DYNAMIC RESPONSE OF EARTHQUAKE EXCITED INELASTIC PRIMARY-SECONDARY SYSTEMS

Christoph ADAM[1] And Peter A FOTIU[2]

SUMMARY

The objective of the paper is to investigate numerically the effect of ductile material behavior on the response of coupled primary-secondary systems. The primary substructure is assumed as a four-storey frame building and elastic-plastic material behavior is considered in the form of a bilinear moment-curvature relation. The secondary structure is taken as a simple SDOF oscillator. Numerical studies are presented for elastic and ideally elastic-plastic SDOF oscillators with different yield levels. The coupled structure is excited by the north-south component of the El Centro earthquake sample. The investigations are performed at different excitation levels. In the examples presented tuning frequencies of the oscillator are varied. Acceleration response spectra of the main structure as well as floor response spectra are generated for various amounts of oscillator masses.

INTRODUCTION

In general, secondary structures denote those components and elements of a building that are not part of the main structural system but may also be subjected by seismic excitation. It is widely recognized that damage of non-structural containments may seriously impair a building's function, and may result in major economic losses. A great deal of research effort has been devoted over the past 3 decades to the development of rational methods for the seismic analysis of secondary systems. State-of-the-art reports of the seismic behavior of non-structural elements attached to primary structures are given by Soong [4] and Villaverde [5]. Majority of the papers is concerned with linear primary-secondary structures without taking into consideration the fact that most of the structures to which secondary systems are attached are designed to yield under the effects of a strong earthquake. Moreover, secondary systems are frequently designed to resist large inelastic deformations, too. As pointed out in the literature (see e.g. [1], [2], [3]) the nonlinear behavior of the main bearing structure and the secondary system may significantly affect the behavior of the latter. However, systematic investigations of inelastic primary-secondary systems appear to be scarce.

The objective of this paper is to study numerically the influence of nonlinear material behavior on the dynamic response of earthquake excited primary-secondary structures. The secondary subsystem is modeled as a SDOF oscillator with varying natural frequencies and its load-deflection diagram is assumed to be elastic-perfectly plastic. Moreover, the material behavior of the main frame is idealized as bilinear with kinematic hardening.

GOVERNING EQUATIONS

Consider a four-storey elastic-plastic frame (as the primary structure) with lumped masses at the joints and equipped with an elastic-plastic SDOF oscillator (secondary structure) attached to the top floor, *Figure* 1(a). The ith primary structural degree of freedom has a lumped mass M_i and the corresponding displacement component W_i represents the primary structure deformation relative to the base. The elastic-plastic reaction behavior of the frame is idealized by a bilinear moment-curvature relation, see *Figure* 1(b). The SDOF oscillator is modeled as a mass-spring-dashpot system with mass m, stiffness k and viscous damping parameter c. The displacement of the SDOF oscillator relative to the base of the primary structure is denoted as w. The entire primary-secondary structure is excited by a ground acceleration $\ddot{w}_g$. In the following, primary and secondary subsystems are

[1] *Department of Civil Engineering, Technical University of Vienna, Vienna, AustriaEmail: ca@allmech9.tuwien.ac.at*
[2] *Department of Mechatronics and Microsystems, Fachhochschule Wr. Neustadt, Wr. Neustadt, AustriaEmail: pf@fhwn.ac.at*

considered together as a single dynamic unit. The equations of motion of the coupled inelastic system with 5 degrees of freedom read as follows,

$$\boldsymbol{M}\ddot{\boldsymbol{W}} + \boldsymbol{C}\dot{\boldsymbol{W}} + \boldsymbol{K}\boldsymbol{W} = -\boldsymbol{M}\boldsymbol{e}\,\ddot{w}_g - \boldsymbol{G}\,\boldsymbol{\kappa}^p + [k\,(w - w^p - W_4) + c\,(\dot{w} - \dot{W}_4)]\,\boldsymbol{g}_4\,, \tag{1.1}$$

$$m\,\ddot{w} + c\,\dot{w} + k\,w = -m\,\ddot{w}_g + k\,(w^p + W_4) + c\,\dot{W}_4\,, \tag{1.2}$$

