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THE DIRECT DISPLACEMENT BASED DESIGN METHOD: A DAMPING PERSPECTIVE

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SUMMARY

The direct displacement based design (DDBD) method, a performance based design approach, has attracted research due to its direct relationship with structural deformations. This paper is a review of the damping theory, on which the DDBD is founded. The equivalent viscous damping and the substitute viscous damping concepts are investigated. Typical values of these forms of damping that may be found in structures that exhibit different hysteretic behaviour and subject to a range of earthquake ground motions are presented. The implications of using these damping concepts in the DDBD approach are discussed and it is concluded that substitute damping is the most appropriate concept for use in the DDBD methodology.

INTRODUCTION

In the last decade there has been considerable research into performance based seismic design methods. Since performance is best gauged through damage, and damage is generically expressed by deformations, it follows that displacements became the main focus of these design philosophies. Understandably, these displacement focused design approaches have their own specific assumptions and rules. These methods are being proposed as a replacement or an alternative to the current force based design approach as they deal more directly with displacements, and hence material strains.

The direct displacement based design (DDBD) [Kowalsky, Priestley and McRae 1994] is one of the performance based design approaches. It has attracted considerable research into its practical advantages and shortcomings. However, less effort has been put into the study of its theoretical background and in particular the concept of equivalent viscous damping that constitutes one of its basic founding assumptions. There is a clear need for this.

Both the current force based seismic design (FBD) and the proposed displacement based methods are founded on the premise that a structure responding in-elastically in an earthquake can be analysed as an associated elastic model. The results of these elastic analyses can be projected back to the yielding structure, which is referred to as the design structure in this paper. This is justified by the complexity of non-linear structural analysis and the requirement for convenience and expediency in design codes. Hence the current force based design approach, for instance assumes that the associated elastic model, which is used for the analysis, has a stiffness equal to the initial stiffness of the design structure. Ultimate deflections are predicted by scaling the elastic values using the equal energy and/or equal displacement hypotheses, which are rules, based on observations of structural response of linear and non-linear systems to earthquake ground motions [Velestos and Newmark 1960].

Direct displacement based design [Kowalsky, Priestley and McRae 1994], uses an associated elastically responding model, which has an elastic stiffness based on a secant value at the point of maximum displacement Fig. (1). Thus for an elastic perfectly plastic design structure with a ductility of 3, the stiffness of the associated elastic model used in the analysis is 1/3rd of the initial stiffness of the design structure. The viscous damping of the associated elastic model is increased to compensate for the hysteretic energy dissipated by the design structure. Rules have been developed to establish the damping value so that the associated elastic model sustains the maximum displacement of the design structure. Damping is also the parameter which enables different modes of hysteretic behaviour to be recognised, such as bi-linear or the multitude of stiffness degrading models. This differentiation is a claimed advantage over the current practice with force based design, where no such

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recognition is made. Hence selecting the correct value of equivalent viscous damping is of key importance in the direct displacement based design method.

This paper deals with the issue of damping as used in the DDBD method and the two different methods of assessing the appropriate value that have been proposed in the literature, namely the equivalent viscous damping and the substitute viscous damping concepts.

SUBSTITUTE VISCOUS DAMPING

Gulkan and Sozen [1974] conducted a series of dynamic tests which involved in-elastic displacements, on one storey, one-bay reinforced concrete frames. These frames were detailed to behave in a ductile manner with plastic hinges forming in the columns. With this form of plastic hinge design the Takeda hysteretic rules provide a good description of the force deflection model of the test units.

From their test results and analytical studies they proposed that the earthquake energy input into a ductile structure is similar to that dissipated by an associated elastic system with substitute viscous damping. The substitute viscous damping coefficient was defined by equating the earthquake input energy, which is on the right hand side of equation (1), to the viscous energy dissipated, giving

$$c \int_0^t \dot{v}^2(\tau)\, d\tau \;=\; -m \int_0^t \ddot{v}_g(\tau)\, \dot{v}(\tau)\, d\tau \qquad (1)$$

where, c is the substitute viscous damping constant, v is the displacement of the structure (the dot refers to the differentiation with respect to time), $\ddot{v}_g$ is the ground acceleration, t is the duration of the excitation and τ is a time variable. Gulkan [1974] assumed that the relative velocity of the associated elastic structure is the same as the ductile structure. Equation (1) can be rearranged to

$$\xi_{substitute} \;=\; \frac{T_{substitute} \int_0^t \ddot{v}_g(\tau)\, \dot{v}(\tau)\, d\tau}{4\,\pi \int_0^t \dot{v}^2(\tau)\, d\tau} \qquad (2)$$

where $\xi_{substitute}$ and $T_{substitute}$ are respectively the substitute viscous damping coefficient and the substitute period of the associated model. Gulkan and Sozen [1974] assumed that the substitute frequency $\omega^2_{substitute}$ can be taken as the ratio of the measured maximum absolute acceleration to absolute measured maximum displacement. Furthermore they used the stiffness of the associated elastic model with this frequency as the secant joining the origin to the maximum excursion of the in-elastic response of the design structure. This is done to model the frequency shift due to the softening of the design structure. The technique of using the secant stiffness is characterised by its simplicity as it is based on the abstract geometry of the hysteretic loop. This approach was later adopted in the DDBD procedure.

