ASEISMIC DESIGN OF TALL STRUCTURES USING VARIABLE FREQUENCY PENDULUM OSCILLATOR

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SUMMARY

Tuned Mass Dampers (TMD) provide an effective technique for vibration control of flexible structures, where base isolation technique is not suitable. However, due to the constant time-period of conventional TMD, they are not usually suitable for aseismic design. The authors have recently proposed a new isolation device, known as Variable Frequency Pendulum Isolator (VFPI) that is also effective for vibration control of structures with wide-band excitations. The VFPI incorporate isolation with both restoring mechanism and energy dissipation. In the present paper, the VFPI are proposed as effective TMD for tall structures. Due to variable frequency of VFPI, they can remain effective even when the structure fundamental frequency changes with response level. The investigations show that TMD using VFPI is more effective than traditional TMD with constant frequency dampers.

INTRODUCTION

Development of control techniques to ensure safety of structures during earthquake excitations is a critical area of continuing research. Most passive control techniques can be categorised as (1) energy dissipator, (2) base isolator, and (3) tuned mass damper. Each of these techniques use fundamentally different approaches for vibration control and are most effective for different types of structures. Of these, the base-isolators are not found suitable for tall or flexible structures wherein the fundamental time-period of the structure may be relatively long (about 0.25 seconds or longer). The energy dissipaters are used to reduce the energy that must be otherwise absorbed by the structure through inelastic deformations, and are very effective if properly designed. However, energy dissipaters are effective only if the deformations are large, and may not be suitable for most structures.

The tuned mass damper (TMD) consists of a single degree-of-freedom (SDOF) oscillator with a small mass, a spring element, and a damping mechanism, usually mounted on the top of the structure. The mass and stiffness are so chosen that the frequency of fixed-base TMD is tuned to the fundamental frequency of the structure. The TMD are very effective in reducing response of the structure if the excitation is sinusoidal or narrow-band. The TMD have been found to be effective in reducing vibration due to wind [McNamara 1977, Kenny 1984]. They have also been found to be effective for resisting seismic forces [Kaynia et al. 1981, Kitamura et al. 1988]. A manor disadvantage of the TMD is that it requires a large mass and substantial amount of space for installation. To overcome this problem, a part of the structure mass itself can be used as the mass of oscillator. Recently, roof isolation system has been proposed by Villaverde and his co-workers [Villaverde 1998, Villaverde and Mosqueda 1999]. In this system, roof of the building to be protected acts as the mass of TMD, and specially installed laminated rubber bearings below the roof level as springs, while external damping mechanism is used for energy dissipation.

The TMD in above studies are essentially SDOF oscillators with a constant time period, which is tuned to the fundamental time period of the structure. There are two main difficulties associated with the use of TMD for aseismic design. (1) The fundamental time period of the structure is different at different excitation levels, and

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hence the TMD is not tuned to the structure at all response levels. (2) In the event of very strong ground motions, the structure may behave inelastically lengthening the time-period of the structure, thereby detuning the TMD. Both these problems inhibit the use of TMD for aseismic design of tall or flexible structures.

Recently, the authors have developed a new isolation system called the Variable Frequency Pendulum Isolator (VFPI) that uses a sliding surface for vibration control [Sinha and Pranesh 1998]. In VFPI, the sliding surface is concave, such that the isolator frequency decreases with increase in sliding displacement and its rate can be controlled by suitable choice of surface parameters. The VFPI take advantage of both the pure-friction system and the Frequency Pendulum System (FPS) while avoiding the disadvantage of these systems. A schematic diagram of VFPI is shown in Figure 1. The parameters of VFPI can be chosen to get the desired initial isolator time-period and the rate of variation of isolator period with sliding displacement. In addition, it is found that friction along the sliding surface also provides an effective energy dissipating mechanism, thereby integrating isolation and energy dissipation in a single unit [Sinha and Pranesh 1998].

