0060

SAFTEY EVALUATION METHOD OF STRUCTURAL SYSTEMS AND ITS APPLICATION TO SEISMIC DESIGN OF RC BRIDGE PIER

Mitsuyoshi AKIYAMA[1] And Motoyuki SUZUKI[2]

SUMMARY

Firstly, a new evaluation method for the probability of failure of a structural system based on reliability theory is proposed. In this proposed method, the multiple limit state is taken into consideration simultaneously. Using numerical examples, it is demonstrated that this method is accurate yet simple to implement. Then, the safety of RC bridge pier against earthquakes is evaluated on the basis of the proposed method. Three limit states flexural capacity, shear capacity and ductility are taken into consideration in the analysis. Particular importance is attached to the shear/flexural capacity ratio of the bridge pier, and the effect of shear capacity of a flexural-failure-dominant bridge pier on its safety is studied. It is shown that that capacity ratio is very important factors in evaluating the seismic performance of RC bridge piers.

INTRODUCTION

Even when based on reliability, conventional design methods apply safety checks separately to each limit state considered relevant by the designer; all likely limit states are not considered simultaneously. Moreover, safety verification is usually carried out only on the limit state which is most likely to arise; i.e. the one with the highest failure probability. According to conventional thinking, in the case of an RC bridge pier designed to undergo flexural failure, it is considered sufficient to ensure that the shear capacity exceeds the shear force at the moment when flexural capacity is reached and to examine safety against flexural failure only. In a case like this, the failure probability of the pier is assumed to be totally unaffected by its shear capacity. However, the ratio of shear capacity to flexural capacity (referred to as the capacity ratio), for example, is closely correlated to the ductility of the bridge pier; if the capacity ratio is low, the bridge pier is subject to brittle fracture. Moreover, the flexural and shear capacities are influenced by differences between the uncertainty levels of models containing equations for calculating the flexural and shear capacities; it can be expected that these values will substantially influence the failure probability of the bridge pier. In general, to use one limit state to represent a failure event which really comprises a plurality of limit states results in a risky evaluation. Even if the design satisfies the target failure probability for that limit state, the resultant structure may not meet the prescribed safety requirements.

Adapting this point of view, this study aims, firstly, to propose a safety evaluation method for structural systems which offers ease of calculation, good accuracy in a practical context, and takes into account multiple limit states while remaining an approximate method. Then, based on the proposed method, an assessment related to the capacities and ductility will be adopted as the ultimate limit state for a RC bridge pier (free-standing, single-column type), and the reliability of the RC bridge pier in an earthquake will be analyzed. In doing so, particular

importance is attached to the capacity ratio of the bridge pier, and the effect of shear capacity of a flexural-failure-dominant bridge pier on its safety is studied.

[1] *Department of Civil Engineering, Tohoku University, Sendai, Japan. Email:akiyama@civil.tohoku.ac.jp*
[2] *Department of Civil Engineering, Tohoku University, Sendai, Japan. Email:suzuki@civil.tohoku.ac.jp*

PROPOSED RELIABLITY EVALUATION METHOD FOR STRUCTURAL SYSTEMS

Reliability Evaluation Method

For combinations of events in set theory, if higher-order events represented by the intersection of three or more joint probabilities are ignored, the probability of failure event E, comprising k separate events, is expressed as follows.

$$P(E) = \sum_{i}^{k} C_i \tag{1}$$

where,

$$C_1 = P(E_1), \quad C_2 = P(E_2) - P(E_2 E_1), \quad C_k = P(E_k) - \sum_{i=1}^{k-1} P(E_k E_i) + \sum_{\substack{m=1,k-2 \\ n=2,k-1}}^{m<n} P(E_k E_m \cap E_k E_n), k > 3$$

In this calculation, there are three forms of failure probability: $P(E_k)$, $P(E_k E_i)$ and $P(E_k E_m \cap E_k E_n)$. $P(E_k)$ are calculated by Rosenblatt transformation [Ang and Tang, 1977] with the following assumptions: [a] The probability distribution of probability variables representing capacities is either normal or logarithmic normal; [b] There is no correlation between probability variables representing capacities and probability variables representing external force; and [c] There is also no correlation among probability variables representing external force.

