

## DISTURBED STATE MODELING OF SATURATED SAND UNDER DYNAMIC LOADS

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### SUMMARY

A fundamental procedure is proposed for the identification of liquefaction in saturated sandy soils based on the disturbed state concept (DSC) which can capture the dynamic instability in the geomaterial's microstructure. DSC provides a unified constitutive model for the characterization of the stress-strain relationship under cyclic loading, and the values of disturbance at threshold states in the deforming geomaterial provides the basis for the assessment of liquefaction potential. The DSC model parameters for saturated Joomunjin sands (Korean standard sand) are evaluated using data from cyclic triaxial test device. The laboratory test results are also used for the verification of the liquefaction assessment method proposed in this paper. It can be stated that the proposed model can provide a fundamental procedure for liquefaction analysis, and as a result, it is considered to be an improvement over the available empirical procedures.

### INTRODUCTION

Relatively loose and saturated sands liquefy during earthquakes and strong ground motions. Such liquefaction can cause damage to foundations, earth retaining structures, dams, and other structures. Observed and well-documented cases of liquefaction damage have been reported extensively, e.g., the Niigata, El Centro, Mexico, and Kobe. The damage to property and life due to liquefaction has spurred considerable research activities over the last 30 years. Among the conventional and empirical procedures for the assessment of liquefaction potential are the SPT-N method, which is perhaps the oldest and often reliable, and those based on equivalent uniform shear stress and leading cycles on laboratory specimens under simulated cyclic loading.

The importance of additional research in the areas of constitutive modeling for dynamic behavior of saturated materials and liquefaction has been emphasized in the recent earthquake reports. A new method based on unified constitutive modeling approach, called the disturbed state concept (DSC), for the dynamic behavior of soils is presented in this paper. This approach is based on fundamental considerations, yet, it can provide a simplified procedure for the assessment of initial liquefaction.

Here, the main attention is given to the identification of liquefaction, with a brief description of material index and test results. A description of the DSC is provided and then details are given on the identification of liquefaction. The procedure is verified with respect to the laboratory behavior of saturated Joomunjin sand tested using a cyclic triaxial device.

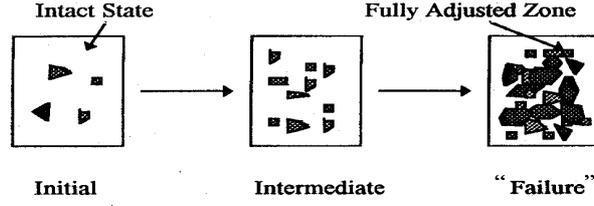
### DISTURBED STATE MODEL

In this study, the *disturbed state concept* (DSC) is developed to model the undrained behavior of Joomunjin sand subjected to cyclic loading including the effects of liquefaction and stress softening. In the DSC, it is assumed that applied forces cause disturbance or change in the material's microstructure. As a result, an initially *relative intact* (**RI**) material modifies continuously, through a process of natural self-adjustment, and a part of it approaches the *fully adjusted* (**FA**) state at randomly disturbed locations in the material as shown in Fig. 1.

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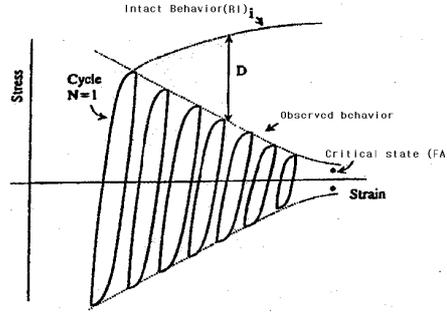
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**Figure 1: Relative intact and fully adjusted state [Desai, 1992]**

The *observed* or *average response* under cyclic loads can be represented in terms of the responses of the material in RI and FA states, which are called reference states, by using the *disturbance (D)*, which provides coupling between the responses of the RI and the FA parts shown in Fig. 2. In this concept, the RI state can be characterized as elastic, elastic-perfectly plastic, or any constitutive model like the HiSS model [Desai, 1980; Desai *et al.* 1986, Desai and Wathugala, 1987]. The FA state can be assumed to imply the state in which the material can continue to carry the shear stress level reached up to that state under given initial hydrostatic stress and can continue to deform in shear with constant volume-- *critical state* [Roscoe *et al.*, 1957].



