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THREE-DIMENSIONAL STRUCTURE - SOIL - STRUCTURE INTERACTION UNDER SEISMIC EXCITATION WITH PARTIAL UPLIFT

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SUMMARY

The present paper deals with the dynamic soil-structure interaction under seismic excitation and especially how the dynamic behaviour of structures (buildings) is influenced by the condition of contact between the foundations and the subsoil. The so called condition of partial contact admits only pressure contact stresses. Tensile contact stresses do not occur, so a partial or full uplift of the foundation from the soil becomes possible. Obviously this problem of soil-structure interaction is a nonlinear one. In this paper a numerical approach will be presented for treating problems in conjunction with partial contact in the time domain by using the substructure method. Here the approach is elaborated for rigid foundations but can also be extended to elastic foundations without difficulties. The results given here show the influence of partial uplift concerning three dimensional structure – soil – structure interaction under seismic excitation.

INTRODUCTION

Since about three decades a lot of research work has been done on the field of soil-structure interaction. A large number of papers and textbooks concerning this topic has been published up to now. Nearly all of these works are based on the assumption of a tension-proof connection between the foundation and the underlying soil. The linearity associated with this kind of connection (subsequently denoted as full contact) leads to considerable simplifications in the analysis, however its justification with regard to real situations seems sometimes to be doubtful. This lack recently led to research works [Savidis, Bode et. al. 1999, Hornig, 1998] with the intention of a closer-to-reality description of the actual contact conditions between the foundation and the subsoil by assuming a one-sided connection between both (partial contact). Hereby tensile contact stresses between the foundations and the soil are basically excluded and a partial or full uplift becomes possible, see Fig. 1. Subject of the present work is to examine the influence of partial contact on structures subjected to a seismic excitation.

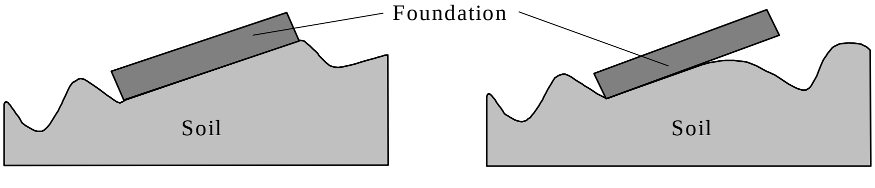

Figure 1 Conditions of contact. Left: full contact, right: partial contact

Due to the nonlinear character of partial contact, here the interaction problem is formulated in the time domain. Thus the approach is also basically applicable to the analysis of any other nonlinear structure.

FORMULATION OF THE PROBLEM

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Substructure method

As nowadays usual in the analysis of soil-structure interaction, the system under consideration has been divided into two substructures: a) the finite structure and b) the unbounded soil. The basic equations describing the dynamic behaviour of each substructure have been derived first by treating both independently. Then, the influence of the unbounded soil on the behaviour of the structure is represented by a boundary condition linked at those degrees of freedom associated with the nodes at the interface between the two substructures (interaction points, Fig. 2). In the present paper this boundary condition is of the form of a displacement-force relationship (flexibility formulation) calculated by using influence functions (Green's functions) for a layered or homogeneous halfspace.

Finite structure

Although the structure itself may be i. g. complex and nonlinear, in the present paper it is assumed to consist of several rigid, rectangular surface foundations. Additional superstructures supported by these foundations are modelled as lumped masses. Doing so, the displacements of one foundation can be condensed to six degrees-of-freedom (DOF) according to the rigid body motions (3 translations and 3 rotations, Fig. 2a). Assembling the rigid body DOF in the generalized displacement vector $\boldsymbol{u}$, the externally applied loads (forces and moments) in the generalized external force vector $\boldsymbol{P}$ and denoting the vector of the resultants of the contact stresses (generalized interaction forces) with $\boldsymbol{Q}$, the equation of motion for the rigid foundations can be written as:

$$\boldsymbol{M} \cdot \ddot{\boldsymbol{u}}(t) = \boldsymbol{P}(t) - \boldsymbol{Q}(t) \tag{1}$$

