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SEISMIC BEHAVIOUR OF EXTENDED STRUCTURES

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SUMMARY

In recent years it is established that spatial variability of earthquake ground motion should be considered in seismic analysis of extended structures such as bridges and lifelines. The problem of the differential motion estimating at adjacent points on the ground surface is very difficult. Utilizing of a random vibration approach general model for spatial correlation of the ground motion by the author permits to introduce different statistical hypothesis. The proposed procedure allows to estimate root mean square and peak seismic responses of extended structures taking into account the spatial correlation of the ground acceleration and displacement components, the strong-motion duration, the relatively long natural periods of the system, very small damping and other factors.

INTRODUCTION

It is recognized that the extended structures seismic response associated with spatial variations in the earthquake ground motion under the supports may differ significantly from those obtained through uniform ground motion. It is suggested in previous studies (Bogdanoff et al, 1965; Petrov, 1974, 1978, 1998; Abdel-Ghaffar, 1980; Abdel-Ghaffar & Rubin 1982; Kahan et al, 1995). For the solution of the problem usually time history or spectral analysis are utilized introducing travelling wave hypothesis (non-dispersive wave) and assuming effective constant horizontal propagation velocity of the wave. Another way to describe the spatial variation of the ground motion is using the sumultaneous records collected by closely spaced arrays of strong-motion instruments. However, at present the necessary seismological data are not sufficient. In this paper the stochastic ground motion model is considered and the general expression for its spatial correlation (Petrov, 1978) is applied. This model permits to introduce different correlation laws, so that travelling wave hypothesis or uncorrelated unput support motions may be selected as particular cases too. Using of a random vibration theory, the simplified approach is obtained to estimate seismic response of extended structures.

GROUND MOTION PARAMETERS

The peak ground acceleration is not a very meaningful measure of classifying the severity of earthquake ground motion. Design earthquake described of this measure alone don't takes into consideration the effect of uncertainties in the ground motion on the structure response. The PGA is only random value of the stochastic process and contains a very little energy. Therefore, a new parameter is needed to describe the structural damage potential of the earthquake at a site. It should be correlated to the energy of the earthquake at the source as well as to the distance of the source from the site. Cumulative square accelerations, or Arias Intensity, may be a measure of the ground motion severity. The root mean square (rms) of the ground accelerations and the suitable value $\bar{t}$ of mean duration of earthquake ground motion are selected using this measure.

The cross-spectral density function between the inputs at points *k* and *l* is given by next general expression:

$$G_{kl}(\omega) = \sigma_k \sigma_l \sqrt{G_k^N(\omega) G_l^N(\omega)} R_{kl}(\omega), \quad (1)$$

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in which σ_k, σ_l = rms of the ground accelerations at points *k* and *l*; $G_k^N(\omega), G_l^N(\omega)$ = normalized power spectral density functions of the ground accelerations at points *k* and *l*; ω = circular frequency, $\sec^{-1}$; $R_{kl}(\omega)$ = cross-correlation function of the ground accelerations at points *k* and *l*, which can be written as it was proposed earlier (Petrov, 1978):

$$R_{kl}(\omega) = \exp\left[-\frac{\omega}{v} L_{kl}(c_1 + c_2 i)\right], \qquad (2)$$

where L_{kl} = length of span between the k-th and l-th supports of the structure; v = shear wave velocity; c_1, c_2 = constants; $i = \sqrt{-1}$.

For example, the full correlation case corresponds to $c_1 = 0, c_2 = 0$; the uncorrelated multiple-support excitations - $c_1 \to \infty, c_2 = 0$; the travelling wave (or "frozen wave") - $c_1 = 0, c_2 = 1$.

The earthquake ground motions are usually multidimensional. In practice only the translation components in three (or two) orthogonal directions are considered. Penzien & Watabe (1975) concluded, that the principal orthogonal axes exist along which the ground motion components are not correlated. The orientation of the principal axes is not known and vary during an earthquake. Then it is necessary to orient the principal excitation axes along the principal response directions to obtain the worst case response.