In the present paper, the use of VFPI as TMD for flexible structures has be investigated. It is shown that the VFPI has advantages over conventional TMD whose time-period is constant. A ten-storey uniform shear building is chosen as an example system to evaluate the effectiveness of the proposed system.

VFPI GEOMETRY

Consider a rigid block of mass, *m*, sliding on a curved surface whose geometry is represented by the parametric curve $y=f(x)$. If the origin of the co-ordinate system is taken at the centre of the surface, where the sliding displacement is zero as shown in Figure 1, the restoring force at any time instant is given by

$$f_R = mg\frac{dy}{dx} \tag{1}$$

Assuming that this restoring force is provided by a spring-mass system, the spring force can be expressed as the product of the spring stiffness and deformation. The spring stiffness in turn can be expressed as the product of mass and square of frequency. So,

$$f_R = m\omega_b^2(x)x \tag{2}$$

where $\omega_b(x)$ can be called as the isolator frequency, which purely depends on geometry of the sliding surface. In case of FPS isolator whose sliding surface is spherical, the frequency is approximately a constant for small sliding displacements, and increases sharply for large displacement. If the isolator surface is parabolic, then the isolator time-period remains constant [Pranesh 1999].

The sliding surface of VFPI has been derived from the basic equation of an ellipse, in which its major axis is expressed as a linear function of sliding displacement. The equation of an ellipse is given by

$$y = b(1-\sqrt{1-x^2/a^2}) \tag{3}$$

Now, in the above equation the major axis *a* is substituted as a linear function of *x* as given below.

$$a = x + d \tag{4}$$

Substituting and simplifying for $x \geq 0$, we get

$$y = b\left[1-\frac{\sqrt{d^2+2dx}}{d+x}\right] \tag{5}$$

To keep the symmetry of sliding surface about the vertical axis, we can write

$$y = b\left[1-\frac{\sqrt{d^2+2dx\,\mathrm{sgn}(x)}}{d+x\,\mathrm{sgn}(x)}\right] \tag{6}$$

The slope at any point on the sliding surface is given by

$$\frac{dy}{dx} = \frac{bd}{(d + x\,\mathrm{sgn}(x))^2 \sqrt{d^2 + 2dx\,\mathrm{sgn}(x)}}\, x \tag{7}$$

Defining a non-dimensional parameter r and the initial frequency (when x = 0), we have

$$r = x\,\mathrm{sgn}(x)/d\,, \qquad \omega_I^2 = gb/d^2$$

$$\omega_b^2(x) = \frac{\omega_I^2}{(1+r)^2\sqrt{1+2r}} \tag{8}$$

In the above equations, b and d are parameters that completely define the isolator properties. The ratio b/d^2 defines the initial frequency of the isolator and the value of d determines the rate of variation of isolator frequency. The variation of isolator frequency with sliding displacement for VFPI, FPS and constant period isolator is shown in Figure 3(a). It can be seen that the frequency of VFPI decreases with sliding displacement and its rate of change can be controlled by choosing proper values of the parameters. In contrast, the frequency of FPS increases sharply for large sliding displacements. From the force-displacement hysteresis loops shown in Figure 3(b), it can also be observed that the isolator force in VFPI has an upper limit after which it decreases. This provides the desirable softening mechanism that is activated for large sliding displacement

RESPONSE ANALYSIS

To illustrate the use of VFPI as a tuned mass damper, the response analysis of a shear structure with TMD at its top has been considered. The mass of TMD can be an additional mass or the floor mass itself. This mass is connected to the main structure through VFPI units. During earthquake ground motions, the restoring force is provided by the sliding of this mass along the isolator surface, while energy dissipation mechanism is provided by frictional forces that are developed on the sliding surface. Due to the action of frictional forces, the motion consists of two phases, namely, non-sliding phase and sliding phase. The equations of motion are different in the two phases and the overall behaviour is very highly nonlinear. Depending on the phase of motion, the corresponding equations need to be evaluated to determine the response of the structure and VFPI.