The generalized diagonal mass matrix $\boldsymbol{M}$ consists of the masses and mass moments of inertia, respectively. It is important to note that in Eq. (1) the seismic excitation is included in the vector $\boldsymbol{Q}$. Given the initial displacements, rotations and their velocities, the equation of motion Eq. (1) can be solved by a time step integration scheme. Making use of the Newmark numerical integration scheme [Bathe & Wilson, 1976], the generalized displacements at the time t^{i+1} can be determined as

$$\boldsymbol{u}^{i+1} = \underbrace{\boldsymbol{u}^{i} + \Delta t\,\dot{\boldsymbol{u}}^{i} + (1/2-\beta)\Delta t^{2}\ddot{\boldsymbol{u}}^{i}}_{\text{Predictor}} + \underbrace{\beta\Delta t^{2}\ddot{\boldsymbol{u}}^{i+1}}_{\text{Corrector}} = \boldsymbol{u}^{i+1}_{pred} + \beta\Delta t^{2}\ddot{\boldsymbol{u}}^{i+1}, \tag{2}$$

with β being one of two Newmark parameters. Starting from Eq. (2), the first step is to predict the generalized displacements based on the quantities at the time t^{i} (Predictor step). Notice that all quantities at the time t^{i} are known, either from the initial values (first time step) or from the previous time step. To perform the corrector step it is necessary to evaluate the equation of motion at the time t^{i+1} to get the still unknown accelerations $\ddot{\boldsymbol{u}}^{i+1}$:

$$\ddot{\boldsymbol{u}}^{i+1} = \boldsymbol{M}^{-1} \cdot \left\{ \boldsymbol{P}^{i+1} - \boldsymbol{Q}^{i+1}\left(\boldsymbol{u}^{i+1}_{pred}\right) \right\} \tag{3}$$

However, this equation contains the unknown interaction forces $\boldsymbol{Q}^{i+1}$ resulting from the contact stresses $\boldsymbol{q}^{i+1}$. Their determination requires the knowledge of the influence of the unbounded soil. That will be addressed next.

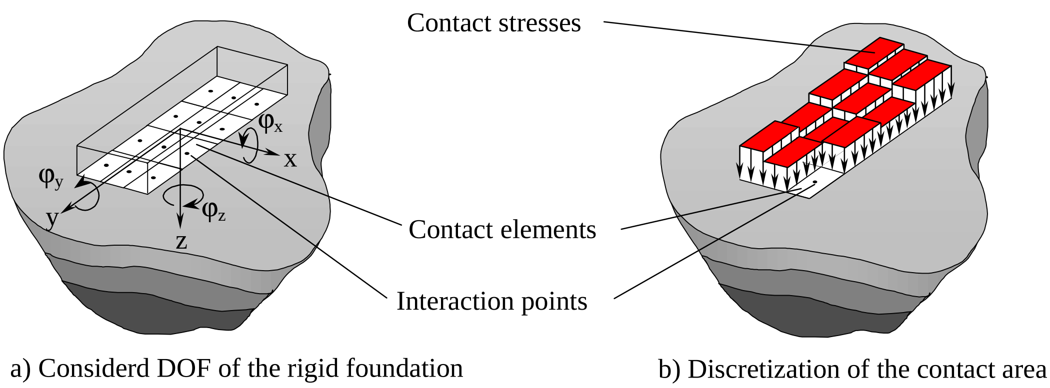


Figure 2 Definition of the problem

Unbounded soil

The description of the unbounded soil starts from the so called influence functions for harmonic point loads acting on the surface of the halfspace. They can be calculated analytically in the case of a homogeneous halfspace [Lamb, 1904] or semi-analytically by means of the Thin-Layer-Method [Kausel, 1986] in the case of a layered one. If influence functions for an arbitrary halfspace are available, such a halfspace can also be treated without difficulties. Now, to obtain the resultant interaction forces $\boldsymbol{Q}$ it is necessary to solve a mixed boundary-value problem in which zero stresses are imposed on the soil surface outside the interface, while at the interface displacements according to the rigid body motions have to be imposed. To overcome this problem, the interface is discretized into N elements of uniform rectangular shape [Savidis & Richter, 1979]. Within each element the contact stresses are assumed to be constant (Fig. 2b). Since influence functions for point loads are used, quite arbitrary shaped geometries as well as arbitrary distributed contact stresses are basically possible.