**Figure 2: Schematic of cyclic behavior and disturbance**

### Relative Intact Behavior

In order to evaluate the disturbance, it is necessary to define the behavior of the RI material, which can be characterized by using a continuum theory such as linear or non-linear elasticity and elasto-plasticity. When the elasto-plastic theory is used, the incremental stress-strain equation is expressed as,

$$d\sigma_{ij}^i = C_{ijkl}^{ep(i)} d\epsilon_{kl}^i \quad (1)$$

Where  $d\sigma_{ij}^i$  and  $d\epsilon_{ij}^i$  are the RI incremental stress and strain tensor, and  $C_{ijkl}^{ep(i)}$  is the elasto-plastic constitutive tensor, which will depend on the model used, e.g. von Mises, Mohr-Coulomb, or hierarchical single surface (HISS) plasticity [Desai, 1980; Desai *et al.*, 1986]. Here, the HISS model is used in which the yield function,  $F$  is given by

$$F = \bar{J}_{2D} - (-\alpha \bar{J}_1^n + \gamma \bar{J}_1^2)(1 - \beta S_r)^{-0.5} = 0 \quad (2)$$

Where the overbar denotes non-dimensional quantity with respect to the atmospheric pressure constant,  $p_a$ ,  $S_r =$  stress ratio  $= \frac{\sqrt{27}}{2} (J_{3D} J_{2D}^{-1.5})$ ,  $J_{2D}$  and  $J_{3D}$  are the second and the third invariants of the deviatoric stress tensor,  $S_{ij}$ ,  $J_1$  is the first invariant of the stress tensor,  $\gamma$  and  $\beta$  are the parameters related to the ultimate yield behavior and shape of yield surfaces, respectively,  $n$  is phase change parameter, and  $\alpha$  is the hardening function expressed as

$$\alpha = \frac{a_1}{\xi^{\eta_1}} \quad (3)$$

Where  $a_1$  and  $\eta_1$  are the hardening parameters and  $\xi$  is the trajectory of plastic strains =  $\int \sqrt{d\epsilon_{ij}^p d\epsilon_{ij}^p}$  and  $d\epsilon_{ij}^p$  is the increment of plastic strain tensor.

### Fully Adjusted Behavior

The FA state is modeled using critical state concept. At the critical state condition, shear deformation can continue without further changes in volume. Such critical state often lies on a straight line called *critical state line* (CSL) with the slope  $\bar{m}$ , given by

$$\sqrt{J_{2D}^c} = \bar{m} J_1^c \quad (4)$$

During shearing, the materials change in volume and initial void ratio changes. Finally, at the critical state the void ratio,  $e^c$ , is given by

$$J_1^c = 3p_a \times e^{\left( \frac{e_0^c - e^c}{\lambda} \right)} \quad (5)$$

Where  $e_0^c$  is the value void ratio at critical state corresponding to  $J_1^c = 3p_a$  and  $\lambda$  is a slope of the critical state line. In this study, the behavior of the material at the FA or critical state is defined using Eq. (4) and (5).