Gulkan and Sozen proposed that a simpler and a more convenient expression could represent the substitute damping concept, namely equivalent damping first proposed by Jacobsen [1930]. They applied this to a force based design procedure, which had a clear focus on deformations. In this approach, the designer set a deformation limit, expressed by a pre-selected ductility level in a very similar manner to the current FBD design practice. The process was intended to result in design structures that would not undergo displacement demands that are in excess of a predetermined limit during the earthquake excitation. This process was a non-iterative procedure. It was expanded later by Shibata and Sozen [1976] who incorporated procedures to cover multi-degree of freedom structures. It was referred to as the “Substitute Structure Method”. Gulkan and Sozen conducted a series of trial designs using their approach and found the results were in satisfactory agreement with their design targets.

The DDBD method [Kowalsky, Priestley and McRae 1994] uses the equivalent damping concept and the secant stiffness for the associated elastic model, as initially proposed by Gulkan and Sozen.

EQUIVALENT VISCOUS DAMPING

Jacobsen [1930] proposed the theory of equivalent damping as a tool in the study of "*steady forced vibration of damped systems of one degree of freedom when under the influence of sinusoidally varying disturbing forces*". He proposed that alternative oscillators (analogous to associated elastic models) could have viscous damping as an attenuating force to replace systems with complex damping mechanisms. From the theory's perspective, the original system with complex damping (which might include in-elastic hysteretic response) and the elastically responding, viscously damped system (associated elastic model) were both in a steady state excitation sustaining the same levels of deformation. Thus Jacobsen's equivalent damping is concerned with equating the energies of the two oscillators in a specific steady state cycle, while substitute damping relates to the equal dissipation of energy of the design and the associated elastically responding structures over an earthquake record.

Jacobsen assumed that a vibrating system influenced by a damping force that is proportional to the n^{th} power of the velocity as described by equation 3, could be replaced by the same system with viscous damping, as described in equation 4.

$$m \frac{d^2v}{dt^2} \pm c_n \left(\frac{dv}{dt}\right)^n + k v = p_o \sin \omega t \qquad (3)$$

$$m \frac{d^2v}{dt^2} \pm c_1 \frac{dv}{dt} + k v = p_o \sin \omega t \qquad (4)$$

The displacements, v, of the two systems are the same. These systems are assumed to be equivalent when the work dissipated by the two oscillators in a steady state cycle is the same. Equating these two values leads to equation 5.

$$Work = 4\, c_n \int_0^{\frac{\pi}{2\omega}} \left(\frac{dv}{dt}\right)^n dv = 4\, c_1 \int_0^{\frac{\pi}{2\omega}} \left(\frac{dv}{dt}\right) dv \qquad (5)$$

Jacobsen conducted studies on mechanical systems under forced steady state vibration. He found his theory was in close agreement with the exact solution of Den Hartog [Jacobsen 1930, Jacobsen and Ayre 1958]. He pointed out that his theory led to appreciable error for highly non-linear systems (that is high ductility) [Jacobsen 1960].

Gulkan and Sozen found that Jacobsen's equivalent viscous damping could be related to substitute damping with sufficient accuracy for practical purposes in earthquake engineering. They found from their experimental work, in which the test structures behaved in a manner reasonably obeying the Takeda hysteretic model, that the two damping concepts gave similar damping values [Gulkan and Sozen, 1974].

Prior to Gulkan and Sozen, Hudson [1965] investigated the substitute and equivalent damping concepts for the case of bi-linear systems. He found, in contrast to the later propositions of Gulkan and Sozen, that these two concepts were not inter-changeable for bi-linear oscillators. In fact, his analyses indicated that for a specific ductility level, the substitute damping levels were approximately 1/3rd of the counterpart equivalent damping levels. This divergence in values he attributed to the fact that the maximum excursion, from which the equivalent damping value is calculated, is a once-in-an-earthquake incident, while substitute damping represents the 'average' energy dissipation for the total earthquake record. As most excursions during the seismic record are appreciably less than the maximum excursion, this discrepancy was understandable. Hudson also noted that the substitute damping varies slightly with the ductility level. His results differ significantly from Gulkan and Sozen's as their work was based primarily on systems exhibiting Takeda type hysteretic behaviour while his related to bi-linear hysteretic behaviour.