Mostaghel & Ahmadi (1982) studies the relationship between the resultant and the component horizontal accelerations, and it is found that the component peak must be increased by about 20 per cent to give the appropriate value for the resultant peak.

Newmark et al (1973) suggested that the average peak vertical-to-horizontal spectral ratio can be taken reasonably as 2/3. But several recent earthquakes have shown that this ratio can exceed the value of 2/3. Elnashai & Papazoglou (1997) establishes the lack of conservatism in current code spectra used to estimate vertical earthquake forces.

Therefore it is necessary to obtain the structure response on the each uncorrelated ground motion component and to combine the structure response components to obtain the resultant response. In this case the structure may be oriented such that its own geometric axes align with the principal excitation axes. The ground motion is assumed along the span axis for in-plane vibrations and perpendicular to the span axis for out-of-plane vibrations. The influence of nonuniform support excitations on the response of a multi-span continuous bridge may be conservative or nonconservative depending on the geometrical parameters, the dynamic characteristics of the structure, soil type etc. But for one-span structures, in particularly suspension bridges, the predictions of the spatially variable wave effects are close to each other.

SEISMIC RESPONSE OF EXTENDED STRUCTURES

The analyses of the extended structures seismic responses are based on the idea by Clough & Penzien (1975), that the total displacement is separated into quasi-static and vibrational components.

The motion of a structure which is subjected to multiple support excitations of each ground motion component can be governed by the differential equations:

$$M\ddot{u} + C\dot{u} + Ku = -MR\ddot{u}_0, \qquad (3)$$

where *M, C, K* = mass, damping and stiffness matrices respectively; *u* = vibrational displacement vector; $\ddot{u}_0$ = ground acceleration vector of support points; *R* = quasi-static influence matrix which represents the displacement of structure due to unit displacement at each support, while the other supports are held fixed.

The total displacement vector (u_t) of the degrees of freedom is the sum of the quasi-static (u_s) and the relative or vibrational displacements:

$$u_t = u + u_s \ , \tag{4}$$

where $u_s = Ru_0 ; u_0 =$ ground displacement.

The vibrational displacement at point *j* of the structure due to displacement at *k*-th support may be decomposed into its modal solution:

$$u_{kj}(t) = \sum_{j=1}^{N} \alpha_{ij} f_{ik}(t) \, , \tag{5}$$

where $\alpha_{ij} = i$ -th mode shape of the structure; $f_{ik}(t) = i$ -th generalized coordinate, which satisfies the decoupled equation:

$$\ddot{f}_{ik}(t) + 2\xi_i p_i \dot{f}_{ik}(t) + p_i^2 f_{ik}(t) = -\frac{\delta_{ik}}{M_i} \ddot{u}_{0k}(t) \, , \tag{6}$$

in which $\xi_i, p_i =$ damping ratio and natural circular frequency, respectively, of the *i*-th mode; $M_i = \sum_{s=1}^{n} m_s \alpha_{is}^2 = i$ -th generalized mass; $m_s =$ mass, concentrated at point *s* of the structure; $\delta_{ik} = \sum_{s=1}^{n} m_s r_{ks} \alpha_{is} ; r_{ks} =$ quasi-static influence function which gives the displacement at the *k*-th support, while the other supports are held fixed.

In right part of the Equation (6) the velocity term are ignored because its contribution to the total response is usually small when compared to that of the acceleration.