For calculation method of $P(E_k E_i)$, It was assumed that for Ditlevsen's bounds, the area of overlap between the regions A and B in Fig. 1 is proportional to $P(E_k E_i)$ [Ramachandran, 1992]. When the limit state equations representing failure events E_k and E_i are $g_k = 0$ and $g_i = 0$, respectively, the direction cosine of the angle formed between the hypersurfaces of the two critical state equations of Figure 1 equals the correlation coefficient ρ_{ki} of the two events. Consequently, the following is obtained as an approximation equation:

$$P(E_k E_i) = \left(1 - \frac{\cos^{-1} \rho_{ki}}{\pi}\right)\left(P(A) + P(B)\right) \tag{2}$$

where,

$$P(A) = \Phi(-\beta_i)\Phi\left(-\frac{\beta_k - \rho_{ki}\beta_i}{\sqrt{1-\rho_{ki}}}\right), P(B) = \Phi(-\beta_k)\Phi\left(-\frac{\beta_i - \rho_{ki}\beta_k}{\sqrt{1-\rho_{ki}}}\right), \rho_{ki} = \frac{Cov(g_k, g_i)}{\sigma_{gk}\sigma_{gi}}$$

Φ : cumulative distribution function of standard normal distribution

Unlike a two-order joint probability, $P(E_k E_m \cap E_k E_n)$ cannot be approximated geometrically. Hence, an approximation is obtained by considering the correlation between the failure events. First, parameter Ω representing the correlation between the events is defined as follows.

$$\Omega = P(E_k E_m \cap E_k E_n) / \min\left(P(E_k E_m), P(E_k E_n)\right) \tag{3}$$

When $\Omega = 1.0$, events $E_k E_m$ and $E_k E_n$ are completely dependent, and when $\max\left(P(E_k E_m), P(E_k E_n)\right)$, events $E_k E_m$ and $E_k E_n$ are independent. Thus, the range of Ω is as follows.

$$\max\left(P(E_k E_m), P(E_k E_n)\right) \le \Omega \le 1.0 \tag{4}$$

However, the correlation between events $E_k E_m$ and $E_k E_n$ cannot be obtained directly. Therefore, an approximation is obtained from the respective correlation coefficients of failure events E_k and E_m, E_k and E_n, and E_m and E_n, using the following equation:

$$\Omega = \left(\min(\rho_{km}, \rho_{kn}, \rho_{mn})\right) \times \left(\sum (\rho_{km} + \rho_{kn} + \rho_{mn}) - \min(\rho_{km}, \rho_{kn}, \rho_{mn})\right) \tag{5}$$

Equation (4) represents the ratio of the correlation between the two most weakly correlated events and the other two events.

Comparison between Proposed Equations and Previous Researches

The 1-level rigid-framed structure shown in Figure 2 was adopted as an example [Ditlevesen, 1979]. The following three equations were worked out as the limit states in mechanism formation:

$$g_1 = M_1 + 2M_3 + 2M_4 + M_5 - Fa - Gb$$

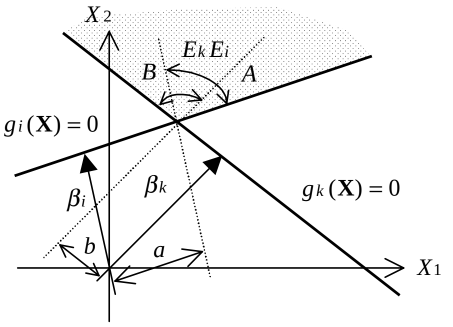


Figure 1: Ditlevesen's bounds

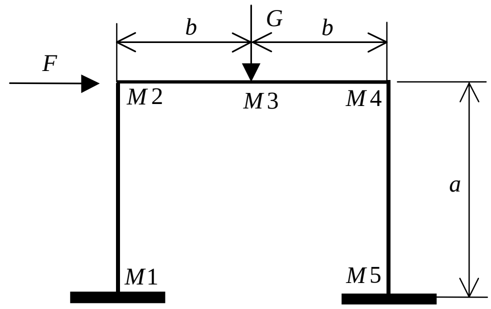


Figure 2: Calculation example [Ditlevsen, 1979]