With regard to the fact, that the occurrence of contact between the foundation and the soil can only be stated at discrete points, the displacements are represented by the corresponding quantities at the interaction points (midpoints, Fig. 2). Introducing the relative soil displacement $r = w - s$, with w being the absolute soil displacement and s the free field motion due to the seismic excitation, the relationship between the vector of the relative soil displacements $\boldsymbol{r}$ at all interaction points and the vector of the uniform contact stresses $\boldsymbol{q}$ of all elements, both arranged in $N \times 1$-vectors, can be expressed by means of a convolution integral:

$$\boldsymbol{w}(t) - \boldsymbol{s}(t) = \boldsymbol{r}(t) = \int_0^t \boldsymbol{F}(t-\tau) \cdot \boldsymbol{q}(\tau) d\tau \qquad \boldsymbol{F}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \boldsymbol{F}^*(\omega) e^{i\omega t} d\omega \qquad (4a,b)$$

with $\boldsymbol{F}(t)$ being the unit impulse response (Dirac impulse). Its entry F_{jk} denotes the displacement (flexibility) at the j-th interaction point due to uniform contact stresses acting on the k-th element. It can be derived from its counterpart in the frequency domain $\boldsymbol{F}^*(\omega)$ (Eq. (4b)). To get $\boldsymbol{F}^*(\omega)$, the aforementioned influence functions for point loads have been integrated numerically over the loaded area by a Gaussian quadrature. Since all elements are of the same size, $\boldsymbol{F}^*(\omega)$ appears to be symmetric.

By evaluating Eq. (4b) by means of the Inverse Fast Fourier Transform (IFFT), the difficulties arising from the high frequency content of $\boldsymbol{F}^*(\omega)$, can be avoided by using a modified unit impulse response instead of Eq. (4b). This modified impulse and its Fourier transform is represented by a Gaussian distribution [Wolf,1988]:

$$g(t) = \frac{1}{2\sigma\sqrt{\pi}} exp\left(-\frac{t^2}{4\sigma^2}\right) \qquad g^*(\omega) = exp\left(-\sigma^2\omega^2\right) \qquad (5a,b)$$

Notice that if the parameter σ tends to zero, the modified impulse $g(t)$ converges to the Dirac impulse $\delta(t)$. Making use of Eq. (5b) yields the following definition for the modified impulse response, which is much more suited for the application of standard IFFT algorithms due to the exponential decay of $g^*(\omega)$:

$$\boldsymbol{F}_{mod}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \boldsymbol{F}^*(\omega) g^*(\omega) e^{i\omega t} d\omega \qquad (6)$$

Replacing $\boldsymbol{F}(t-\tau)$ by $\boldsymbol{F}_{mod}(t-\tau)$ then leads to a similar expression to that given in Eq. (4a). For the sake of simplicity the subscript '*mod*' will be dropped subsequently. To evaluate the expression for the soil displacements, the convolution integral in Eq. (4a) is discretized into $i+1$ intervals of equal length Δt. The time step Δt is chosen equal to that given in Eq. (2). The discretization in the time domain can be regarded as a representation of the continuous contact stresses by a sequence of discrete impulses as shown in Fig. 3.

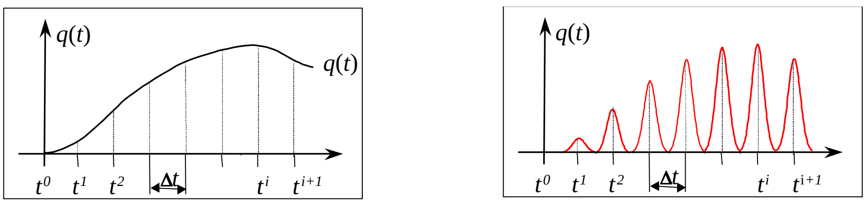


Figure 3 Representation of the continuous contact stresses by a sequence of modified impulses

Rearranging Eq. (4a) and writing it in a discretized form leads to the absolute soil displacements at the time t^{i+1}:

$$\boldsymbol{w}^{i+1} = \underbrace{\Delta t\boldsymbol{F}^{i} \cdot \boldsymbol{q}^{1} + ... + \Delta t\boldsymbol{F}^{1} \cdot \boldsymbol{q}^{i}}_{\text{History part}} + \underbrace{\Delta t\boldsymbol{F}^{0} \cdot \boldsymbol{q}^{i+1}}_{\text{Current part}} + \underbrace{\boldsymbol{s}^{i+1}}_{\text{Seismic part}} \tag{7}$$