### Disturbance Function

The disturbance can be dependent on microstructural deformations, initial conditions (mean effective pressure, density), temperature, and moisture content. When the value of D reaches the threshold state,  $D_c$ , in Fig. 3, initial liquefaction occurs, and when it reaches  $D_f$ , which is a state between  $D_c$  and  $D_u$ , final liquefaction occurs. The disturbance function can be evaluated approximately based on effective stresses [park, 1998]:

$$D_\sigma = \frac{\bar{\sigma}^{(i)} - \bar{\sigma}^{(a)}}{\bar{\sigma}^{(i)} - \bar{\sigma}^{(c)}} \quad (6)$$

where  $\bar{\sigma}$  is the measured stress such as  $\sqrt{J_{2D}}$  or the principal stress difference  $(\sigma'_1 - \sigma'_2)$ , or shear stress,  $\tau$ ,  $e$  is the void ratio, and  $\sigma'$  is the effective stress; the latter is used for undrained behavior. Assuming that the shear strains cause the predominant microstructural changes, D can be expressed in terms of the deviatoric plastic strain trajectory,  $\xi_D$ , as

$$D = D_u (1 - e^{-A \xi_D^Z}) \quad (7)$$

where  $D_u (< 1)$  is the ultimate or residual disturbance, Figure 3, ( $D=1$  is the asymptotic state and is not measurable in laboratory tests),  $A$  and  $Z$  are parameters that will dependent on the initial effective confining pressure, initial relative density, fine content, and number of cycles (N):

$$\xi_D = \int \sqrt{dE_{ij}^p dE_{ij}^p} \quad (8)$$

where  $dE_{ij}^p$  = tensor of incremental deviatoric plastic strains. For cyclic loading,  $\xi_D$  is evaluated as function number of cycles (N) based on the  $dE_{ij}^p$  corresponding to peak stresses at different cycles (see Fig. 2). Then disturbance function, D, is expressed as a function of N. Figure 3 shows the schematic of D as a function of  $\xi_D(N)$ .

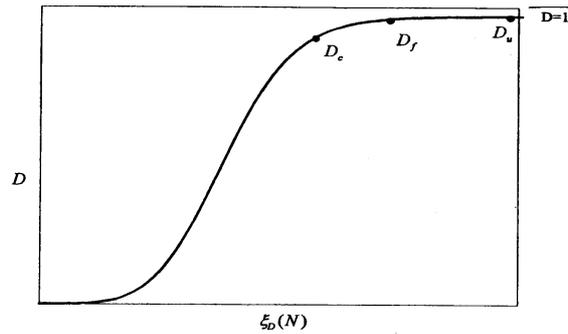


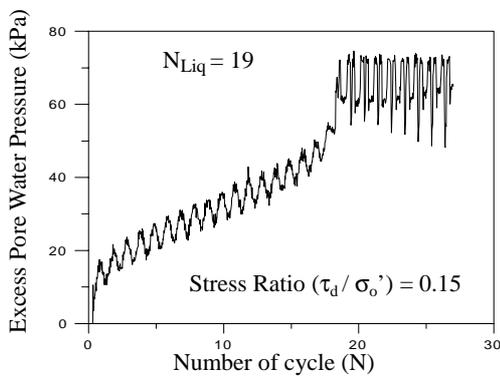
Figure 3: Schematic of disturbance (D) vs.  $\xi_D(N)$

### ANALYSIS

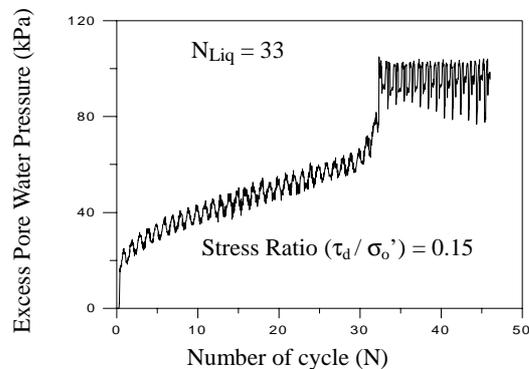
The saturated Joomunjin sand was tested using a cyclic triaxial device under stress controlled loading with a frequency  $\omega = 1.0\text{Hz}$ . Some index properties and test conditions are shown in Table 1. The tests were conducted with a relative density,  $D_r = 60\%$ , under three initial effective confining pressures,  $\sigma'_0 = 70, 100, 150\text{kPa}$ , and a initial effective confining pressures,  $\sigma'_0 = 100\text{kPa}$ , under three relative density,  $D_r = 50, 60, 70\%$ . Fig. 4 (a)-4(b) show typical test data for excess pore water pressure vs. time under constant stress ratio,  $\frac{\tau_d}{\sigma'_0} = 0.15$ . In this results when excess pore water pressure suddenly increases up to initial effective confining pressure, initial liquefaction may occur in saturated samples under cyclic loading.  $N_{liq}$  in Fig. 4(a)-4(b) refers the number of cycles when initial liquefaction occurs.