$$g_2 = M_1 + M_3 + M_4 - Gb$$
$$g_3 = M_1 + M_2 + M_4 + M_5 - Fa$$

where,

a, b : fixed values, with the assumption $a = b = 2$, F, G : probability variables representing external forces; mean values assumed to be $\mu_F = \mu_G = 1.0$; standard deviations assumed to be $\sigma_F = \sigma_G = 1.0$, M_i : probability variables representing capacity (plastic moment); mean value assumed to be $\mu_{Mi} = 1.0$ and standard deviation $\sigma_{M}i = 0.5$ for all.

Failure probabilities $P(E)$ computed with the proposed equations and from the earlier results are as follows.

(a) First-order bounds $0.173 \leq P(E) \leq 0.316$
(b) Ditlevsen's bounds $0.173 \leq P(E) \leq 0.264$
(c) Monte Calro simulations $P(E) = 0.230$
(d) Proposed method $P(E) = 0.230$

From some calculated examples, we verify that the proposed method (referred to hereafter as the reliability evaluation method for structural systems) is as easy to use as any of the earlier methods, and gives results in accord with those obtained by the Monte Carlo simulation.

Now, the failure probability calculated by the reliability evaluation method for structural systems is converted into the safety index $\beta\left(\equiv \Phi^{-1}(P(E))\right)$. The correspondence between failure probability and safety index is shown in Table 1.

Table 1: Relation between failure probability and safety index

Failure Probability	0.5	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Safety Index	0.0	1.28	2.33	3.72	4.75

OUTLINE OF SEISMIC SAFTEY ANALYSIS OF RC BRIDGE PIER

Limit State Equations

In evaluating the safety of RC bridge pier using the reliability evaluation method for structural systems as proposed above, a limit state equation has to be developed. In general, this limit state equation is set up as the "capacity term" minus the "external force term". In this study, capacity and ductility were chosen as items to be examined in investigating the ultimate limit state of the RC bridge pier. This study adopts the ductility evaluation formula which was drived through the extensive research and consolidation of earlier studies [JSCE, 1996]. Accordingly, the limit state equations are set up as follows

$$g_1 = \alpha_1 M_u - \left(P_{\max} \cdot a + N \cdot \delta_{\max}\right) \tag{6}$$
$$g_2 = \alpha_2 \left(V_c + V_s\right) - P_{\max} \tag{7}$$

$$g_3 = \alpha_3\left[\frac{N}{N_B} + \left(1 - \frac{N}{N_B}\right)\left\{12\left(\frac{0.5V_c + V_s}{M_u / a}\right)\right\} - 3\right] - \frac{\delta_{max}}{\delta_y} \tag{8}$$

where,

$\alpha_1, \alpha_2, \alpha_3$: probability variable to deal with variations in the equations for calculating capacity terms.

M_u : flexural capacity, V_c, V_s :shear capacity without hoop ties and contributed by hoop ties [Ishibashi and Yoshino, 1988], N, N_B : axial compressive force and axial compressive force when equilibrium breaks down, a : shear span, δ_y : deformation at yielding of the tension reinforcement, P_{max}, δ_{max} : maximum values of active inertial force and response displacement obtained from dynamic analysis

Seismic Response Analysis Model and RC Bridge Pier Used in Analysis

P_{max}, δ_{max} are obtained from dynamic analysis. In this study, RC bridge pier used in analysis was modeled as shown in Figure 3. The bridge pier and superstructure were modeled in one dimension, and the nonlinear hysteresis characteristics of RC bridge pier were taken into consideration. As a nonlinear model for the RC bridge pier, the stiffness degradation tri-linear model was used. The nonlinearity of the ground around the base was taken into account by using the bilinear restoring force model [Harada et al., 1988] for ground spring.