$\boldsymbol{F}^k$ stands for $\boldsymbol{F}(k\Delta t)$ while the superscript at $\boldsymbol{q}$ and $\boldsymbol{s}$ indicates the corresponding time. For the following considerations it proves to be appropriate to split the expression containing all the contact stresses into one part caused by the known contact stresses up to the time t^i (history part) and a second part caused by the unknown contact stresses $\boldsymbol{q}^{i+1}$, which have to be determined. Denoting the history part of the relative soil displacements with $\boldsymbol{r}^{hist}$ and interpreting $\Delta t\boldsymbol{F}^0$ as the actual flexibility matrix $\boldsymbol{F}^{act}$, Eq. (7) can be rewritten as:

$$\boldsymbol{w}^{i+1} = \boldsymbol{r}^{hist} + \boldsymbol{F}^{act} \cdot \boldsymbol{q}^{i+1} + \boldsymbol{s}^{i+1} \tag{8}$$

Finally, it remains the task to relate the motion of the structure, given with Eq. (2), to that of the soil, given with Eq. (8). With this relationship, the still unknown contact stresses $\boldsymbol{q}^{i+1}$ and their resultants $\boldsymbol{Q}^{i+1}$ have to be determined depending on the condition of contact.

Condition of contact

With regard to the condition of contact formulated only at the interaction points, the (predicted) rigid body motion Eq. (2) has to be transformed into an expression for the displacements at all interaction points. Summarizing these quantities at the time t^{i+1} in the vector $\boldsymbol{v}^{i+1}$, the displacements at all interaction points belonging to the foundations follow from the kinematics of rigid bodies by means of a transformation matrix $\boldsymbol{T}$:

$$\boldsymbol{v}^{i+1} = \boldsymbol{T} \cdot \boldsymbol{u}^{i+1}_{pred} \tag{9}$$

Full Contact

In the case of full contact the compatibility between the foundations and the soil has to be guaranteed at each interaction point (no penetrations as well as no gaps are allowed). This means that the displacements of the foundations and the soil have to be equal at all interaction points:

$$\boldsymbol{w}^{i+1} = \boldsymbol{v}^{i+1} \quad \Rightarrow \quad \boldsymbol{F}^{act} \cdot \boldsymbol{q}^{i+1} = \boldsymbol{v}^{i+1} - \boldsymbol{s}^{i+1} - \boldsymbol{r}^{hist} \tag{10}$$

With Eq. (10) a set of linear equations is obtained for the determination of the unknown contact stresses $\boldsymbol{q}^{i+1}$. If they are determined their resultants $\boldsymbol{Q}^{i+1}$ follow with ΔA being the area of an element as:

$$\boldsymbol{Q}^{i+1} = \Delta A \boldsymbol{T}^{T} \cdot \boldsymbol{q}^{i+1} \tag{11}$$

Partial Contact

In the case of partial contact the determination of the unknown contact stresses $\boldsymbol{q}^{i+1}$, that means the fitting of the impulse $\boldsymbol{q}^{i+1}\Delta t$, is based on the demand that no tensile contact stresses may occur. So first, for each interaction point it has to be checked whether contact occurs or not since for those elements being not in contact the contact stresses have to be set equal zero. That's just the case if the soil displacement at any interaction point, caused by the history of the contact stresses, is „greater" than the displacement at the corresponding point of the foundation. This can be stated in the form:

$$r^{hist}_j > v^{i+1}_j - s^{i+1}_j \quad \Rightarrow \quad q^{i+1}_j = 0 \tag{12}$$

Notice that „greater" is related to the definition of „positive" displacements which are directed inwards the subsoil, see Fig. 2a. For the remaining interaction points, those being in contact, again the compatibility of the displacements has to be guaranteed, which leads to a reduced set of linear equations analogous to Eq. (10):

$$\boldsymbol{w}^{i+1}_{red} = \boldsymbol{v}^{i+1}_{red} \quad \Rightarrow \quad \boldsymbol{F}^{act}_{red} \cdot \boldsymbol{q}^{i+1}_{red} = \boldsymbol{v}^{i+1}_{red} - \boldsymbol{s}^{i+1}_{red} - \boldsymbol{r}^{hist}_{red} \tag{13}$$

The subscript '*red*' (= reduced) indicates, that the vectors comprise only the quantities of those interaction points where contact occurs. Finally the time step is finished by evaluating the equation of motion Eq. (3), correcting the displacements and then calculating the (generalized) velocities from a formula similar to that given in Eq. (2), followed by an iteration process until an accuracy goal is achieved.