Non-sliding Phase

In this phase, there is no relative motion between the structure and the TMD, and the structure behaves conventionally but with the additional mass of TMD added to the top floor mass. The equations of motion governing this phase are

$$\mathbf{M}_0\ddot{\mathbf{x}}_0 + \mathbf{C}_0\dot{\mathbf{x}}_0 + \mathbf{K}_0\mathbf{x}_0 = -\mathbf{M}_0\mathbf{r}_0\ddot{x}_g \tag{9}$$

and

$$\ddot{x}_b = \dot{x}_b = 0 \quad \text{and} \quad x_b = \text{constant} \tag{10}$$

with

$$\left|(\ddot{x}_1 + \ddot{x}_g) + \omega_b^2 x_b\right| < \mu g \tag{11}$$

In the above equations, $\mathbf{M_0}$, $\mathbf{C_0}$ and $\mathbf{K_0}$ are the $n{\times}n$ mass, damping and stiffness matrices of the structure (excluding TMD degree of freedom), $\mathbf{x_0}$ is the n-vector of relative displacements of the structure with respect to ground, x_b is the sliding displacement of VFPI, $\mathbf{r_0}$ is the force influence vector, x_1 is the relative displacement of the TMD mass relative to its base and m_b is the additional mass used for TMD. The coefficient of friction is denoted by μ and overdots indicate derivatives with respect to time. The left-hand side of equation (11) is the absolute value of sum of the inertial force of TMD mass and the restoring force at the isolator level while the right-hand side is the frictional force that must be overcome for sliding motion to take place.

Sliding Phase

The structure starts sliding when the forces acting on the tuned mass exceed the static frictional force along the sliding surface. The structure now has one additional degree-of-freedom. The equations of motion are given by

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{r}\ddot{x}_g - \mathbf{r_f}\, m_b \mu g\, \mathrm{sgn}(\dot{x}_b) \tag{12}$$

where

$$\mathbf{M} = \begin{bmatrix} \mathbf{M_0} & \mathbf{M_b} \\ \mathbf{M_b^T} & m_b \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \mathbf{C_0} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix}, \mathbf{K} = \begin{bmatrix} \mathbf{K_0} & \mathbf{0} \\ \mathbf{0} & k_b \end{bmatrix}, \mathbf{r} = \begin{Bmatrix} \mathbf{r_0} \\ 0 \end{Bmatrix}, \mathbf{r_f} = \begin{Bmatrix} \mathbf{0} \\ 1 \end{Bmatrix}$$

with

$$\mathbf{M_b^T} = \{m_b \quad 0 \quad . \quad . \quad 0\}, \mathbf{x^T} = \{\mathbf{x_0} \quad x_b\}$$

In the above equations, sgn(.) is the signum function, which assumes a value of +1 for positive sliding velocity and –1 for negative sliding velocity. This value is determined from the sign of the sum of total inertia force of the oscillator mass and isolator restoring force as given below.

$$\mathrm{sgn}(\dot{x}_b) = -\frac{\ddot{x}_1 + \ddot{x}_b + \ddot{x}_g}{\left|\ddot{x}_1 + \ddot{x}_b + \ddot{x}_g\right|} \tag{13}$$

The value of signum function remains constant until the sliding velocity becomes zero. The end of sliding phase is governed by the condition that the sliding velocity of the TMD mass is equal to zero. As soon as this condition is satisfied, equation (9) and (10) corresponding to the non-sliding phase are used to evaluate the response and check the validity of inequality (11). This decides whether, during the next time step, the structure continues in the sliding phase or enters a non-sliding phase [Sinha and Pranesh 1998].