For reliability evaluations such as the ones carried out in this study, it is preferable to choose RC bridge piers designed using the same design standards, hence the choice of the three single-column RC bridge piers (piers A, B, and C with capacity ratios of 1.18, 1.32, and 1.84, respectively) shown in specifications [Japan Road Association, 1990]. In table 2, sectional particulars of the bridge piers are shown.

In this study, the seismic waveform observed at the Kaihoku Bridge during Miyagiken-Oki earthquake (observation at the bedrock surface; maximum acceleration = 293gal) was used as the seismic input. The waveform was input into the bedrock of the ground being analyzed and the response at the bottom of the footing was estimated using multiple reflection theory. The basic natural period of the subsurface ground used in analysis was 0.084(s).

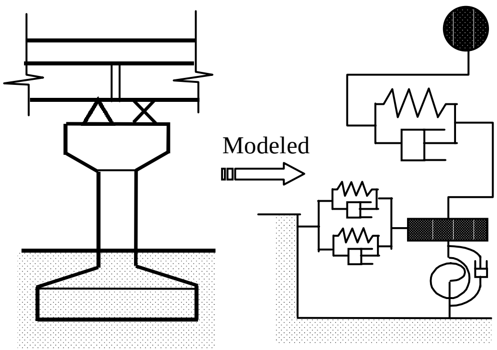


Figure 3: Numerical analysis

Table 2: Bridge pier used in analysis

Pier A capacity ratio =1.18	Section	4.0m in diameter
	Axial reinforcing bars	D51× 72
	Hoop ties	D25ctc150mm
Pier B capacity ratio =1.32	Section	4.0m in diameter
	Axial reinforcing bars	D38× 78
	Hoop ties	D22ctc125mm
Pier C capacity ratio =1.84	Section	3.2m× 3.7m
	Axial reinforcing bars	D32× 23
	Hoop ties	D25ctc150mm

SEISMIC SAFTEY EVALUATION OF C BRIDGE PIERS BY THE RELIABLITY EVALUATION METHOD FOR STRUCTURAL SYSTEMS

Effect of Material Strength Uncertainty on Capacity

The compressive strength of the concrete and the yield strength of the reinforcing bars were adapted as uncertainties affecting material strength. On the basis of survey results, the upper limits of the coefficients of variation, which represent the degree of uncertainty, were assumed to be 20% for concrete compressive strength and 7% for reinforcing bars yield strength. The variability in material strength was assumed to be normally distributed. To begin with, the effect of variations in material strength on flexural capacity was evaluated by Monte Carlo simulation. A certain number of specimens simulating actual bridge piers were selected and analyzed. The results are shown in Figure 4 (a). In this graph, the horizontal axis represents mean values of the flexural capacity (converted into shear force) for the selected specimens. Although each specimen had its own value for the coefficient of variation, in this study, one of flexural capacities were 8% or less without exception.

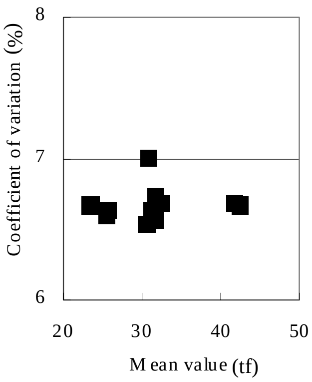
Accordingly, when analyzing the reliability of the RC bridge piers, the calculated flexural capacity was used as a mean value and treated as a probability variable having the coefficient of variation of 8%.

Next, the effect of variations in material strength on shear capacity was calculated. The respective coefficients of variation δ were computed using the equation below.

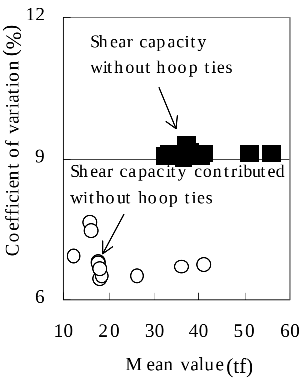
$$\delta = \frac{\sqrt{\sum_{i=1}^{j}\left[\left(\frac{\partial V}{\partial X_i}\right)\right]\cdot\sigma_{\overline{Xi}}^{2}}}{V\left(\overline{X1},\cdots,\overline{Xj}\right)+\sum_{i=1}^{j}\left(\frac{\partial V}{\partial Xi}\right)_{\overline{Xi}}\left(Xi-\overline{Xi}\right)} \tag{9}$$

where,
X_i : probability variables related to material strength, $\sigma_{\overline{Xi}}$: standard deviations of probability variables, $V(\cdot)$: equation to calculate shear capacities