Table 1: Description of index properties of Joomunjin sand and CTC test condition

Index property of Joomunjin Sand		CTC Test Condition	
$e_{max}$	0.84	Frequency	1 Hz Sinusoidal
$e_{min}$	0.074	Stress Ratio ( $\tau_d / \sigma'_0$ )	0.15
$D_{50}$ (mm)	0.49	Method of Saturation	Water Sedimentation
$C_u$	1.35	Sample Condition	Fully Saturated and Undrained
$C_c$	1.14	Loading Control	Stress Control



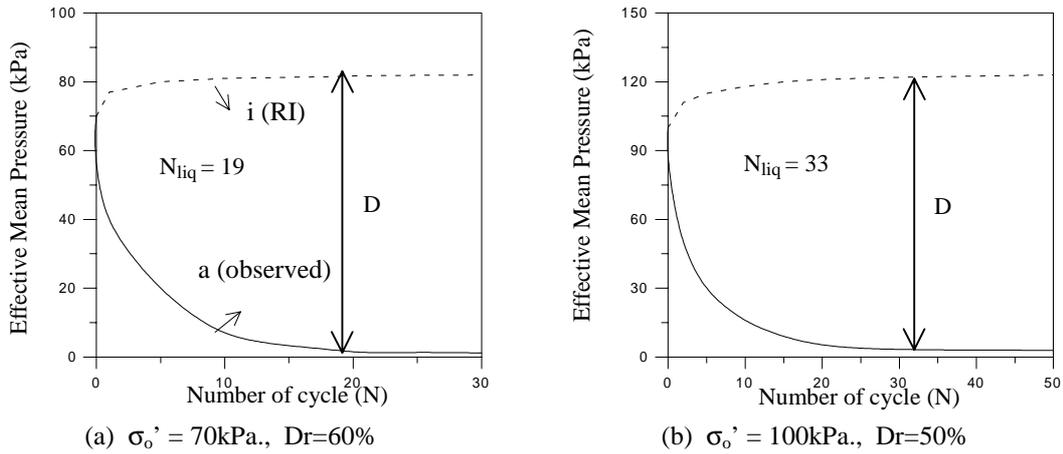
(a)  $\sigma'_0 = 70\text{kPa}$ ,  $Dr=60\%$



(b)  $\sigma'_0 = 100\text{kPa}$ ,  $Dr=50\%$

Figure 4: Plot of excess pore water pressure vs. time(number of cycle) for CTC test with Joomunjin sand

