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A STUDY ON MODAL STRAIN ENERGY METHOD FOR VISCOELASTICALLY DAMPED STRUCTURES

M H TSAI[1] And K C CHANG[2]

SUMMARY

The modal strain energy (MSE) method has been proposed to estimate the modal loss factors or modal damping ratios of structures with viscoelastic (VE) dampers. However, there are certain assumptions made in deriving the MSE method, such as the computation of the modal loss factor directly from the ratio of the imaginary and real part of the eigenvalue, and the neglect of the influence of the imaginary mode shapes, *etc*. In this study, the effect of the assumptions made by the MSE method is investigated, and modified formulations of the MSE method are derived. The modified MSE method lifts the assumptions made in the original MSE method. The earthquake responses of a complex stiffness system and a linear viscous damping system, of which the modal damping ratios are estimated by the MSE method, are compared in order to illustrate the difference of these two systems. Analytical results indicate that the difference arising from the assumptions becomes significant when the damping ratio is larger than 20%. Likewise, for the illustrated non-proportionally damped system, the effect of imaginary mode shapes on the MSE method can be neglected when the damping ratio is smaller than 20%. It is then concluded that, for most engineering applications with design damping ratio smaller than 20%, the conventional linear viscous damping model with the MSE method may result in solutions in good agreement with that obtained by the more rigorous complex stiffness model.

INTRODUCTION

Viscoelastic (VE) damper has been shown as an effective energy dissipator. Since the VE dampers are used for providing extra damping to the structures, the composite damping ratio of the viscoelastically damped structures is one of the key parameters to be determined for dynamic analyses. The modal strain energy (MSE) method has been proposed to estimate the modal loss factors or modal damping ratios of structures with VE dampers [Johnson and Kienholz, 1982]. Its accuracy has been demonstrated through experimental studies [Chang *et al*., 1995] on model structures and analytical studies on the exact and approximate mean-square responses of structures with added VE dampers [Zambrano *et al*., 1996]. Also, in earthquake resistant applications, the MSE method has also been applied to design the structures with added VE dampers [Chang *et al*., 1998a; Shen *et al*., 1995]. However, the MSE method used to estimate the damping ratio of a linear viscous damping model is derived from the eigenvalue analysis of a complex stiffness model. Consequently, there are some assumptions made in the original derivation, such as the computation of the modal loss factor directly from the ratio of the imaginary and real part of the eigenvalue, and the neglect of the influence of the imaginary mode shapes, *etc*. In this study, the effect of such assumptions on the predicted modal damping ratio is investigated, and modified MSE methods are derived. The modified MSE methods suggest the range of application that the original MSE method may apply for modal damping ratio prediction. Finally, the seismic responses of a complex stiffness system and a linear viscous damping system whose modal damping ratios are estimated by the MSE method are compared to illustrate the difference.

[1] *Department of Civil Engineering, National Taiwan University, Taipei, Taiwan. Email: mhtsai@vip.url.com.tw*
[2] *Department of Civil Engineering, National Taiwan University, Taipei, Taiwan. Email: mhtsai@vip.url.com.tw*

2. MODAL STRAIN ENERGY METHOD

2.1 MSE1

The differential equation of a complex stiffness system under free vibration in the frequency domain is expressed as

$$M\ddot{X}(j\omega)+(K_1+jK_2)X(j\omega)=0 \tag{1}$$

where M is the mass matrix; K_1 and K_2 are the elastic and loss stiffness matrices of the system, respectively, and X is the displacement vector in the frequency domain. Eq.(1) can be converted to an eigenvalue problem by assuming a solution in the time domain in the form as:

$$x(t)=\phi_i^* e^{j\lambda_i^* t}\equiv\phi_i^* q_i(t) \tag{2}$$

where ϕ_i^*, λ_i^* and $q_i(t)$ are the i-th complex mode shape, the square root of the eigenvalue and the modal coordinate, respectively. Therefore, the eigenvalue problem becomes

$$(K_1+jK_2)\phi_i^*=\lambda_i^{*2}M\phi_i^* \tag{3}$$

Now, drop the modal index i and express λ_i^{*2} as

$$\lambda^{*2}=\lambda^2(1+j\eta) \tag{4}$$

In addition, approximate ϕ^* by its real part ϕ_R, which is obtained from the eigenvalue analysis by neglecting the loss stiffness K_2. By premultiplying ϕ_R^T on both sides of Eq.(3) we have

$$\lambda^2(1+j\eta)=\frac{\phi_R^T K_1\phi_R}{\phi_R^T M\phi_R}+j\frac{\phi_R^T K_2\phi_R}{\phi_R^T M\phi_R} \tag{5}$$