EXAMPLE SYSTEM

A ten-storey shear building is chosen as an example for this investigation (Figure 2). The VFPI system is attached to the top floor of the structure and carries an additional mass that acts as the TMD mass (m_b). This additional mass is assumed to be 10% of the total mass of the structure. The lumped storey-mass is taken as 60080 kg for all the floors. The initial values of storey stiffness are each equal to 4.25×10^8 N/m, such that the fundamental time period of the structure without the TMD is 0.5 s. The modal properties of first five modes of the structure without the TMD are given in Table 1. When the structure is subjected to a high intensity base excitation, the structure deforms inelastically and the fundamental time period of the structure lengthens due to stiffness degradation of the resisting elements. Accurate modelling of this behaviour is very complicated. However, the seismic force that is attracted in the structure with inelastic deformations can be estimated by considered the inelastic structure as equivalent to an elastic structure with lower values of storey stiffness resulting in a higher fundamental time period. Such simulations are used in this study to investigate the effect of detuning of the structural time period with that of the TMD due to inelastic deformations during strong earthquake. The VFPI system acting as TMD has its initial time period equal to that of the original structure (0.5 s), and an FVF of 1.0. It is observed from the analytical studies that for a given fundamental period of the degraded structure and the initial time period of the isolator (equal to the fundamental time period of the original structure), there is an optimum value of FVF that increases with the increase in the time period of the degraded structure [Pranesh 1999]. The FVF chosen in the present study corresponds to the optimum FVF for a degraded structure with time period of 1.0 s. Since isolation devices are most effective with low coefficient of friction, the coefficient of friction is chosen as 0.02. The damping in structure is assumed equal to 5% of the critical for all the structural modes.

The example structure with VFPI acting as TMD system is analysed for NS component of El Centro 1940 ground motion. Several different storey stiffness of the structure have been considered to represent the effect of stiffness degradation. The performance of structure with VFPI based TMD is compared to that with TMD using FPS (spherical sliding surface) and constant period isolator (laminated rubber bearings). The plots of the maximum acceleration and displacement of the top storey, maximum base shear and maximum sliding displacement of the TMD are shown in Figure 4. The response quantities in Figure 4 have been normalised with respect to those of the structure without a TMD. From these figures it can be observed that there is a substantial reduction in the top floor acceleration for VFPI based TMD when compared to both constant period and FPS based TMDs. However, the corresponding displacement of top floor and other structure response quantities do not show any increase when VFPI is used. It is also interesting to note that the conventional FPS may not be practical as a TMD due to its characteristics of increase in time period with sliding displacement.

It is also seen that the relative displacement of top mass and base shear are not substantially reduced for any of the TMD. This clearly implies that passive control of flexible and inelastic structures using TMD may not be effective for these response quantities. However, in situations where the acceleration response needs to be reduced, for instance in primary secondary systems, VFPI based TMD can be very effective for all structural properties. It is also seen that the sliding displacement in VFPI is greater than that in other systems.

CONCLUSIONS

In this paper, the effectiveness of a recently proposed isolation system as a tuned mass damper has been investigated. The response of VFPI is highly complex due to its nonlinear restoring force characteristics; however, they can be evaluated using suitable numerical techniques. Since the fundamental time period of a real structure is excitation dependent, the effectiveness of TMD for different time periods of an example shear building has been investigated. For comparison purposes, the response of the same structure with TMD using FPS and linear bearings have also been considered. In addition, response of the corresponding structure without a TMD has also evaluated. From these investigations following conclusions have been drawn:

1. Flexible structures with tuned mass dampers can experience reduced response during seismic excitation even if the structural properties change due to inelastic behaviour.
2. The VFPI can be very effective for TMD due to its inherent ability to control oscillation time period.
3. Conventional TMD (roof isolation system) using linear bearings and FPS are less effective than VFPI.

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TABLE 1. Modal properties of the example system without TMD.