Figure 4 (b) shows the results of the analysis of coefficients of variation of each equation for shear capacity. From this graph, it was decided to treat shear capacities without hoop ties as probability variables having 8% coefficient of variation and shear capacities contributed by hoop ties as ones with 10% coefficient of variation, the calculated shear capacities being taken as mean values.



(a) Effect on flexural capacity



(b) Effect on shear capacity

Figure 4: Effect on variance in material strength

Table 3: Random variable

Variable	Distri-bution	Mean	Coefficient of variation
α_1	normal	1.0	10%
α_2	normal	1.0	10%
α_3	normal	1.0	40%
N	normal	Design value	5%
δ_y	normal	Calculated value	10%
P_{max}	normal	Result of response	30%
δ_{max}	normal	Result of response	30%

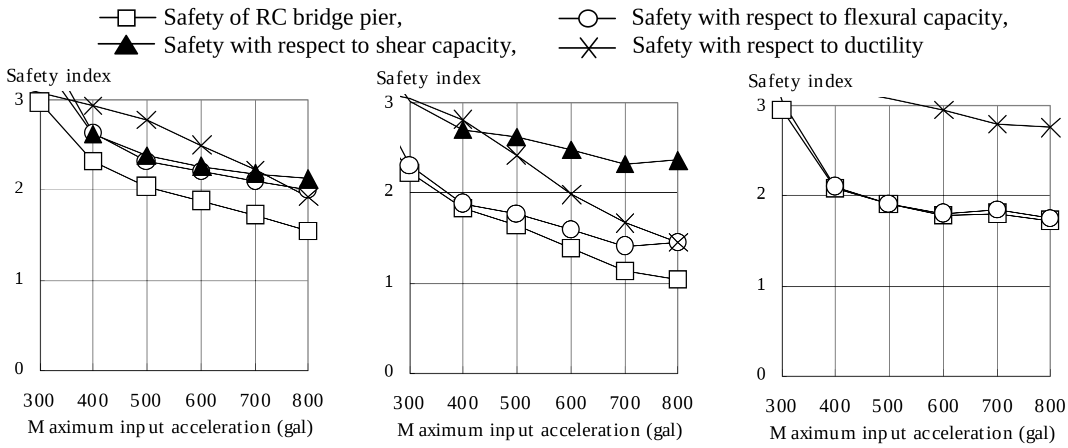
Evaluation of Uncertainties in the Equation and in the Structural Analysis

The equations for calculating shear capacity without hoop ties and the equations for evaluating ductility were based on experimental results. As a result, calculated capacity values were influenced by uncertainty in the equations themselves as well as by variations in material strengths. The influence of such uncertainties is taken into consideration by treating coefficients $\alpha_i\,(i=1,2,3)$ in the limit state equation as probability variables. Even when the active inertial force and response displacement due to an earthquake are set at their maximum values, it is not possible to eliminate uncertainties when modeling a structure. Accordingly, for coefficients of variation other than those representing material strength uncertainties, the assumed probability distributions and parameters listed in Table 3 were used in the safety evaluation of the RC bridge piers as described below.

Safety Evaluation of RC Bridge Piers

The seismic waveforms were amplified to give maximum input accelerations at the bedrock of between 300gal and 800gal. These were input, in 100gal increments, into the models shown in Figure 3. The relationships between maximum acceleration and safety index for bridge piers A, B, and C are shown respectively in Figure 5 (a), (b), and (c). These graphs individual safety indices obtained by assessing safety with respect to flexural capacity, shear capacities, and ductility, as well as safety indices for the RC bridge piers as calculated by the reliability evaluation method of structural systems proposed in section 2 (henceforth, the index given by the proposed evaluation method is referred to as the 'RC bridge pier safety index').