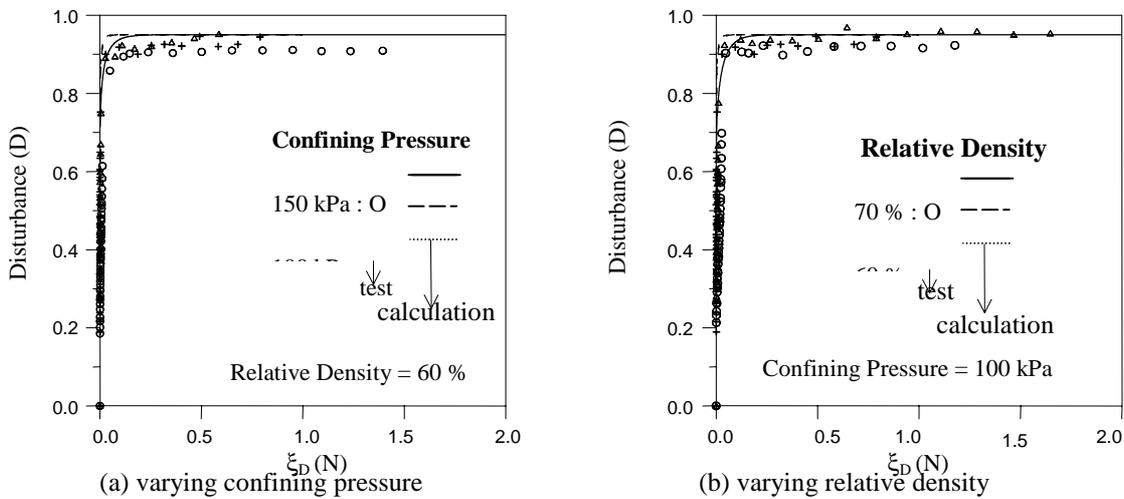
Fig. 5(a)-5(b) represent typical test results for effective stress plots used to find the disturbance by using Eq. (6), by employing the procedure described in section 2 to define the i (RI) response in Fig. 5. Then the parameters A and Z were found by using Eq. (7) and (8) and summarized in Table 2. The value of  $D_u$  was adopted to be 0.97 as a simplification. It is possible to include different values of  $D_u$  based on residual stresses. Fig. 6(a)-6(b) show plots of  $D$  vs  $\xi_D(N)$  for the tests on the sand under three confining pressures=70, 100, and 150 kPa and three relative density=50, 60, and 70 %. Also,  $D$  calculated from Eq. (7) and (8) is in good agreement with that obtained from results of cyclic loading tests (see Fig. 6(a)-6(b)).



**Figure 5: Observed and intact effective stress responses: Joomunjin sand**

**Table 2: Summary of parameter A and Z**

Parameter	Varying Confining Pressure			Varying Relative Density		
	70 kPa	100 kPa	150 kPa	50 %	60 %	70 %
A	4.522	4.043	3.153	3.812	4.043	3.468
Z	0.289	0.278	0.327	0.310	0.278	0.395



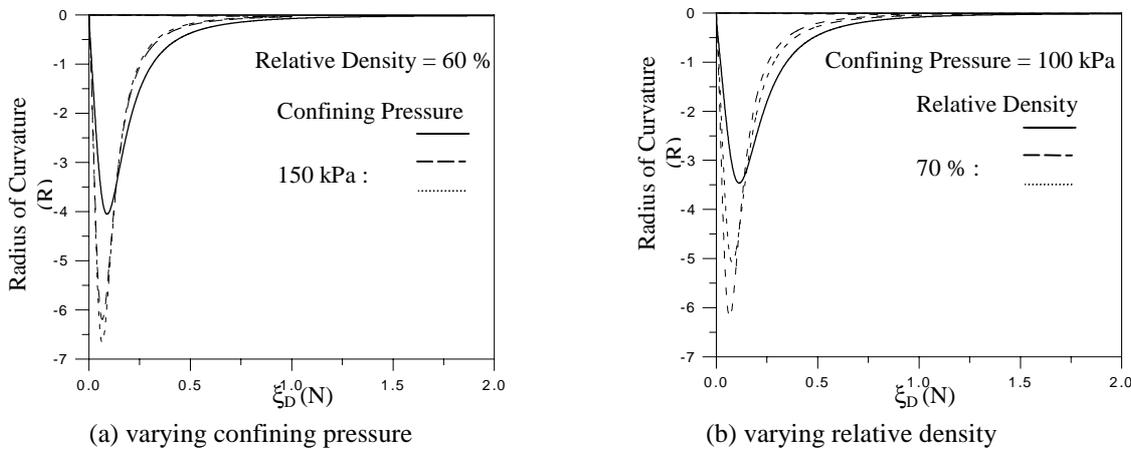
**Figure 6: Plot of disturbance (D) vs.  $\xi_D(N)$**