Equating the real part and imaginary part of Eq.(5) gives

$$\lambda^2=\frac{\phi_R^T K_1\phi_R}{\phi_R^T M\phi_R} \tag{6}$$

and $\eta=\dfrac{\phi_R^T K_2\phi_R}{\phi_R^T K_1\phi_R}$. (7)

The value of • obtained by Eq.(6) is the undamped modal frequency based on the real part of the stiffness matrix. Note that K_2 is the loss stiffness contributed by the added VE dampers only, and K_1 is the composite stiffness composed of the storage stiffness of the VE dampers and the elastic stiffness of the primary system, which is defined as the structural system without VE dampers. The value of η obtained from Eq.(7) is the so-called modal loss factor [Johnson and Kienholz, 1982], and the modal damping ratio ξ is equal to $\eta/2$. Moreover, it is easy to prove that the usual adoption of $Ceq=K_2/\lambda$ for the damping matrix leads to the same modal loss factor as expected by the MSE1 method.

2.2 Proportionally Damped System (MSE2)

Although the MSE1 method is derived from the characteristic equation of a complex stiffness system, the modal damping ratio obtained is an approximation due to the definition of η in Eq.(4). Now, if λ^* in Eq.(4) is expressed as

$$\lambda^*=\alpha+j\beta \tag{8}$$

then the solution of $x(t)$ becomes

$$x(t)=\phi^* e^{j(\alpha+j\beta)t}\equiv\phi^* e^{-\beta t}e^{j\alpha t} \tag{9}$$

Hence, according to the theory of structural dynamics, the real part and imaginary part of λ^* are related to the modal damping ratio ξ and undamped modal frequency ω_n as

$$\alpha=\omega_n\sqrt{1-\xi^2} \text{ and } \beta=\omega_n\xi \tag{10a, b}$$

Also, from Eq.(4), it is seen that α and β are the real and imaginary parts of $\lambda\sqrt{1+j\eta}$, and the roof of the eigenvalue has the form of

$$\alpha + j\beta = \lambda\sqrt{1+j\eta} = \frac{\lambda}{\sqrt{2}}\left\{(\sqrt{1+\eta^2}+1)^{1/2} + j(\sqrt{1+\eta^2}-1)^{1/2}\right\} \tag{11}$$

The final result gives

$$\xi = \left[\frac{1}{2}(1-\frac{1}{\sqrt{1+\eta^2}})\right]^{1/2} \tag{12}$$

This modal damping ratio is obtained from the eigenvalue analysis of a complex stiffness system. The value of η in the above equation is the same as that in Eq.(7), in which the assumption of proportional damping is implied.

The modal damping ratios calculated by Eq.(12) with various loss factor, η, which is based on the MSE1 method, is shown in Fig.1. It is obvious that the difference becomes significant as the modal damping ratio obtained from the MSE1 method increases. As seen in the figure, both methods give almost identical values when the modal damping ratio is smaller than 20%. However, when the modal damping ratios expected by the MSE1 method increase to 30% and 40%, the corresponding values obtained by the modified method (MSE2) are 26% and 33%, respectively. Alternatively, when the variation of the stiffness ratio (the ratio of the storage stiffness of the dampers to the elastic stiffness of the primary structure) is considered, the difference of the predicted modal damping ratios is shown in Fig.2 based on a damper loss factor $\eta_v = 1$. In Fig.2, it is realized that increasing the stiffness ratio will enhance the contribution of VE dampers to the modal damping ratio. Furthermore, the MSE1 method predicts larger modal damping ratios than that of the MSE2 method under the same stiffness ratio, especially when the stiffness ratios are larger than 1.0. The difference of predicted modal damping ratios increases gradually as the stiffness ratio increases.

2.3 Non-proportionally Damped System (MSE3)

The previous formulations are based on a proportionally damped system which neglects the effect of imaginary mode shapes. Since a multiple degrees-of-freedom structure may be non-proportionally damped, it is necessary to investigate the effect of imaginary mode shapes on the MSE1 method. In the cases of non-proportionally damped systems, the derivation of η may be proceeded without the assumption of suppressing the imaginary mode shapes as follows.