Figure 5 (a) shows the results for bridge pier A. Compared to the safety indices of the three limit states, the RC bridge pier safety index is lower for all maximum input accelerations. This means that no dominant limit state exists in this case; pier safety can be evaluated properly only by considering on three limit states at the same time. However, in the case of bridge pier B, shown in Figure 5 (b), the difference between the safety index with respect to flexural capacity and RC bridge pier safety index is smaller, while for bridge pier C in Figure 5 (c), the two are almost the same. Hence, the safety of this final RC bridge pier can be considered as well approximated by flexural capacity. Thus, even when the shear capacity exceeds flexural capacity due to variability in the equations used to calculate capacity, it may not in some cases be possible to ignore an assessment of safety with respect to shear capacity. We believe that the issue of whether safety can be examined for a single limit state, or whether the correlation among multiple limit states needs to be taken into account in the assessment of safety, depends on the capacity ratio of the structure.

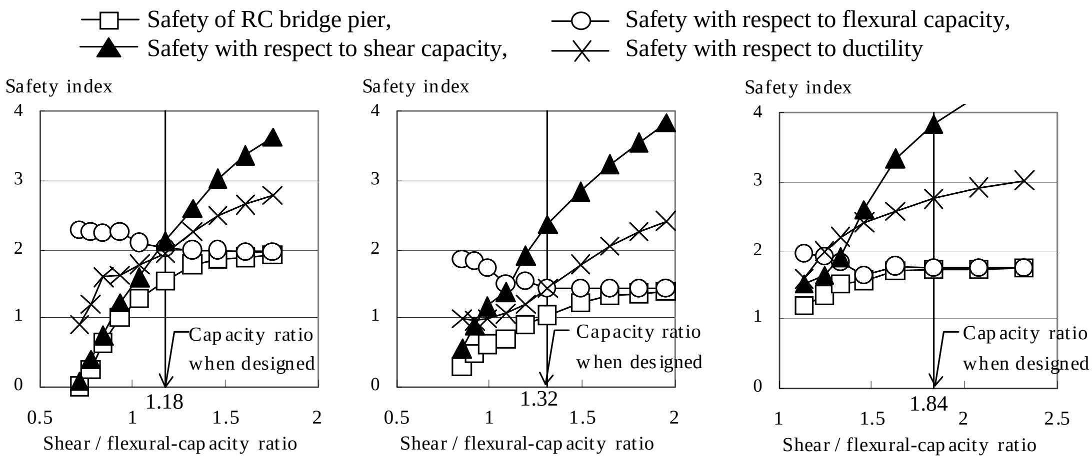


(a) Bridge pier A **(b) Bridge pier B** **(c) Bridge pier C**

Figure 5: Relation between maximum input acceleration and safety index

Effect of the Capacity Ratio on RC Bridge Pier Safety

In this section, the effect of the capacity ratio on RC bridge pier safety is discussed by changing the number of axial reinforcing bars and hoop ties in the RC bridges.



(a) Bridge pier A **(b) Bridge pier B** **(c) Bridge pier C**

Figure 6: Relation between capacity ratio and safety index

First, each RC bridge pier was modified by increasing the number of axial reinforcing bars in steps, each time by 20% of the original number in each of the cited design examples, while keeping the number of hoop ties unchanged. This increases the flexural capacity and decreases the capacity ratio. Next, the number of hoop ties was raised, each time by 25%, while keeping the number of axial reinforcing bars unchanged, thus increasing the shear capacity and raising the capacity ratio. The relationship between capacity ratio and the RC bridge pier safety index was investigated when seismic waves amplified to 800gal were input into the bedrock. The results

of the analysis for bridge piers A, B, and C are shown in Figure 6 (a), (b), and (c), respectively. Referring to Figure 6 (a) and (b), when the capacity ratio is raised by increasing the shear capacity (above 1.18 for pier A in

Figure 6 (a) and above 1.32 for pier B in Figure 6 (b)), the safety index also rises. It can be concluded from this analytical result that for capacity ratios above about 1.7, however, the value of bridge pier safety converges at the level of safety with respect to flexural capacity, and so a further increase in shear capacity does nothing more than impart excessive capacity. Furthermore, if the capacity ratio is decreased by increasing the flexural capacity (below 1.18 for pier A in Figure 6 (a) and below 1.32 for pier B in Figure 6 (b)), RC bridge pier safety decrease in all cases, due to rising active shear force during an earthquake and a decrease in ductility of the pier.

