beta \ln 4\overline{N} - \ln \mu^*) < 1|I_k] \tag{5}$$

It is observed that reliability of a story subjected to seismic action depends on earthquake intensity I_k, maximum story ductility $\mu_{\max}$, and equivalent hysteretic loop Number $\overline{N}$ and ultimate story ductility μ_p.

PROBABILISTIC DISTRIBUTION OF EARTHQUAKE INTENSITY

Bao(Aibing Bao,1985) has made seismic risk analysis of 45 cities in China and 20 towns in Xinjiang in China respectively. Probabilistic index of occurrence of earthquakes of different intensity levels is presented. Gao(Xiaowang Gao,1985) defines the probability distribution of the earthquake intensity in China using extreme value III type function on the basis of regressive results of seismic risk evaluation:

$$F_T(i) = \exp[-(\frac{\omega - i}{\omega - \varepsilon})^k] \tag{6}$$

In which ω =upper bound value, ω =12 for earthquake intensity;

ε =mode intensity, its exceed probability within 50 years is 0.632;

k =shape function, see Table 1 in Gao'paper.

Thus, the probability of a structure attacked by ground motion inducing earthquake intensity I_k in its serviceability life T can be described as

$$P_E = (I_k, T) = F_T(I_k + \frac{1}{2}) - F_T(I_k - \frac{1}{2}) \tag{7}$$

STATISTICAL CHARACTERISTICS OF MAXIMUM STORY DUCTILITY

It is found that the story yield strength coefficient ξ_y has much greater effect on the maximum story ductility than other variables, hence uniform structures with different ξ_y are taken into account for statistical analysis. Hysteretic model is the bilinear model with stiffness degradation. Structural parameters are shown in Table 1

.The author have gotten the histograms of maximum ductility. Except those 150 seismic waves are employed for structures with ξ_y equal to 0.3 or o.4, 30 seismic waves are used for the structures with other ξ_y. It is shown that logarithmic normal distribution fits all the histograms. The probabilistic density function is

$$f(\mu_{\max}) = \frac{1}{\mu_{\max}\sigma_{\ln \mu_{\max}}\sqrt{2\pi}} \exp[\frac{-(\ln \mu_{\max} - \overline{\ln \mu_{\max}})^2}{2\sigma^2_{\ln \mu_{\max}}}] \tag{8}$$

Table1: Structural Parameter

Story (n)	Fundamental Period ,T(s)	Mass, m $kN \cdot \sec^2 / m$	The First stiffness (kN/m)	Degradation Stiffness Co.	ξ_y
8	0.8	1×10^3	9.05659×10^3	0.00	0.2~0.6

Some analytical results of $\mu_{\max}$ are shown in table 2.

Table2: Some Statistical Results of $\mu_{\max}$

Index ξ_y	$\overline{\mu_{\max}}$	$\overline{\ln \mu_{\max}}$	$\sigma^2_{\mu_{\max}}$	$\sigma^2_{\ln \mu_{\max}}$
0.2	17.37	2.77	30.42	0.154
0.3	10.16	2.26	11.43	0.133
0.4	6.13	1.81	3.92	0.075
0.5	3.97	1.35	1.01	o.o69
0.55	3.29	1.15	0.75	0.067

From the histograms and best-fit curve of distribution, it is shown that the fir results are appropriate for mid-structures considering statistical characteristics of the sample and best-fit distribution. It may stem from the much greater scatter in nonlinear responses of the low-strength structures.

Significance test is carried out with the assumption of logarithmic normal distribution of the analytical results, employing K-S test.

Comparing the empirical distribution function $F_N(x)$ with the assumed theoretical distribution function $F(x)$, the following statistical value can be put forward:

$$D_n = \sup_{-\infty<x<\infty} \left| F_n(x) - F(x) \right| = \max_{1<k<n} \left\{ \left| F_n(x_k) - F(x_k) \right|, \left| F_n(x_{k-1}) - F(x) \right| \right\} \tag{9}$$

According to assigned significance level α, the critical $D_{N,\alpha}$ can be obtained for certain samples from the $D_{N,\alpha}$ table (Yongqi Chen,1986) . When the computed D_N is less than $D_{N,\alpha}$, little difference exists between distribution and the original assumptions. Thus there is no reason to reject the original assumptions.

The K-S test results for structures with different ξ_y are showed in table 3.

Table3: K-S Test Result $(\mu_{\max})$

ξ_y	D_N	$D_{N,\alpha}$		Sample Size. N
		α=0.05	α=0.01	
0.20	0.128	0.243	0.291	30
0.30	0.082	0.111	0.133	150
0.40	0.071	0.111	0.133	150
0.50	0.061	0.243	0.291	30
0.55	0.054	0.243	0.291	30
0.60	0.048	0.243	0.291	30

From the table, the original assumptions are not rejected for all structures with the confidence being 0.05 and 0.01. Therefore, it is convincing that the maximum story ductility response of shear-type structures under seismic is fit with logarithmic normal distribution.