The mean square response at point *j* of the structure is given by

$$\overline{u_j^2(t)} = \sum_{i,r=1}^{N} \alpha_{ij} \alpha_{rj} \overline{f_i(t) f_r(t)} \ , \tag{7}$$

where

$$\overline{f_i(t) f_r(t)} = \frac{1}{2\pi M_i M_r} \int_{-\infty}^{\infty} S_{ir}(\omega) \Phi_i(i\omega, t) \Phi_r(-i\omega, t) d\omega \ \ ; \tag{8}$$

$\Phi_i(i\omega,t), \Phi_r(-i\omega,t) = i$ -th complex frequency response and *r*-th complex conjugate frequency response, respectively; if $t \to \infty$, $\Phi_i(i\omega,t) \to (p_i^2 - \omega^2 - 2\xi_i p_i i\omega)^{-1} ; i\omega = \sqrt{-1}\omega$.

The cross-spectral density function of the generalized forces Q_i and Q_r is equal

$$S_{ir}(\omega) = \sum_{k,l=1}^{n_0} G_{kl}(\omega) \delta_{ik} \delta_{rl} \ \ , \tag{9}$$

(n_o = number of supports).

Substituting Equations (1,2) & (9) into Equation (8), yields

$$\overline{f_i(t) f_r(t)} = \frac{1}{M_i M_r} \sum_{k,l=1}^{n_0} \sigma_k \sigma_l \delta_{ik} \delta_{rl} I_{irkl} \quad , \tag{10}$$

where

$$I_{irkl} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sqrt{G_k^N(\omega) G_l^N(\omega)} R_{kl}(\omega) \Phi_i(i\omega, t) \Phi_r(-i\omega, t) d\omega \, . \tag{11}$$

Then mean square of vibrational displacement can be represented as

$$\overline{u_j^2(t)} = \sum_{i,r=1}^{N} \sum_{k,l=1}^{n_0} \frac{\alpha_{ij} \alpha_{rj} \delta_{ik} \delta_{rl}}{M_i M_r p_i^2 p_r^2} \sigma_k \sigma_l \beta_{ik}(t) \beta_{rl}(t) A_{ir} C_{irkl} \quad , \tag{12}$$

where

$$\overline{\beta_{ik}^2(t)} = \frac{p_i^4}{2\pi} \int_{-\infty}^{\infty} G_k^N(\omega) \left| \Phi_i(i\omega, t) \right|^2 d\omega \quad ; \tag{13}$$

$$A_{ir} = \frac{I_{irkk}}{\sqrt{I_{iikk} I_{rrkk}}} \quad ; \qquad C_{irkl} = \frac{I_{irkl}}{\sqrt{I_{irkk} I_{irll}}} \quad . \tag{14}$$

In the low frequency range the mean square resonant magnification factor is equal

$$\overline{\beta_{ik}^2(t)} \approx \frac{p_i}{4\xi_i} G_k^N(p_i) \left[1 - \exp(-2\xi_i p_i t) \right] . \tag{15}$$

If $\xi_i \to 0, \qquad \overline{\beta_{ik}^2(t)} = 0.5 p_i^2 t G_k^N(p_i)$. (16)

The factor A_{ir} describes the cross-correlation of *i*-th and *r*-th modal coordinates. The solution of integral I_{irkk} can be computed using the concept "white noise" for the random loading (Petrov & Bazilevsky, 1978; Kiureghian, 1980).

If $\left| \frac{p_r}{p_i} - 1 \right| \frac{1}{\xi_i \xi_r} < \varepsilon \ (\varepsilon \to 0) , \qquad A_{ir} \approx \frac{2\sqrt{\xi_i \xi_r}}{\xi_i + \xi_r}$. (17)

The factor C_{irkl} accounts the spatial correlation of the accelerations at the k-th and l-th supports. Assuming $R_{kl}(\omega)$ in according to Equation (2), from Equations (12) & (14) it is obtained for full correlation case C_{irkl} =1, for uncorrelated case $C_{irkk} = 1, C_{irkl} = 0$ ($k \neq l$), for travelling wave case $C_{irkl} = \cos(p_{ir} L_{kl} v^{-1})$, where $p_{ir} = 0.5(p_i + p_r)$.