NUMERICAL RESULTS

The verification of the proposed method was given in an earlier paper [Savidis, Bode et. al. 1999] for a linear system subjected to a direct excitation. It is based on a comparison with results obtained by well developed methods working in the frequency domain [Savidis & Hirschauer, 1997] and making use of the inverse Fourier transform afterwards. The results confirm the working of the method.

Influence of partial contact on a seismic excited foundation

First, to study the influence of the condition of contact, the dynamic behaviour of a rigid square foundation (side length a = 1.0m) resting on a homogeneous soil and subjected to a seismic excitation is investigated. Two similar types of seismic excitation are considered, consisting of a horizontal propagating plane wave with vertical components only (sv-wave) and a sinusoidal shape of one wavelength. The first has a wavelength of λ = 50m (long-wave excitation) with a peak acceleration of 10m/s² while the second one has a wavelength of λ = 0.5m (short-wave excitation) with a peak acceleration of 100m/s². The phase velocity in both cases is c = 200m/s, leading to the corresponding time histories $s(t)$ as shown in Fig. 4.

To determine the dynamic response, the interface between the foundation and the soil is discretized into 20x20 uniform square elements. The time step is selected as Δt = 0.2ms. The parameter σ, characterizing the sharpness of the modified impulse, Eq. (5), is chosen as σ = 0.1ms. Notice that the accuracy can be improved, by increasing the number of elements or by decreasing the parameter σ of the modified impulse, however a larger frequency range for the IFFT of the modified impulse response has to be accepted, see Eq. (6).