Since the capacity ratio of bridge pier C is as high as 1.84 even before the number of reinforcing bars is changed, increasing the shear capacity while keeping the flexural capacity fixed has no effect on safety (as represented by the capacity ratio range above 1.84 in Figure 6 (c)). If the flexural capacity is raised in order to reduce the capacity ratio, as with piers A and B, a tendency for safety to begin falling when the ratio reaches 1.7 is seen.

In this analysis, increasing the flexural capacity results in falling pier safety. However, it is demonstrated that increasing the shear capacity while keeping the capacity ratio at a predetermined value allows for effective raising of RC bridge pier safety. We next turned our attention to bridge pier B and developed piers for which the number of axial reinforcing bars was 1.0-fold, 1.2- fold, 1.6-fold, and 2.0-fold greater than in the cited sample. In these piers with varying amounts of axial reinforcement, the number of hoop ties was increased by 25% so as to raise the shear capacity. Figure 7 shows the relationship between safety index and capacity ratio, when seismic motion amplified to 800gal was input into the bedrock. This graph shows only the RC bridge pier safety indices obtained using the reliability safety evaluation method for structural systems.

It can be seen from Figure 7 that an increase in flexural capacity elevates the RC bridge pier safety if the shear capacity is also increased simultaneously. Nevertheless, in this case too, for all combinations of reinforcing bars, safety changes little once the capacity ratio exceeds 1.7 or so. This is thought to be due to the fact that RC bridge pier safety is governed by the assessment of safety with respect to flexural capacity because, although not shown in the graph, this value becomes almost constant at a point close to this capacity ratio despite increasing shear capacity. Accordingly, the safety index of RC bridge piers converges to the safety index with respect to flexural capacity (and becomes almost constant) when the capacity ratio exceeds a certain value. Consequently, it may be said that adjusting the amount of reinforcement so as to maintain the capacity ratio of approximately 1.7 is an appropriate way to enhance bridge pier safety. Judging from this analysis, capacity ratio can be considered a useful index of the earthquake resistance of RC bridge piers.

The value of capacity ratio indicated here depends on the way in which we set up the limit states and on the parameters of the probability variables listed in Table 3. The emphasis in this study is not on the actual value of the capacity ratio, but rather on the fact that capacity and ductility are adopted as two items assessed in considering the limit state of RC bridge piers in an earthquake, and that RC bridge pier safety can be evaluated quantitatively by introducing a reliability evaluation method based on a common yardstick, i.e. the safety index. This allows consideration of whether a structure reaches the target safety level, whether safety must be assessed by considering a multiplicity of limit states, and whether excessive capacity is incorporated, etc.

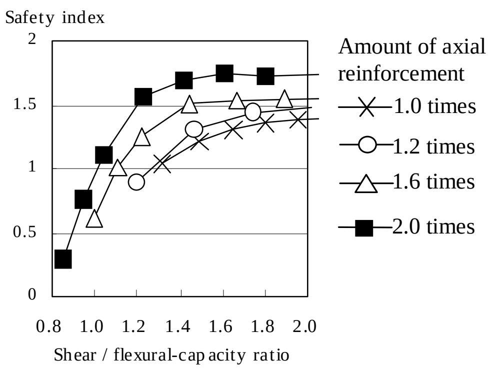


Figure 7: Influence of capacity ratio on safety of RC bridge pier

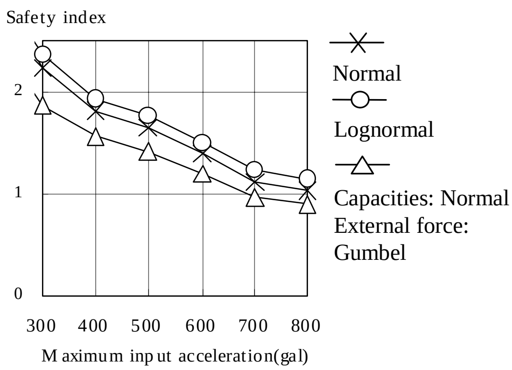


Figure 8: Influence of form of probability distribution on safety of RC bridge

Influence of the Form of Probability Distribution on the Safety of RC Bridge Piers