Premultiplying ϕ^{*T} on both sides of Eq.(3) and dropping the modal index *i* result in

$$\phi^{*T}(K_1 + jK_2)\phi^* = \lambda^{*2}\phi^{*T}M\phi^* \tag{13}$$

where ϕ^{*T} is the conjugate transpose row vector of ϕ^*. In a similar way as Eqs. (5)~(7), the modal loss factor, η, can be estimated by

$$\eta = \frac{\phi_R^T K_2 \phi_R + \phi_I^T K_2 \phi_I}{\phi_R^T K_1 \phi_R + \phi_I^T K_1 \phi_I} \tag{14}$$

where ϕ_R and ϕ_I are the real and imaginary part of the complex mode shape, respectively. It is clear that the contribution of the imaginary mode shape depends on the value of $(\phi_I^T\phi_I)/(\phi_R^T\phi_R)$, which can be used as a measure of non-proportionality of damping in a system. Now, the value of η determined by Eq.(14) has taken the complex mode shapes into account, and the modal damping ratio obtained from the characteristic equation can be determined by substituting η into Eq.(12).

Now considering a shear-type two degrees-of-freedom model with added VE dampers, the mode shape of this system can be expressed as

$$\phi^* = \begin{Bmatrix} 1 \\ \phi_{1r} \end{Bmatrix} + j\begin{Bmatrix} 0 \\ \phi_{1i} \end{Bmatrix} \tag{15}$$

where the amplitude of the top floor is set to 1, and ϕ_{1r}, ϕ_{1i} are the real and imaginary part of the first floor mode shape, respectively. Defining the mass ratio and elastic stiffness ratio, respectively, as

$\gamma = m_2 / m_1$ and $R_s = k_2^{'} / k_1^{'}$ (16a, b) where m_1 and m_2 are the floor masses of the lower story and top story, respectively. $k_1^{'}$ and $k_1^{''}$ are the storage and loss stiffness of the lower story, while $k_2^{'}$ and $k_2^{''}$ are those of the top story. The modal damping ratios of this two degrees-of-freedom system can be calculated based on specified values of•, R_s and loss factor of each story. Now, consider an extreme case that the VE dampers are installed at the first story only. Let the mass ratio and floor stiffness ratio are both equal to 1.0, and the top story has an inherent loss factor, denoted by η_2 , equal to 0.1. The influence of the imaginary mode shape on the MSE1 method can be examined by increasing the loss factor of the lower story, denoted by η_1, from 0.1 to 1.0. The modal damping ratios, estimated by Eqs.(7) and (14), are compared in Fig.3a. The contribution of $\phi_{1i}^2 / \phi_{1r}^2$ with increasing η_1 is shown in Fig.3b. It is seen that as the ratio of η_1 / η_2 increases, the value of $\phi_{1i}^2 / \phi_{1r}^2$ increases. The difference becomes significant as the loss factor ratio is greater that 6.0, at which the value of $\phi_{1i}^2 / \phi_{1r}^2$ is near to 0.05. Furthermore, since the 2nd modal damping ratios are small as compared to the 1st ones and below 20%, the difference is not significant even when the value of η_1 / η_2 increases to 10, as seen in Fig.3a. Hence, the imaginary mode shape will affect the results predicted by the MSE1 method only when both the modal damping ratio and $\phi_{1i}^2 / \phi_{1r}^2$ are large.

Following the same procedure, it is also easy to demonstrate that for ordinary base isolated structures, the imaginary mode shapes have negligible effect on the modal loss factors or modal damping ratios predicted by the MSE1 method, even if the values of $\phi_{1i}^2 / \phi_{1r}^2$ are large. For a typical base isolated building model, if the mass ratio and elastic stiffness ratio are set to 1.5 and 24, respectively, and the values of η_1 represent the loss factor of the isolation system, the effect of imaginary mode shapes on modal damping ratios of the base isolated building and the corresponding values of $\phi_{1i}^2 / \phi_{1r}^2$ are shown in Figs.4a and 4b. It is seen that both results are nearly identical and the values of $\phi_{1i}^2 / \phi_{1r}^2$ are small enough to be neglected. On the other hand, when the mass ratio and elastic stiffness ratio of a typical seismically isolated bridge model are selected to be 20 and 0.24, respectively, the results are shown in Figs.5a and 5b. Note that the loss factor of the isolation system is denoted by η_2 for the seismically isolated bridge model, and η_1 represents the inherent loss factor of the bridge piers. As shown in the figures, the values of $\phi_{1i}^2 / \phi_{1r}^2$ for the seismically isolated bridge model are larger than that for the building model with added VE dampers as shown in Fig.3b. However, the effect of imaginary mode shape on modal damping ratios is less significant for the typical seismically isolated bridge than for the non-proportionally damped buildings as shown in Fig.3a.