where $\boldsymbol{M}$, $\boldsymbol{K}$, $\boldsymbol{C}$, respectively, denote the mass, initial stiffness and proportional damping matrix of the primary structure, and $\boldsymbol{W}$ is the vector sampling the primary displacements. For a system as in *Figure* 1(a) $\boldsymbol{e} = (1, 1, 1, 1)^T$ for single point ground excitation and $\boldsymbol{g}_k = (0, 0, 0, 1)^T$ when the oscillator is attached to the fourth floor. The vector $\boldsymbol{\kappa}^p$ contains the imposed inelastic curvatures, which may either be distributed along the frame or, in a plastic hinge model, are confined to the joints of columns and girders. Furthermore, $\boldsymbol{G}$ represents constraint forces at horizontally fixed storey masses due to a singular unit curvature (that is, actually, a kink of unit angle) in the primary frame structure. In case of distributed plastic zones the dimension of $\boldsymbol{\kappa}^p$ is much larger than that of $\boldsymbol{W}$, and therefore, the matrix $\boldsymbol{G}$ becomes rectangular. If an elastic-plastic secondary system is considered w^p denotes the plastic deflection of the SDOF oscillator. It should be noted, that considering a proportional damping matrix $\boldsymbol{C}$ for the primary structure only, will leave the coupled system non-classically damped. Thus, the analysis of (1) as a single dynamic unit would require the solution of a complex eigenvalue problem. This can be avoided if equations (1) are solved by an iterative synthesis technique suggest by the authors [1]. Thereby, $\boldsymbol{W}$ is decomposed into the mode shapes of the primary structure leaving the equations of motion in modal form. The modal coefficients appearing in these equations are separated into an elastic part and a remainder containing contributions from inelastic deformation and coupling terms. The latter portion is determined in a time stepping procedure by iteration. Convergence of this iteration procedure is usually fast. For further details see Adam and Fotiu [1].

PARAMETRIC NUMERICAL STUDY

The primary substructure is assumed to be classically damped with damping ratios of $\zeta_i = 0.03$, $i = 1, ..., 4$. All relevant parameters and eigenfrequencies of the frame are given in *Table* 1 and *Figure* 1(a). In *Table* 1 EJ_{ci}, EJ_{gi} denote the initial bending stiffness of the columns and girders of the ith storey, respectively, and EJ_{ci}^*, EJ_{gi}^* are the corresponding post-yielding stiffnesses. The P-δ effect is taken into account in the columns to determine the stiffness matrix and the corresponding axial forces N_i in the columns are assumed to be constant in each storey. The damping coefficient of the SDOF oscillator is set to $\zeta_0 = 0.003$. The composite system, therefore will be non-classically damped. Numerical studies are presented for an elastic and an ideally elastic-plastic SDOF oscillator with two different yield levels $f_{y1} / mg = 2$ (yield level 1) and $f_{y2} / mg = 1$ (yield level 2), where f_y denotes the yield force. The four-storey building is excited by the north-south component of horizontal ground acceleration recorded at the Imperial Valley Irrigation District substation, El Centro, California, during the Imperial Valley earthquake of May 18, 1940.

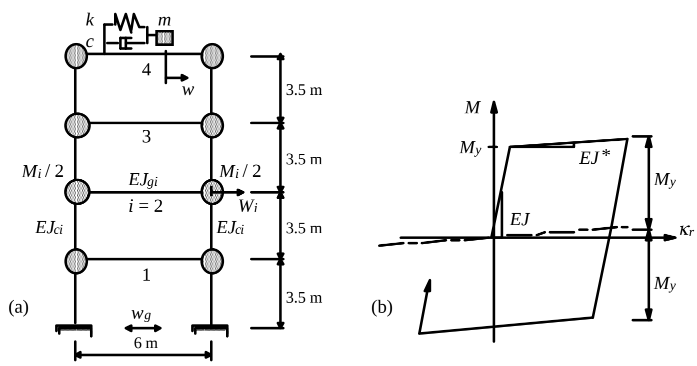

Figure 1. (a) Four storey frame building with SDOF oscillator attached to the top floor. (b) Bilinear elastic-plastic moment-curvature relation.

Table 1: Parameters and natural frequencies of the primary system.