In this section, we evaluate the safety of RC Bridge piers when the form of the probability distribution for each probability variable in Table 3 is changed, and we discuss the effect of such changes. The probability distributions considered include cases where all variables are assumed to be logarithmic normally distributed, where the external force term, i.e. the maximum inertia force P_{max} and maximum response displacement δ_{max}, are assumed to be of the extreme-I type (Gumbel), and probability variables expressing the capacity term are normally distributed. Bridge pier B was selected as the subject of the analysis. Seismic waves amplified to be between 300gal and 800gal in maximum input acceleration were input, at 100gal increments, into the bedrock. The result of the analysis is shown in Figure 8.

In the range of high maximum input acceleration, the effect of the distribution form on RC bridge pier safety is lower. Accordingly, when the acceleration range - which is a problem of seismic design - is estimated, the influence of differences in the form of the probability distribution of the various probability variables used in this study on the safety evaluation is rather small. Hence it is considered that the analytical result mentioned above, obtained under the assumption that all are normally distributed, is generally applicable.

CONCLUSIONS

1. A method for calculating the failure probability of a structural system was established for in a situation where a failure event comprises multiple limit states correlated with each other. The proposed technique substantially facilitates calculation; its superiority, both in time and accuracy, over calculation methods in previous studies was confirmed.
2. The influence of uncertainties (variations in the compressive strength of the concrete and in the yield strength of the reinforcing bars) included in the calculated capacities was evaluated. This influence was represented by modifying the calculated capacities by coefficients of variation, and its upper limit was evaluated.
3. The safety of RC bridge pier in an earthquake was evaluated using the reliability evaluation method for structural systems proposed in this study. The results showed that when the capacity ratios assigned to the RC bridge piers are in the low range, the evaluation errs on the side of risk if shear force and ductility are not taken into consideration, even in the case of a pier designed for flexural failure.

The safety of RC bridge pier was analyzed with different capacity ratios by changing the number of main reinforcing bars. The results showed that when the shear capacity is about 1.7 times the flexural capacity, a pier's safety can be ascertained by performing a safety assessment of the flexural capacity and active bending moment; also that for a certain flexural capacity, optimum safety is attained when this capacity ratio is assigned to the bridge pier.

ACKNOWLEDGEMENT

This study was financially supported by Grand-in-Aid for scientific research No.11750419 from The Ministry of Education of The Japanese Government.

REFERENCES

Ang, A. H-S. and Tang, W.H. (1977), *Probability Concepts in Engineering Planning And Design, Volume II Decision, Risk and Reliability* , John Whiley & Sons, Inc., New York.

Ditlevsen, O. (1979), “Narrow Reliability Bounds for Structural Systems”, *Journal of Structural Mechanics*, 7, 4, pp453-472.

Harada, T., Hirose, T., and Yamada, S. (1988), “*A Winkler Model Approach to the Evaluation of Nonlinear Dynamic Soil Reactions*”, The 34th Report of Miyazaki University, pp17~25. (in Japanese)

Ishibashi, T. and Yoshino, S. (1988), “Study on Deformation Capacity of Reinforced Concrete Bridge Piers Under Earthquake”, Proc. of JSCE,390,8, pp57-66.(in Japanese)

Japan Road Association. (1990), *Japan Road Association: Specifications for Highway Bridges, V Seismic Design*, Maruzen, Japan. (in Japanese)

JSCE. (1996), *Damage Analysis by the Hanshin-awaji Earthquake and Ductility Evaluation Function, Concrete Engineering Series No.12*, Japan. (in Japanese)