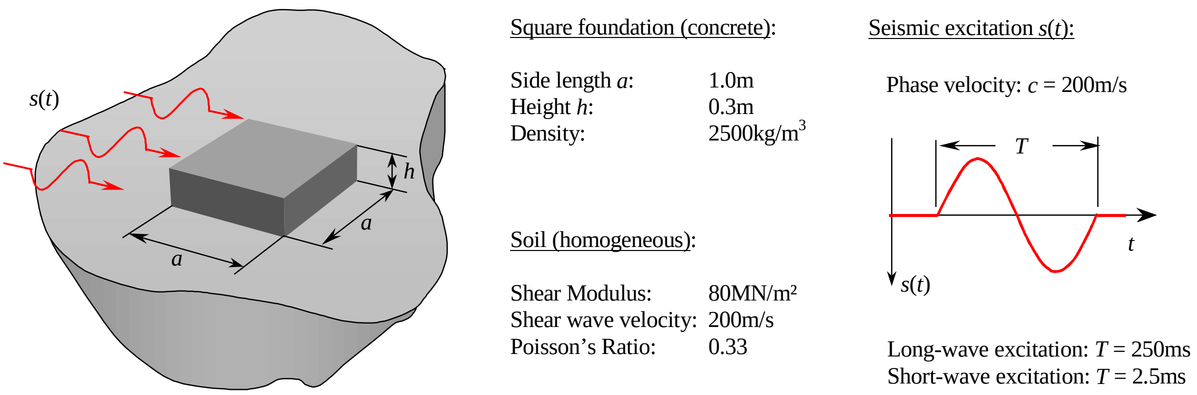
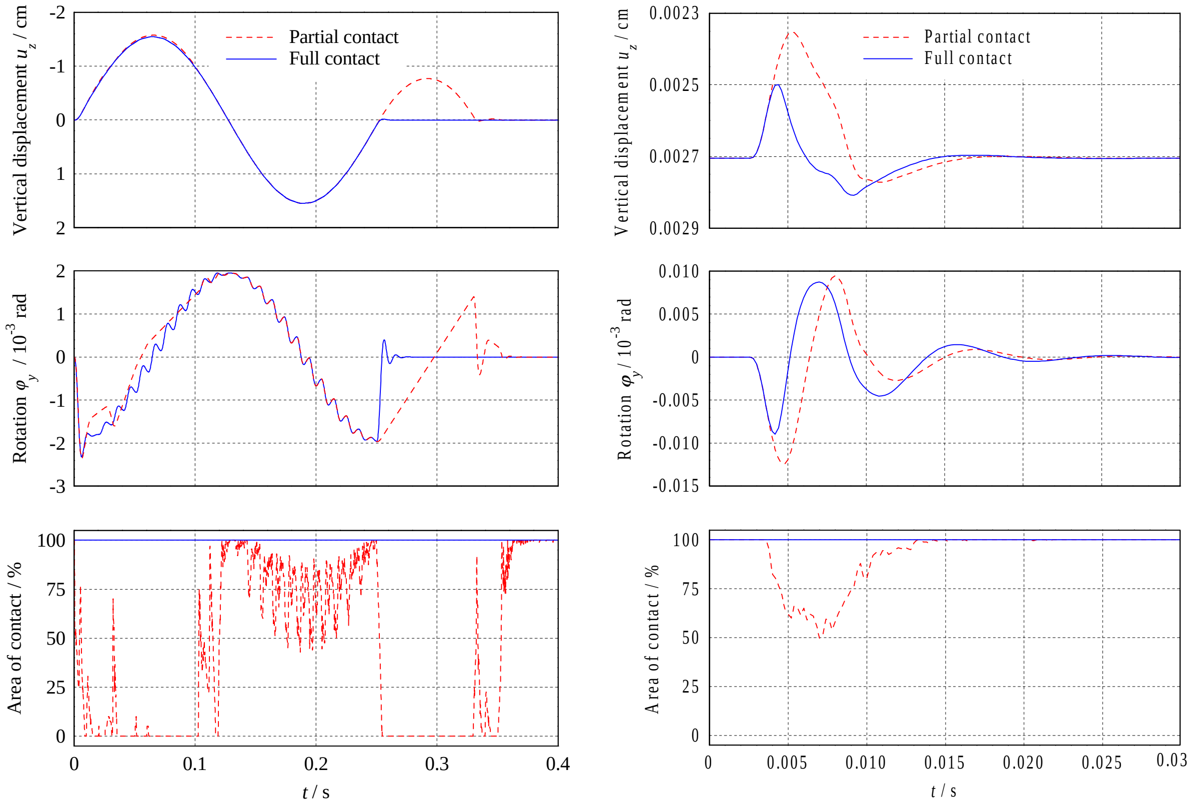


Figure 4 System under investigation and time history of the seismic excitation

In Fig. 5 the vertical and rotational DOFs of the foundation as well as the percentage of the area being in contact are given for both types of excitation. The through lines denote the results for full contact, whereas the dashed lines denote the results for partial contact. Regarding the vertical displacement of the foundation subjected to the long-wave excitation (Fig. 5a) nearly no difference between full and partial contact can be seen during the passage time (up to 0.25s). Immediately after the wave has passed, the foundation lifts up and hence it moves under the influence of the gravity which is confirmed by the parabolic shape of the dashed line between 0.25s and 0.33s. Afterwards the impact happens, which can be seen in the sudden increase of the contact area at about 0.33s. Note that there is also a period at the beginning (up to 0.1s) where only a few or none elements are in contact without having a significant influence on the vertical displacement. Regarding the rotational DOF, a superposition of a higher frequency signal due to the rotational eigenfrequency of the system can be observed.

Another interesting effect is, that for the short-wave excitation, the difference between full and partial contact is more distinct (Fig. 5b), although the area being not in contact is much more smaller than in the first case.



a) Long-wave excitation: λ = 50m

b) Short-wave excitation: λ = 0.5m

Figure 5 Square foundation subjected to a seismic excitation: Influence of full and partial contact

Influence of adjacent structures with partial contact

The system analyzed here is shown in Fig. 6. It consists of two superstructures based on rigid plate foundations resting on a homogeneous soil. Both foundations have a side length of a = 25m and the distance between both is d = 5m. The superstructures are modelled by lumped masses connected by rigid massless rods. The lumped masses m, their mass moments of inertia Θ and their heights h above the ground are summarized in Fig. 6. The soil properties are the same as given before. Both foundations are excited by a seismic base motion according to a horizontal propagating sv-wave (phase velocity c = 200m/s). As time input function a corrected data set of vertical displacements based on a recorded accelerogram of the Imperial Valley earthquake (1979) is chosen.