## Critical Disturbance and Liquefaction

During cyclic loading, loose saturated sandy soil experiences particle motions such as sliding and rotation; these can lead to instabilities in the microstructures of the material. The disturbance grows during the loading, and it can be used as the assessment of the liquefaction potential. It has been found that when the disturbance reaches the threshold value,  $D_c$  (see in Fig. 3), its rate of change experiences transition, that is, it increases with a significant decrease in rate, toward the ultimate condition ( $D_u$ ). Microstructural instability leading to the initiation of liquefaction and failure [Desai, *et al.*, 1998] can occur at the critical disturbance,  $D_c$ . This state can be evaluated by examining the derivatives of  $D$ , Eq. (7). For instance,  $D_c$  occurs when the second derivative of the  $D$ , is the minimum [Park, 1997]. Fig. 7(a)- 7(b) show the plot of  $R$  (=second derivative of  $D$ ) vs  $\xi_D(N)$  together with three initial effective confining pressures and three relative densities. Minimum of  $R$  shown in Fig. 7(a)-7(b) can be defined as critical disturbance,  $D_c$ .

The state of  $D_c$  at which saturated sand under dynamic load reaches can indicate a state of instability that may identify initial liquefaction. Table 3 shows the useful data from this research. The relationship between  $D_c$  and initial effective confining pressure,  $\bar{\sigma}_0$ , is found.  $D_c$  increases with the increase of  $\bar{\sigma}_0$ . In these tests, the value of the  $D_c$  is in the range from 0.935 to 0.955. Also, the relationship between  $D_c$  and relative density,  $D_r$ , is defined from this result. The values of  $D_r$  appear to increase with  $D_c$ , indicating the range from 0.947 to 0.951. Based on these results, it can be concluded that the effect of  $\bar{\sigma}_0$  on liquefaction is more sensitive than that of  $D_r$ .

$N_c$  shown in Table 3 is defined as the number of cycles at  $D_c$ . Based on the definition of  $D_c$  discussed above,  $N_c$  can be defined as the number of cycle at initial liquefaction. Therefore, the identification of the initial liquefaction can be made from defining  $D_c$  and  $N_c$  by using model proposed in this paper. In Table 3,  $N_c$  and  $N_{Liq}$  are compared for verification of procedure for the identification of the initial liquefaction, where  $N_{Liq}$  is defined as the number of cycles at initial liquefaction from laboratory tests (see Fig. 4(a)-4(b)). It may be concluded from Table 3 that procedure can allow identification of liquefaction in a fundamental manner.



**Figure 7: Plot of radian for curvature ( $R$ ) vs.  $\xi_D(N)$**

**Table 3: Summary of Results for Joomunjin Sand**

	Initial Confining Pressure			Relative Density		
	70 kPa	100 kPa	150 kPa	50 %	60 %	70 %
$D_c$ (Critical Disturbance)	0.935	0.949	0.955	0.947	0.949	0.951
$N_c$ (From Model)	18	35	40	33	35	40
$N_{Liq}$ (From Lab. Test)	19	35	40	33	35	40

## CONCLUSIONS

A unified constitutive modeling approach based on the disturbed state concept (DSC) is used to characterize the cyclic response of saturated sands (liquefaction) and to define threshold states in the deforming microstructure. The threshold state, when the disturbance tends toward stabilization, leads to the critical disturbance,  $D_c$ , which identifies initial liquefaction.

The results presented here involve effects of initial effective confining pressure and relative density for constant frequency of excitation and limited stress paths. Although they establish the validity of the DSC for liquefaction analysis, additional research would be required to analyze the effects of factors such as fine content and stress path.

The DSC can provide a fundamental procedure, which can be simplified for practical analysis and design, as well as for implementation in finite element procedure. The approach allows calculation of  $D_c$  as an integral part of the procedure, leading to the identification and growth of liquefaction during dynamic (earthquake) loading. It is believed that the proposed procedure can provide a mechanistic, improved, and simplified methodology for liquefaction analysis.

## ACKNOWLEDGMENTS

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