3. STRUCTURAL RESPONSE ANALYSIS

In order to explore the differences on seismic structural responses between a complex stiffness system and a linear viscous damping system, the response spectra of both systems under El Centro NS-direction earthquake record are compared. The required VE dampers are designed based on the MSE1 method for the specified design damping ratios. From the derivation of the MSE1 method, the assumption of proportional damping is implied in Eq.(5). Therefore, the formulation of the damping matrix is based on Rayleigh's damping assumption [Clough and Penzien, 1993; Chopra, 1995] in the linear viscous damping system. The structural response can be determined either by the step-by-step integration technique in the time domain or by the discrete Fourier transform method in the frequency domain.

While for the complex stiffness system, instead of forming the damping coefficient matrix, the structural response can be obtained by solving the equation of motion (Eq.(1)) directly in the time domain [Inaudi and Makris, 1996] or the frequency domain [Inaudi and Kelly, 1995; Aprile *et al.*, 1997; Chang *et al.*, 1998b]. For simplicity, the structural responses of both systems are calculated by the discrete Fourier transform method and neglecting the temperature rise effect of the VE dampers.

3.1 Proportional damping case

Now, let's consider a single degree-of-freedom system with added VE dampers. Two selected values of design damping ratios are 20% and 35% corresponding to a medium damping case and a high damping case, respectively. When the VE dampers are designed by the MSE1 method for these two specified damping ratios, the damping ratios predicted by the MSE2 method would be 18.9%and 30.0%, respectively. The response spectra of both the linear viscous damping and complex stiffness system are shown in Figs.6a and 6b, where the undamped natural period ranges from 0.1 to 3 seconds.

As seen in Fig.6b, the displacement response of the linear viscous damping system is generally smaller than that of the complex damping system because the damping ratio predicted by the MSE1 method is larger than that estimated by the MSE2 method. However, the acceleration response predicted by the linear viscous damping system is regularly larger than that predicted by the complex damping system as displayed in Fig.6a, especially for the high damping case. Moreover, Fig.6a shows that the benefit of increasing damping is more significant for the complex stiffness system than for the linear viscous damping system.

From the observation stated above, it is realized that as the damping ratio increases the structural response predicted by the linear viscous damping system, of which the damping ratio is estimated by the MSE1 method, may overestimate the acceleration response and underestimate the displacement response as compared to those predicted by the complex stiffness system.

3.2 Non-proportional damping case

For the same two degrees-of-freedom non-proportionally damped building model described earlier, the design damping ratio, which is now referred to the first modal damping ratio, is likewise equal to 20% and 35%, respectively, for the moderate and high damping case. The modal damping ratios predicted by the MSE1 method for the linear viscous damping model and by the MSE3 method for the complex stiffness model are listed in Table1. As shown in the table, in the high damping case, the first modal damping ratio estimated by the MSE3 method is smaller than that of the previous proportional damping case due to the consideration of the imaginary mode shape. This is consistent with that observed in Fig.3a.

The responses of this non-proportionally damped system are presented in Figs.7a to 7d. From Figs.7a and 7b, it is noted that, the increase of damping ratio effectively reduces the acceleration response of the top floor throughout the whole spectral periods. It is, however, only effective within the region of peak spectral response for the lower floor. For the acceleration response of the top floor, the benefit of increasing damping is similar for both systems, as shown in Fig.7a. The top floor acceleration predicted by the complex stiffness system is somewhat larger than that predicted by the linear viscous damping system, while the lower floor acceleration predicted by the complex stiffness system is obviously smaller than that predicted the linear viscous damping system. The difference predicted by both models for the displacement response of the top floor is more significant than that of the lower floor, as displayed in Figs.7c and 7d. The difference for the top floor accelerations is much smaller than for the lower floor, while for the displacement response, the conclusion is reversed.

From the investigation described above, it is realized that, the dynamic behavior of a linear viscous damping model incorporated with the MSE1 method is similar to that of a complex stiffness model for the top floor. The discrepancy is mainly caused by different damping ratios for these two systems. Besides, a linear viscous damping system incorporated with the MSE1 method may overestimate the acceleration response of lower stories but underestimate the displacement response of top stories when compare to the responses predicted by the complex stiffness system.