Storey *i*	**1**	**2**	**3**	**4**
M_i [kg]	38 000	38 000	27 000	27 000
EJ_{ci} [kNm^2]	90 700	90 700	64 720	64 720
EJ_{gi} [kNm^2]	64 720	64 720	64 720	64 720
EJ_{ci}^* / EJ_{ci}	0.08	0.08	0.08	0.08
EJ_{gi}^* / EJ_{gi}	0.08	0.08	0.08	0.08
M_{yci} [kNm]	600	600	400	400
M_{ygi} [kNm]	400	400	400	400
N_i [kN]	1100	800	500	200
Ω_i [rad/s]	7.26	21.74	42.74	61.64

At first, the response of the main structure is investigated. In particular the acceleration response spectrum S_{a4} of the fourth floor is generated for an oscillator mass of m = 1000 *kg*. The response spectrum S_{a4} is defined as the peak acceleration of the fourth floor of the frame building for a range of oscillator frequencies ω. In *Figure* 2 results for six different cases are shown. The response spectrum of the coupled elastic structure is displayed by a full line, whereas the decoupled elastic response is represented by the dashed line. It is readily observed that in the range of tuned frequencies, that is around 7 rad/s, the peak acceleration is considerably reduced compared to a decoupled analysis, especially for an elastically vibrating oscillator. This effect is particularly enhanced, if the SDOF is tuned to the first primary frequency, whereas a tuning to the next higher frequency Ω_2 = 21.7 rad/s only results in a minor feedback. For comparison *Figure* 2 also shows the response of an elastic-plastic frame building with material parameters according to *Table* 1. Plastic deformation leads to an overall reduction of the

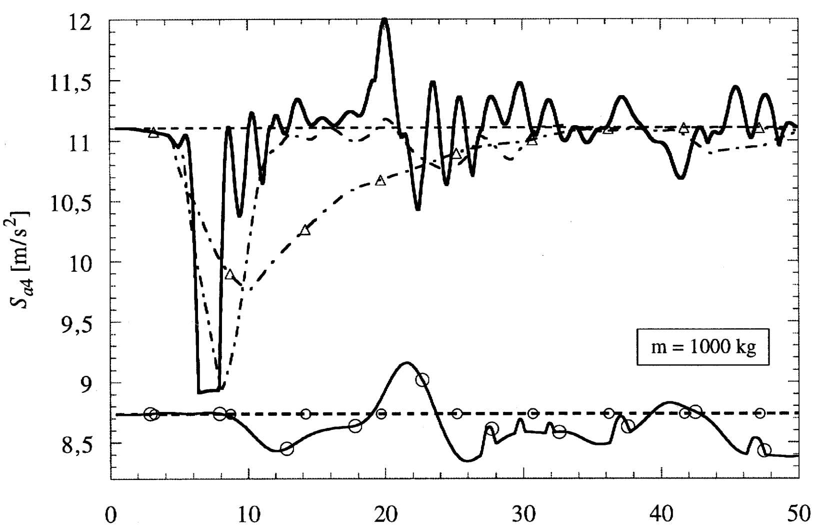

Figure 2. Acceleration primary response spectra of the fourth floor; peak response; various elastic and inelastic substructure properties; coupled and decoupled analysis. The legend is displayed in Figure 3.

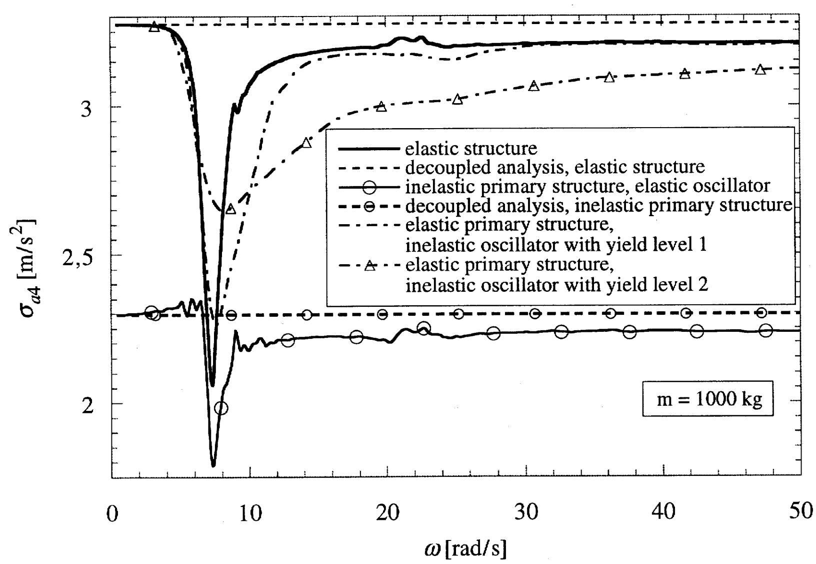

Figure 3. Acceleration primary response spectra of the fourth floor; standard deviation; various elastic and inelastic substructure properties; coupled and decoupled analysis.