Assuming $\sigma_k = \sigma_l = \sigma, G_k(\omega) = G_l(\omega), A_{ii} = 1, A_{ir} = 0 (i \neq r), \delta_{i1} = \pm \delta_{i2}$, for symmetrical bridge with one span length L $(n_0 = 2)$ the mean square of the vibrational displacement at point *j* can be expressed as

$$\overline{u_j^2(t)} = \sigma^2 \sum_{i=1}^{N} \frac{\alpha_{ij}^2 \delta_{i1}^2}{M_i^2 p_i^4} \overline{\beta_i^2(t)} \overline{\mu_i^2} \quad , \tag{18}$$

where

$$\overline{\mu_i^2} = 2(1 \pm C_{ii12}) \,. \tag{19}$$

For full correlation case $\mu_i^{(+)} = 2; \mu_i^{(-)} = 0$.

For uncorrelated case $\mu_i = \sqrt{2}$.

For travelling wave $\overline{\mu_i^2} = 2\left[1 \pm \cos(p_i L v^{-1})\right]$.

The mean square quasi-static displacement can be obtained

$$\overline{u_{j(s)}^2} = \sum_{k,l=1}^{n_0} \sigma_{0k} \sigma_{0l} r_{kj} r_{lj} R_{kl} \quad . \tag{20}$$

where R_{kl} *is* cross-correlation function of the ground displacements at points *k* and *l* which can be obtain from data of SMART-1 arrays (Harada, 1984).

For symmetrical bridge with one span at $\sigma_{01} = \sigma_{02} = \sigma_0, r_{1j} = \pm r_{2j}$ the mean square of the quasi-static displacement at point *j* will be equal

$$\overline{u_{j(s)}^2} = \sigma_0^2 r_{1j}^2 \overline{\mu_s^2} \quad , \tag{21}$$

where

$$\overline{\mu_s^2} = 2(1 \pm R_{12}) \quad . \tag{22}$$

The mean squares of the bending moments, shearing forces and other response values can be obtained by analogical method, replacing α_{ij}, α_{rj} in Equations (12) or (18) and r_{kj}, r_{lj} in Equations (20) or (21) with the equivalent values responses accordingly.

The expected spectral peak value of responses is equal to *rms* responses multiplied by peak factors, which are usually equal 2.5÷4.

The total response due to excitation along the each principal direction, for example, along axis *X*, can be written ignoring cross-correlation of vibrational and quasi-static components of response

$$\overline{u_{tx}^2} = \overline{u_x^2} + \overline{u_{sx}^2} \quad . \tag{23}$$

The total structure response due to multidimensional earthquake ground motions will be equal

$$\overline{u_t^2} = \overline{u_{tx}^2} + \overline{u_{ty}^2} + \overline{u_{tz}^2} \,. \tag{24}$$

The proposed approach is utilized for analysis of some extended systems. Among those there are the suspension bridge across the Amudarya-river with span length *L* = 660 m; the several suspension bridges for mountain regions of Tajikistan; the Rogun multispan highway bridge; the arch bridge across the Arpa-river (*L* = 120 m) in Armenia and other structures.

CONCLUSIONS

It is suggested that uniform ground motion is not a good assumption for extended structures. The total seismic response of general multiple-degrees-of-freedom system is combined from uncorrelated response components at multidimensional nonstationary random seismic excitation of multiple supports. The analyses of the extended structures seismic response are based on the idea by Clough & Penzien (1975), that the total displacement is separated into quasi-static and vibrational components. Utilizing of a random vibration theory, the proposed by author (1978) general model for spatial correlation of ground motion is used.

It was found that the quasi-static effects usually are small in flexible systems, but those are likely to be larger in case of stiff structures. The contribution of simmetrical modes usually decreases as the span length increases in comparison with the effective wave length and contribution of anti-symmetrical modes increases accordingly. The efficiency and simplicity of the proposed procedure are obtained without inducing significant errors, if realistic data are provided.

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