4. CONCLUSION

In this study, the effect of the assumptions made in the modal strain energy method (MSE1) [Johnson and Kienholz, 1982] on the prediction of modal damping ratio is first investigated. Modified formulations of the modal strain energy method for a proportionally damped system (MSE2) and a non-proportionally damped system (MSE3) are derived. Analytical results indicate that the difference between the MSE1 and MSE2 methods becomes significant when the damping ratio is larger than 20%. Likewise, for the illustrated non-proportionally damped system by MSE3, the effect of imaginary mode shapes on the MSE1 can be neglected when the damping ratio is smaller than 20%. Otherwise, the MSE1 method may overestimate the modal damping ratio without considering the effect of imaginary mode shapes. However, the imaginary mode shapes may have only minor effect on the modal damping ratios of typical seismic isolated buildings and bridges.

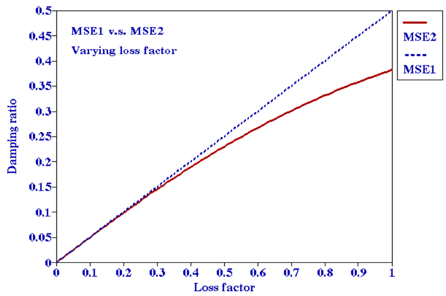
In addition, from the comparison between the structural responses of a linear viscous damping model incorporated with the MSE1 method and a complex stiffness model, it is found that the dynamic behavior of a complex stiffness system can be approximated by a linear viscous damping system incorporated with the MSE1 method when the design damping ratio is below 20%. Thus, it may be recommended that, for most engineering application with design damping ratio smaller than 20%, the conventional linear viscous damping model combined with the MSE1 method will result in solutions in good agreement with that obtained by the more rigorous complex stiffness model.

5. TABLES

Table 1 Predicted modal damping ratios of the non-proportionally damped system

	MSE1 method		MSE3 method	
1st mode	20%	35%	19.0%	28.7%
2nd mode	10.7%	16.5%	10.9%	17.4%

6. FIGURES



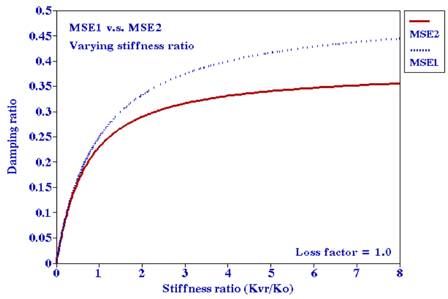


Figure 1(left) and 2(right): Comparison of predicted damping ratios

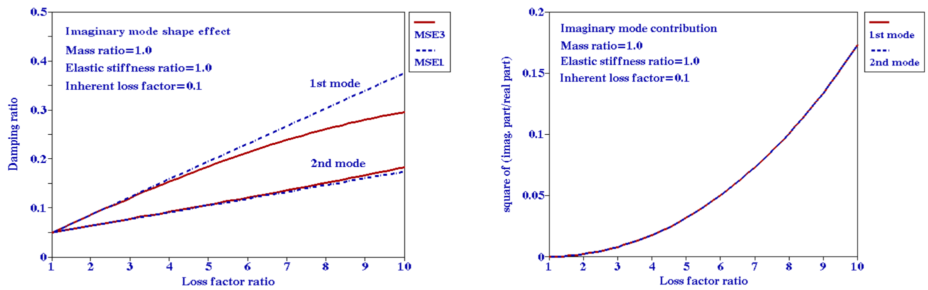


Figure 3a (left): Effect of imaginary mode shape on predicted damping ratio
Figure 3b(right): A measure of imaginary mode contribution

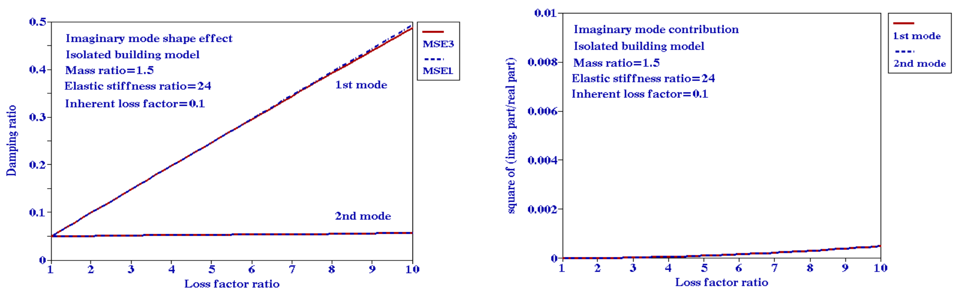
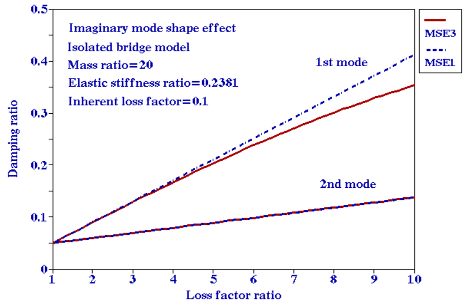