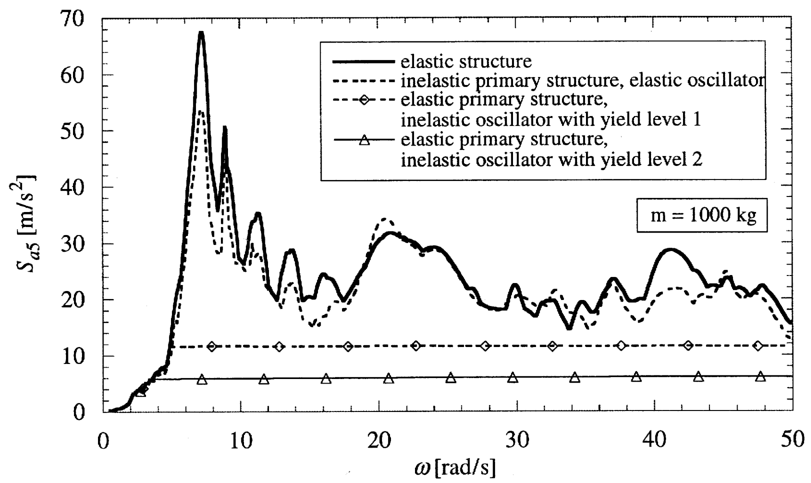

Figure 4. Acceleration floor response spectra; peak response; various elastic and inelastic substructure properties; coupled analysis.

peak accelerations by 20 % except in the tuned domain. The interaction effect in the neighborhood of $\omega \approx \Omega_1$ appears to be substantially smaller than in the elastic case. The dot-dashed lines in *Figure* 2 illustrate the effect of an ideally elastic-plastic secondary structure on the overall response of the elastic primary structure. Two different yield levels f_{y1} and f_{y2} of the SDOF oscillator are considered. It can be observed that lowering the yield level leads to a less significant drop of S_{a4} in the tuned frequency range. However, the acceleration reduction becomes increasingly wide-banded.

As another important quantity in the qualitative description of the response the standard deviation σ_a of the acceleration evolution is considered. In particular, the standard deviation of the *i*th primary acceleration is defined as

$$\sigma_{ai} = \{ \frac{1}{T}\int_O^T (Wi + wg)^2 dt - [\frac{1}{T}\int_O^T (Wi + wg)^2 dt]^2 \}^{1/2} \qquad (2)$$

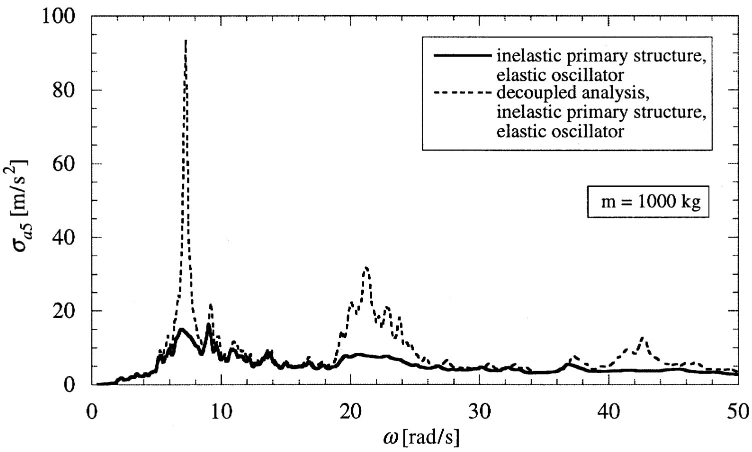

Figure 5. Acceleration floor response spectra; standard deviation; inelastic primary structure and elastic secondary structure; coupled and decoupled analysis.