	Node No.	m [t]	Θ [t m^2]	h [m]
Structure A	A1	3100	6.5 10^5	0
	A2	3400	7.0 10^5	12
	A3	500	2.5 10^5	30
	Total weight:		7000t	
	Center of gravity:		7.97m	
	Side length a:		25m	
Structure B	B1	10500	12.6 10^5	0
	B2	11400	12.6 10^5	10
	B3	6000	3.6 10^5	22
	B4	2100	2.4 10^5	35
	Total weight:		30000t	
	Center of gravity:		10.65m	
	Side length a:		25m	
	Distance d:		5m	

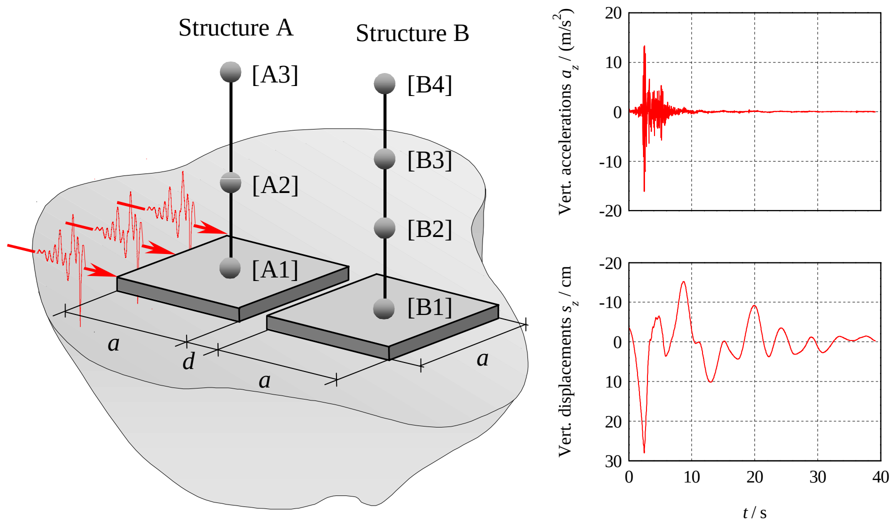
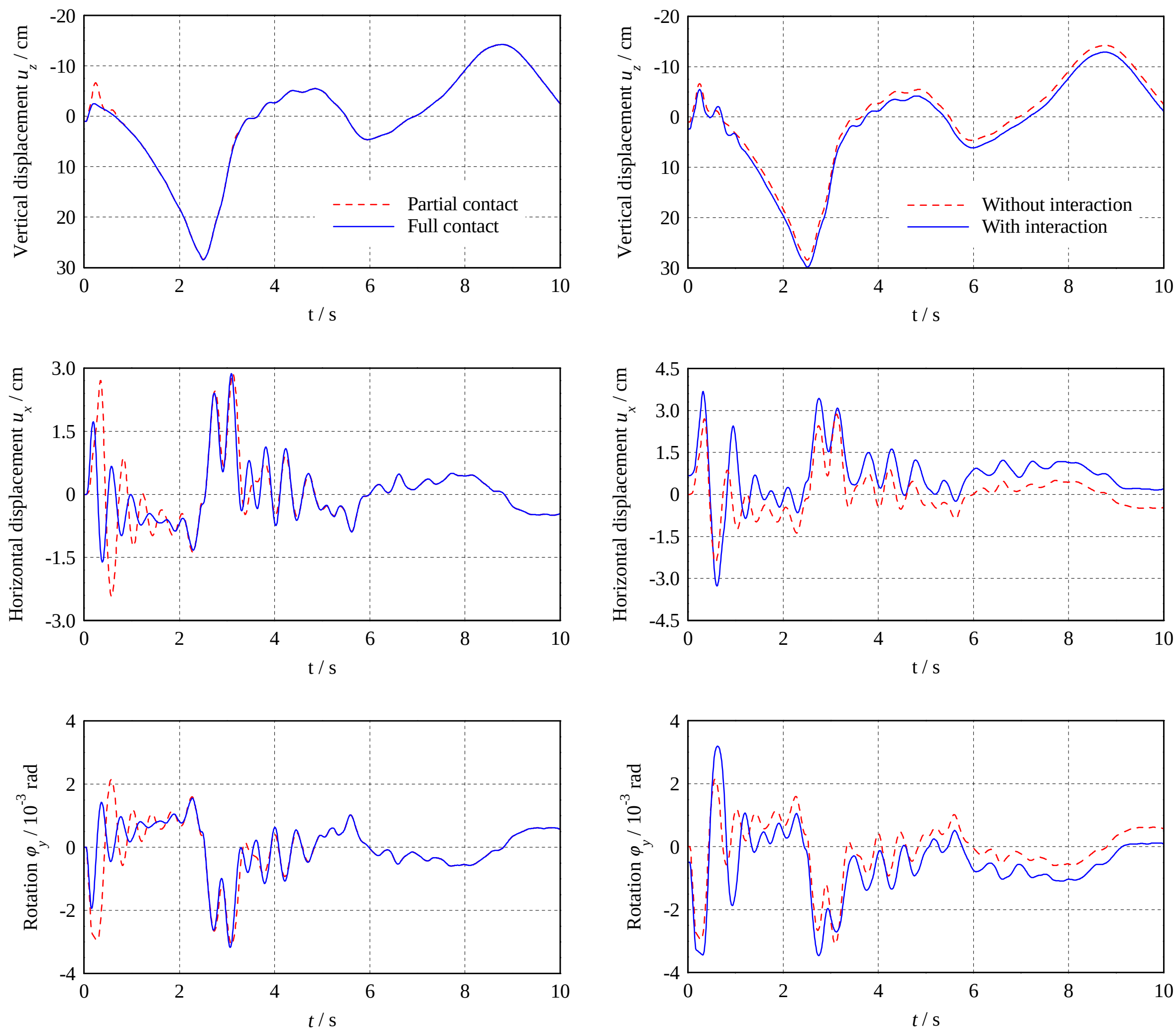