Mode	1	2	3	4	5
Frequency (Hz)	2.00	5.97	9.78	13.39	16.57
Cumulative Mass Participation (%)	84.79	93.93	97.02	98.45	99.20

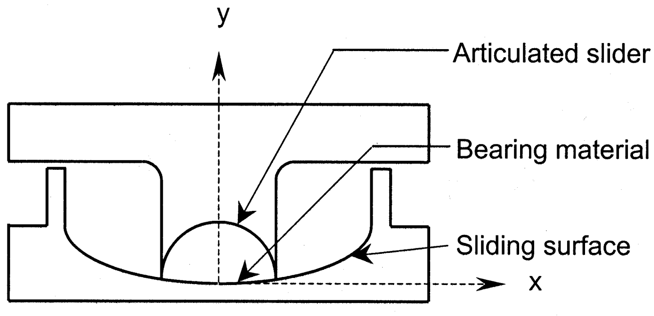

Figure 1. Schematic view of a VFPI, showing the main components.

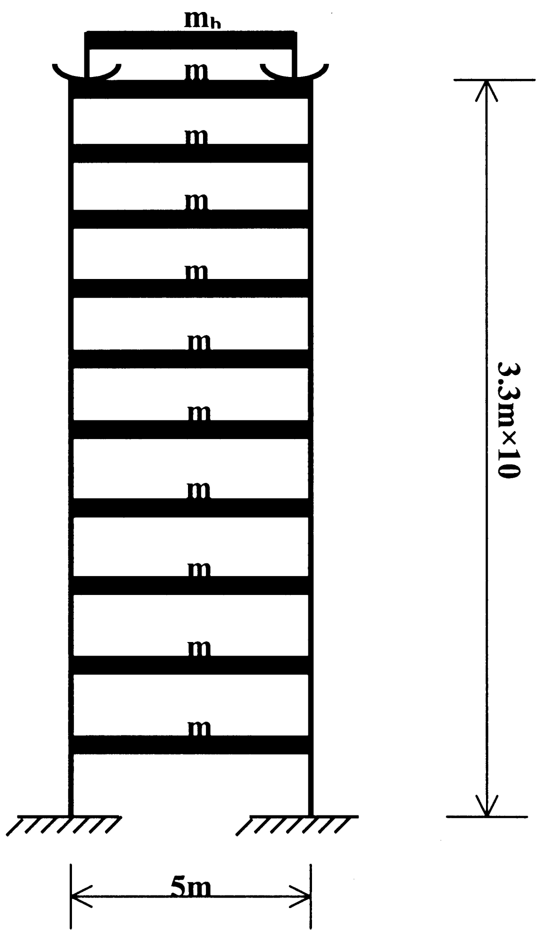

Figure 2. Example shear structure with TMD at top.

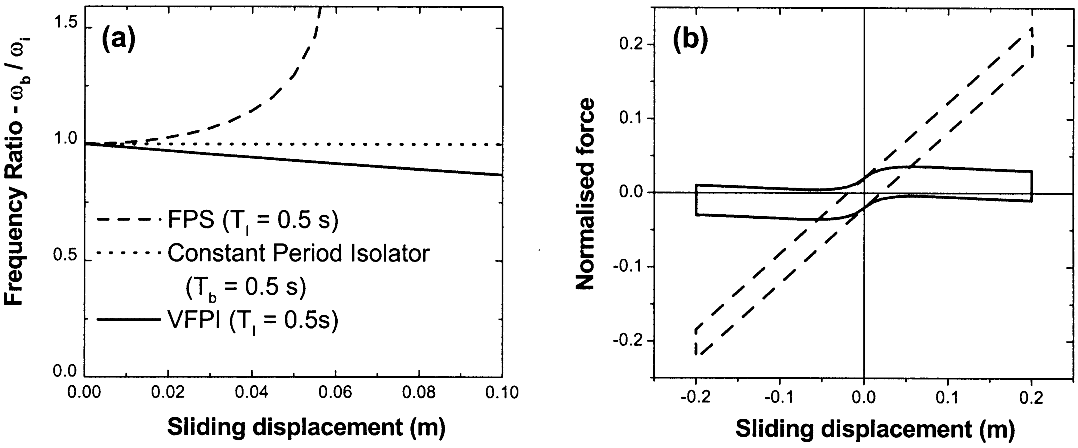

Figure 3. Properties of different isolator based TMDs.