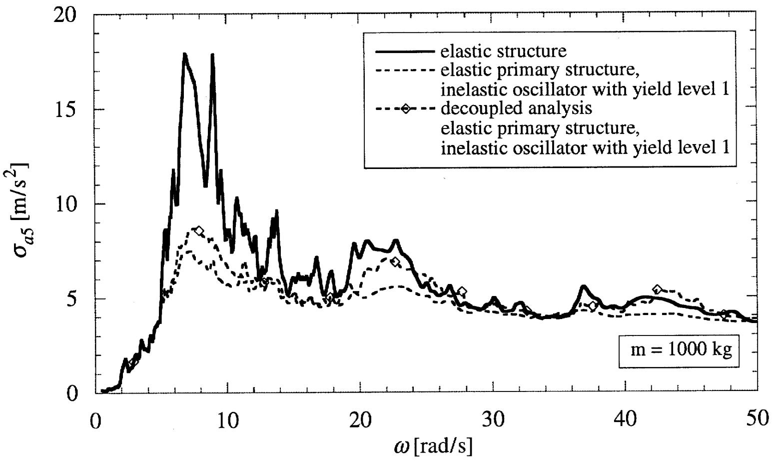

Figure 6. Acceleration floor response spectra; standard deviation; elastic primary structure and inelastic secondary structure; coupled and decoupled analysis.

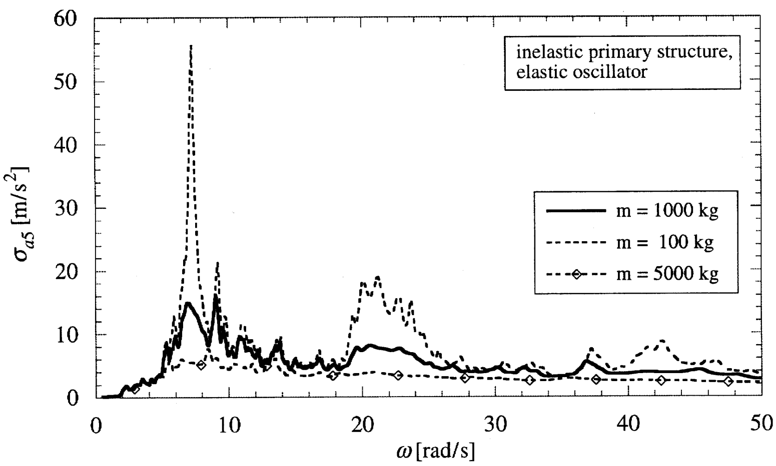

Figure 7. Acceleration floor response spectra; standard deviation; inelastic primary structure and elastic secondary structure; 3 different oscillator masses; coupled analysis.

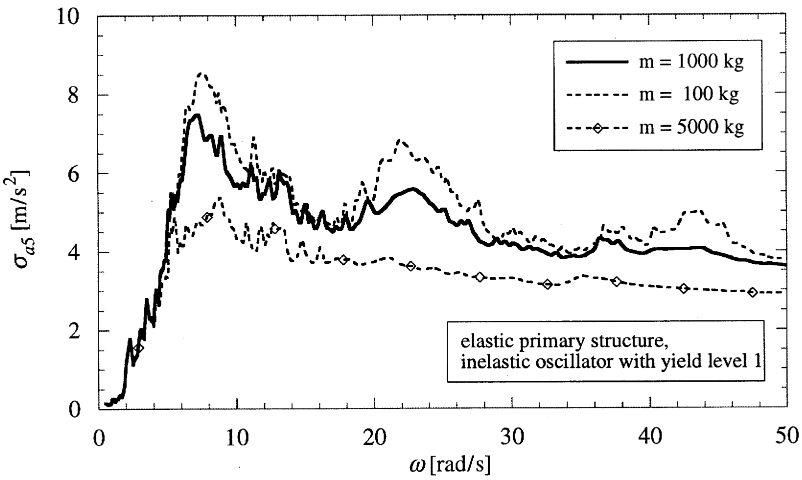

Figure 8. Acceleration floor response spectra; standard deviation; elastic primary structure and inelastic secondary structure; 3 different oscillator masses; coupled analysis.