Figure 6 System under investigation and time history of the Imperial Valley earthquake

First, before studying the influence of the adjacent structure B on the dynamic behaviour of structure A, once more the influence of partial contact is investigated. Therefore, structure A is calculated separately for the condition of full and partial contact. Although the excitation acts over 40s, in Fig. 7a the vertical, horizontal and rotational DOFs of structure A are shown only for the first 10s in order to focus the differences which are limited in that case to this period. Except for a small hump in the very beginning, the vertical displacements for both conditions are nearly the same. However, if the horizontal and rotational DOFs are considered, the differences become more evident, whereby the calculation based on the condition of partial contact leads to bigger values compared with those assuming full contact. As before, these differences mainly appear during the first seconds, where the excitation exhibits the most rapid changes.



a) Influence of partial contact (without structure B)

b) Influence of the adjacent structure B (partial contact)

Figure 7 Translational and rotational DOFs of structure A with respect to its center of gravity

Finally, as already mentioned, the influence of the interaction between structure A and B will be discussed. For this reason, in Fig. 7b again the rigid body DOFs for structure A (without interaction, dashed lines) are compared with those obtained by taking the interaction with structure B into account (through lines). Hereby partial contact is assumed. The differences between both curves are clearly recognizable, even in the range after 10s which is not displayed. Moreover, it is important to note, that here the interaction with structure B almost ever leads to bigger values for each DOF. It is also interesting to note, that both curves run mostly parallel, except those periods where rapid changes happen. The continuous offset is essentially caused by the static dead weight of the heavier structure B, whereas the derivations are due to large gradients in the seismic excitation.

CONCLUSION

A numerical procedure for the time domain analysis of the dynamic soil-structure interaction under seismic excitation with the nonlinear condition of partial contact between the soil and the foundations is proposed. Considering full and partial contact, respectively, there is no big difference in the numerical effort. The investigation of a rigid square foundation resting on a homogeneous soil, subjected to a horizontal propagating plane wave of sinusoidal shape, yields different results for the rigid body motions for full and partial contact, respectively. The differences between full an partial contact depend among others on the shape and intensity of the seismic excitation as well as on the geometry and the mass of the structure. Furthermore, investigating the influence of the interaction between adjacent structures under the condition of partial contact, subjected to a seismic base motion according to the Imperial Valley earthquake, also results in differences which are not neglectable.

Therefore, regarding sensible buildings the condition of partial contact as well as the influence of adjacent structures should be taken into consideration.

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