STATISTICAL CHARACTERISTICS OF EQUIVALENT HYSTERETIC LOOP NUMBER

The equivalent hysteretic loop number $\overline{N}$ is statistically analyzed along with the maximum story ductility μ_{max}. The response curves of the corresponding to μ_{max} are considered.

Statistical analysis is carried out for structures with ξ_y ranging 0.2~0.6. Structural parameters are shown in Table 1. Assuming the distribution is logarithmic normal and employing K-S method for significance test, the results are summarized in Table 4. It is shown that at the significance levels of confidence of 0.05 and 0.01, no significant difference can be observed between the distribution of equivalent hysteretic loop numbers $\overline{N}$ and the assumed logarithmic normal distribution. But the scatter is greater and the fitness is worse than that of μ_{max}. Statistical results of $\overline{N}$ are showed in Table 5.

Table 4 K-S Examination results ($\overline{N}$)

ξ_y	D_N	$D_{N,a}$		Sample Size, N
		$\alpha = 0.05$	$\alpha = 0.01$	
0.20	0.226	0.243	0.291	30
0.30	0.102	0.111	0.133	150
0.40	0.092	0.111	0.133	150
0.50	0.078	0.243	0.291	30
0.55	0.065	0.243	0.291	30
0.60	0.054	0.243	0.291	30

Table 5 Statistical results of $\overline{N}$

Index ξ_y	$m\overline{N}$	$m_{\ln \overline{N}}$	$\sigma^2_{\overline{N}}$	$\sigma^2_{\ln \overline{N}}$
0.2	9.22	2.16	4.67	0.082
0.3	4.72	1.42	2.12	0.071
0.4	2.11	0.65	0.59	0.061
0.5	1.09	0.08	0.46	0.059
0.55	0.84	0.06	0.41	0.043

($m_{\overline{N}}$, $m_{\ln \overline{N}}$ are average values of $\overline{N}$ and $\ln \overline{N}$ respectively)

PROBABILISTIC DISTRIBUTION OF ULTIMATE STORY DUCTILITY

Gao(Xiaowang Gao,1973) excluded normal distribution under confidence of 5%, employing K-S statistical test method on the basis of 230 reinforced concrete column experiments, and didn't reject logarithmic normal distribution and extreme value I distribution. For practical convenience, μ_p is treated as logarithmic normal distribution.

SEISMIC RELIABILITY OF SHEAR-TYPE STRUCTURES

Damage reliability of ith story of a structure in its serviceability limit [0,T] under seismic action causing I_k intensity can be expressed as follow according to Eq. (5)

$$P_{yi}(D<1|I_k) = P_{yi}(\ln\mu_{\max} + \beta\ln 4\overline{N} - \ln\mu_p < 0|I_k) \tag{10}$$

Although it is not completely comply with real situations, for convenience it is assumed that $\mu_{\max}$, $\overline{N}$ and μ_p are independent. Thus the reliability can be obtain when their distributions are known.

To avoid complicated numerical integration, the first-order and second-moment point estimation method (JC method) is used to obtain the above probability:

$$P_{yi}(D<1|I_k) = \Phi(\beta) \tag{11}$$

Now that $\mu_{\max}$, $\overline{N}$ and μ_p all belong to logarithmic normal distribution, the reliability index β is

$$\beta = \frac{\ln m_{\mu_p} - \ln m_{\mu_{\max}} - \beta\ln(4m_{\overline{N}})}{[(\frac{\sigma_{\mu_p}}{m_{\mu_p}})^2 + (\frac{\sigma_{\mu_{\max}}}{m_{\mu_{\max}}})^2 + (\frac{\beta_{\sigma_{\overline{N}}}}{m_{\overline{N}}})^2]^{\frac{1}{2}}} = \frac{\ln m_{\mu_p} - \ln m_{\mu_{\max}} - \beta\ln(4m_{\overline{N}})}{(V_{\mu_p}^2 + V_{\mu_{\max}}^2 + \beta^2 V_{\overline{N}}^2)^{\frac{1}{2}}} \tag{12}$$

Where $m_{\mu_p}, m_{\mu_{\max}}, m_{\overline{N}}, \sigma_{\mu_p}, \sigma_{\mu_{\max}}, V_{\mu_p}, V_{\mu_{\max}}, V_{\overline{N}}$ are mean values, standard deviation and variability coefficients of μ_p, $\mu_{\max}$ and $\overline{N}$.

For shear-type structures, it is assumed that the overall structure will fails if any story fails. Then the probability of the structure that no damage occurs in its serviceability limit state subjected to earthquake of I_k intensity is

$$P_T(D<1|I_k) = \prod_{j}^{n} P_{yi}(D<1|I_k) \tag{13}$$

$i =$ 1,2,…, n, n is story numbers. For practical application, it is convenient to determine the reliability of the weakest story (where $\mu_{\max}$ occurs) to take the place of that of the overall structure.

When evaluating the reliability of the structures under major earthquakes in which no collapse occurs, I_k in Eq(13) my be replaced by corresponding acceleration.

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