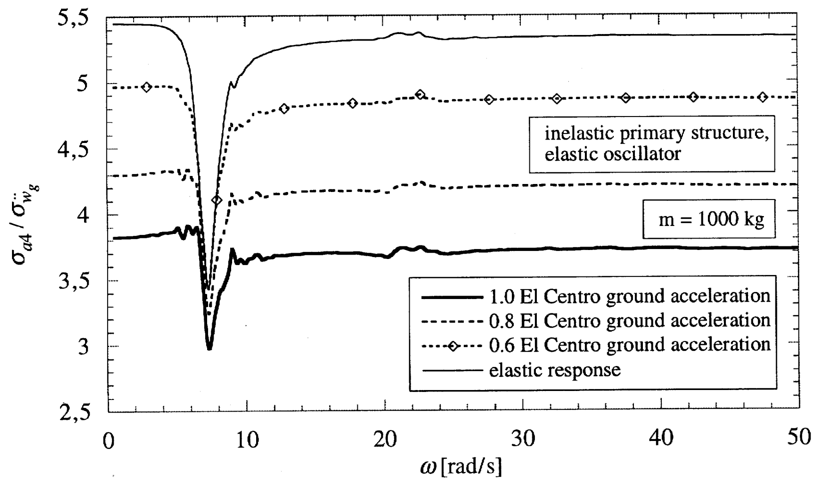

Figure 9. Acceleration primary response spectra of the fourth floor; standard deviation; inelastic primary structure and elastic secondary structure; different excitation levels; coupled analysis.

Unlike the response spectrum which indicates only the peak response, the standard deviation includes information of the entire time evolution. Two response histories with about the same peak values and, hence, similar response spectra, may nevertheless be qualitatively different. Then, this difference will be indicated by σ_a. In all the examples presented in this paper, T is taken as T = 35 s, which is the duration of the El Centro earthquake sample. *Figure* 3 displays the standard deviations σ_{a4} of the fourth storey acceleration over frequency ω. These spectra reveal the same tendency as S_{a4} in *Figure* 2, except the combination of an inelastic primary structure and an elastic SDOF oscillator, where the difference between coupled and decoupled response is amplified when $\omega \approx \Omega_1$. *Figure* 4 shows floor response spectra S_{a5} of the top floor, that is, the peak acceleration of the SDOF oscillator, which is given the number 5. Only coupled solutions are considered in this picture. Between $\omega = 0$ and $\omega \approx \Omega_1$ all peak responses nearly coincide. At $\omega \approx \Omega_1$ there is a significant amplification of the elastic oscillator response, resulting in a spike in the floor response spectra. The magnitude and width of this spike depends on the elastic and inelastic properties of the substructures. In case of inelastic oscillators the peak acceleration of the oscillator is nearly constant over the whole range of frequency.

In order to demonstrate the effect of coupling terms on the oscillator response, *Figure* 5 compares the coupled floor response spectra with results from a decoupled analysis of the inelastic primary structure. While the decoupled results are quite accurate at detuned frequencies, there is a substantial overestimation of the peak response at tuned frequencies. No such an overestimation occurs, if an inelastic SDOF oscillator is considered, see *Figure* 6 (yield level 1). Throughout the entire frequency range the coupled and decoupled results are nearly identical. It is emphasized, that this is not true for the displacements of the primary structure, where significant differences between the coupled and decoupled response spectra can be observed, see *Figures* 2, 3 with respect to response spectra.

Next, the effect of different oscillator masses on primary-secondary structure interaction is examined. This is done by comparing response spectra for the following values of oscillator masses: m = 5000 *kg*, m = 1000 *kg*, m = 100 *kg*. Acceleration spectra σ_{a5} are displayed in *Figures* 7 and 8. As the m is increased, the response of the SDOF oscillator is reduced. The latter effect becomes especially dominant around the tuning frequencies for an elastic oscillator, restricted, however, to a rather narrow band. From *Figure* 8 it can be concluded that the inelastic secondary substructure response is not much effected by different mass ratios.

Finally, the structural response is studied at different strong-motion earthquake excitation levels. The acceleration of the El Centro earthquake sample is reduced to 60 % and 80 %, respectively, of its original amount. Again, an inelastic primary-elastic secondary structure as well an elastic primary-inelastic secondary structure are considered. An oscillator mass of m = 1000 *kg* is taken. Normalized acceleration spectra σ_{a4} and σ_{a5} are displayed in *Figures* 9 - 11, whereby $\sigma_{\ddot{w}_g}$ denotes the standard deviation of the ground acceleration. From *Figures* 9 it is evident that the normalized inelastic primary response increases when the magnitude of excitation decreases, whereas the corresponding elastic oscillator acceleration (*Figure* 10) is quite unaffected from the excitation level. Contrary, between the elastic primary acceleration spectra (inelastic oscillator) as shown in *Figure* 11 no significant differences can be observed for different magnitudes of the ground acceleration.