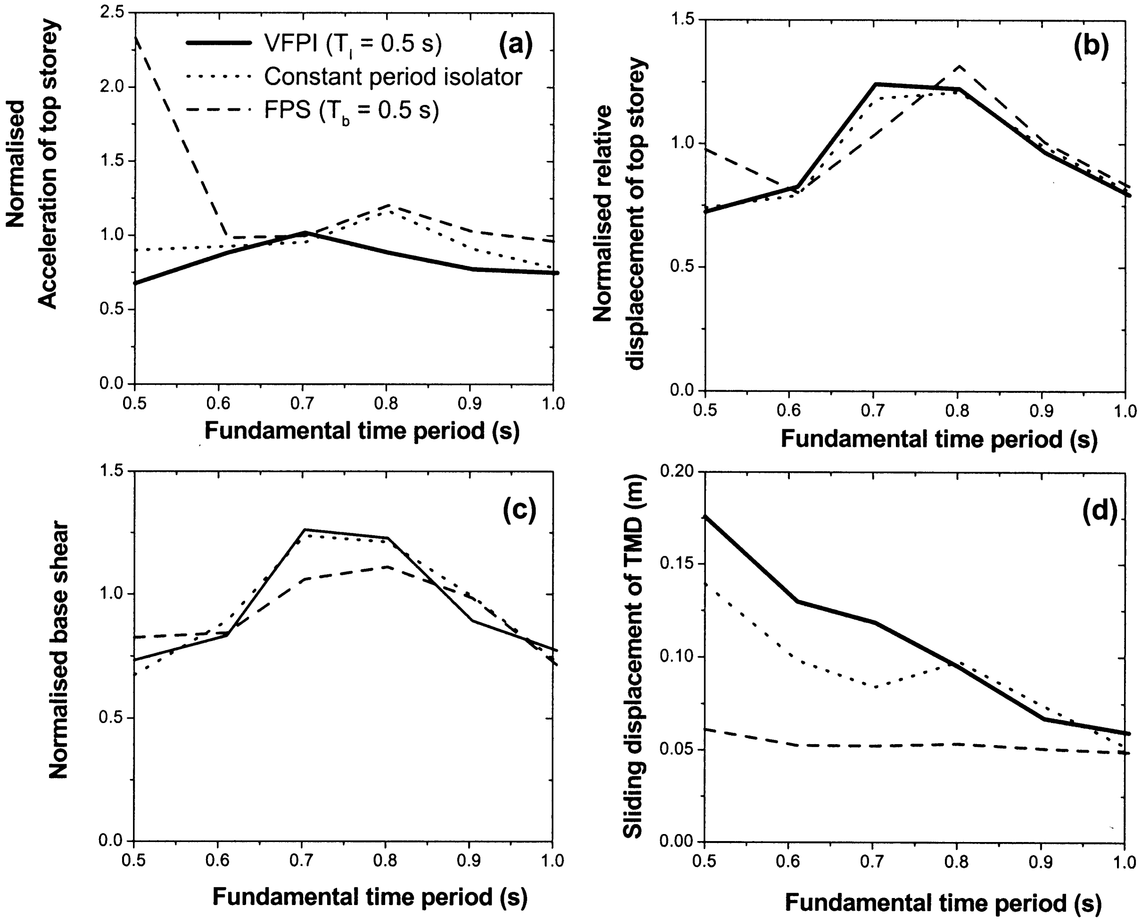

Figure 4. Structural response of example building with TMD subjected to El Centro 1940(NS) ground motion (T_I = 0.5s, FVF = 1.0, μ = 0.02, ξ = 5%).