Figure 4a (left): Predicted damping ratios for a typical isolated building model
Figure 4b (right): A measure of imaginary mode contribution for a typical isolated building model



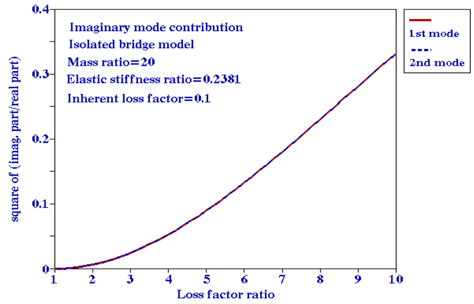
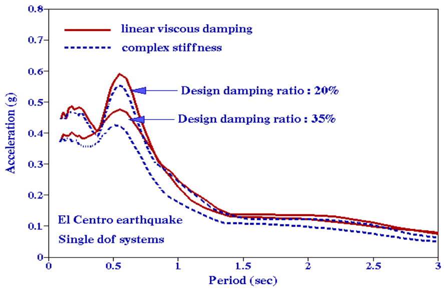


Figure 5a (left): Predicted damping ratios for a typical isolated bridge model
Figure 5b (right): A measure of imaginary mode contribution for a typical isolated bridge model



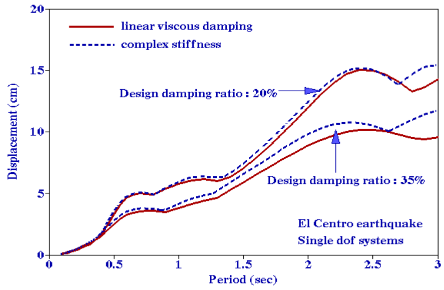
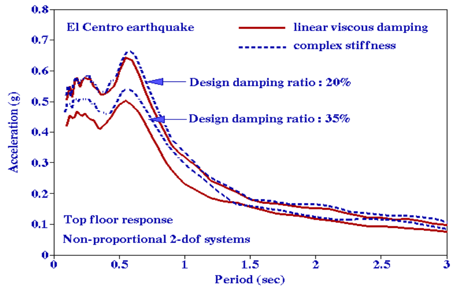


Figure 6a (left): Acceleration response spectra of the SDOF systems
Figure 6b (right): Displacement response spectra of the SDOF systems



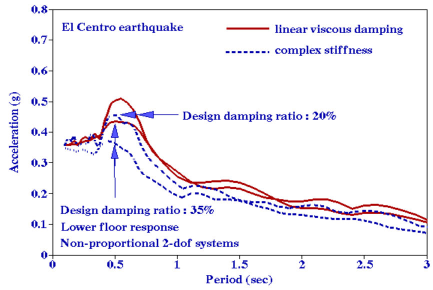
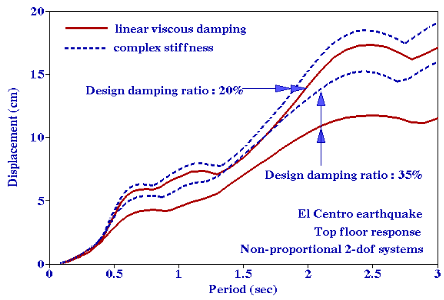


Figure 7a (left): Acceleration response spectra of the non-proportional systems (top floor)
Figure 7b (right): Acceleration response spectra of the non-proportional systems (lower floor)



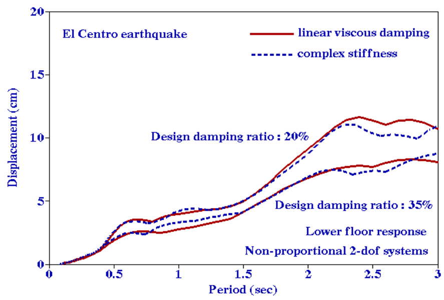


Figure 7c (left): Displacement response spectra of the non-proportional systems (top floor)
Figure 7d (right): Displacement response spectra of the non-proportional systems (lower floor)

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