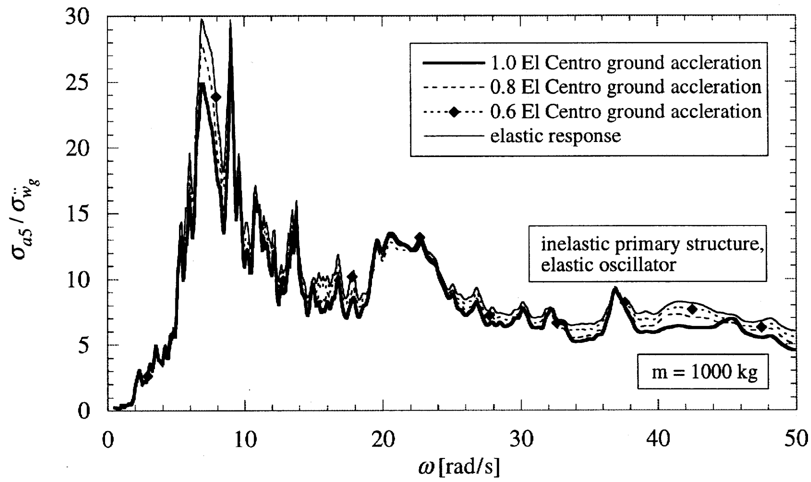

Figure 10. Acceleration floor response spectra; standard deviation; inelastic primary structure and elastic secondary structure; different excitation levels; coupled analysis.

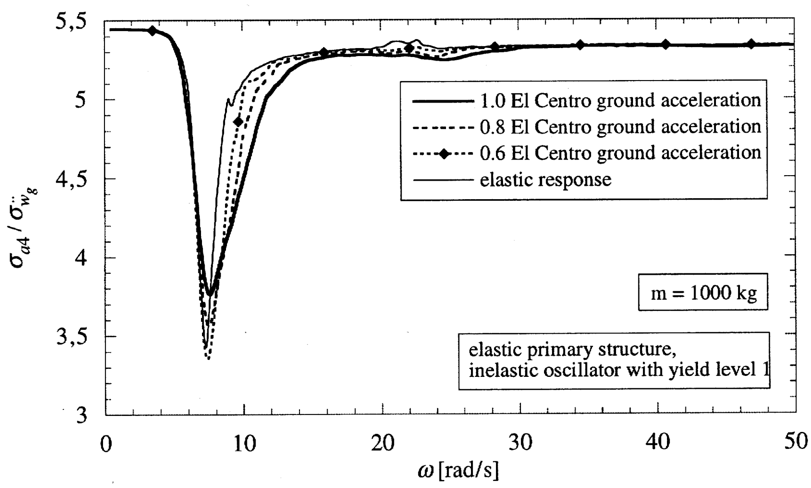

Figure 11. Acceleration primary response spectra of the fourth floor; standard deviation; elastic primary structure and inelastic secondary structure; different excitation levels; coupled analysis.

CONCLUSIONS

In a parametric study examples of SDOF oscillators attached to a four storey frame building are treated. As seismic input the north-south component of the El Centro earthquake sample is used. Various examples demonstrate the importance of effects such as tuning and interaction for elastic as well as inelastic systems. Especially, when an inelastic SDOF oscillator with a low yield level is considered there is an impact on the primary response throughout a large range of frequencies. In order to demonstrate the effect of coupling terms on the oscillator response results from coupled and decoupled analyses of the inelastic primary structure are compared. While the decoupled results are quite accurate at detuned frequencies, there is a substantial overestimation of the peak response at the first tuned frequency and only a slight deviation is noticed at the second tuned frequency. The large differences between coupled and decoupled solutions are effectively demonstrated. No such an overestimation occurs, if an inelastic SDOF oscillator is considered. Throughout the entire frequency range coupled and decoupled results are nearly identical. It is emphasized, that this is not true for the acceleration of the primary structure, where significant differences between the coupled and decoupled response spectra can be observed. The sensitivity of the primary structure response to large mass ratios even within the detuned frequency domain is indicated. Moreover, it can be concluded that the inelastic secondary substructure response is not much effected by different mass ratios.

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