DDBD DESIGN

In this section the DDBD procedure is reviewed for single degree of freedom structures to highlight the rationale behind the approach. The basic steps are set out below [Kowalsky, Priestley and McRae 1994].

Selection of a suitable displacement design spectrum.

As the method has a prime focus on deformations, displacement spectra are used in contrast with acceleration spectra in the conventional FBD method. Since the viscous damping for the associated elastic model varies with the hysteretic form and the ductility of the yielding design structure, a series of spectra are required with differing damping levels.

Selection of an acceptable maximum displacement.

This displacement limit is derived from either acceptable material strains or drift ratios.

Selection of a suitable damping relationship.

With DDBD, the viscous damping level of the associated elastically responding model is chosen as a function of the maximum displacement ductility and the hysteretic form of response. Different relationships have been proposed for these functions. Loeding et al. [Loeding, Kowalsky and Priestley 1998] proposed two different functions for design purposes, one for reinforced concrete beams and the other for reinforced concrete columns. These are similar to the design values suggested by Priestley which are shown in Fig. (2) for use in the proposed New Zealand/Australian loadings code. These exhibit damping values that are independent of ductility at design ductilities greater than 3

4. With a preliminary ductility level derived from the pre-selected ultimate displacement of Step 2, and an assumed yield displacement, a damping value, following Step 3, can be found.

5. From the damping level and the ultimate displacement, a period for the associated elastic model can be read off from the displacement spectra. This period corresponds to the secant stiffness of the hysteretic loop as shown in Fig. (1).

Knowing the equivalent secant stiffness and the maximum displacement of the associated model, a value for its base shear can be determined. This value is the same as for the maximum base shear sustained by the design structure. Hence the yield force of the design structure can be found, see Fig. (1).

The design structure is proportioned and an effective cracked stiffness can be determined. This value can now be used together with the yield force to produce a more accurate assessment of the yield displacement. This enables an improved estimate of the ductility to be made, which allows a further iterative cycle to be made starting at Step 4. This process is followed until convergence is achieved.

ANALYSIS AND DISCUSSION

Substitute damping values were evaluated using Eq. (2) for a series of oscillators with different ductility levels and compared with the corresponding values of equivalent damping. This process was conducted using four earthquake records; El Centro NS 1940, Taft N21E 1952, Matahina Dam Base 1987, Hachinohe NS 1968 and four artificial earthquakes. The artificial earthquakes were developed by modifying the initial four earthquakes so that their 5% spectra matched the New Zealand loadings code spectrum for intermediate soils. Three sets of analyses were made.

The first set used the original ground motions for design structures with an elasto-plastic hysteretic response and a 5% base level viscous damping.

The second set used the artificial ground motion records that had been normalised to the New Zealand loadings code. Design structures behaved in an elastoplastic behaviour with 5% base level viscous damping.

The final set used the artificial records for design structures with a stiffness degrading hysteretic model developed by Davidson and Fenwick [1995] to represent the behaviour of reinforced concrete columns with 5% base level viscous damping.

The principal observations from these analyses are given in the following paragraphs.

Fig. (3) illustrates typical substitute damping values for an elasto-plastic system as calculated for the normalised El Centro record. In this plot are displayed the calculated values of substitute damping and the smoothed 'trend' line for ductilities 2, 4 and 6 behaviour. Similar results have been obtained for the other earthquake records [Judi 1999], and for a specific hysteretic form they show a significant dependence on ductility and only a slight variation with period. As a consequence, the mean values of damping for each of the eight earthquake records are able to be grouped for the period ranges; T<1sec and 1<T<4sec and these are presented in Table 1. For each of the chosen ductility:period groups, it can be observed that the variation in the evaluated substitute damping values is small between the different earthquake records. This observation is reinforced by the low values of coefficient of variation (COV). A similar set of results has been obtained for stiffness degrading systems and a summary set of substitute damping values is presented in Table 2. for the column model.

In Fig. (2) three sets of damping versus ductility values are plotted for elasto-plastic and stiffness degrading column systems. One set of values are those recommended by Priestley, the second have been calculated using the equivalent damping formulation, and the third are substitute damping values evaluated using mean data from the eight earthquake records. The substitute damping relationships were established using the mean values listed in Tables 1 and 2. From Fig.2 it can be seen that the equivalent damping values for elasto-plastic systems are approximately double those for the column model with degrading stiffness which is in contrast with the results presented from the other two methods. It was found that for the data range that they were evaluated, the substitute damping values could be adequately represented by a straight line over the ductility range 2 to 6. These values are similar to the Priestley values in the ductility range 3 to 4 but they are approximately 50% larger at ductility 6.

To investigate the suitability of the presented substitute damping/ductility relationship for DDBD, a large number of single degree of freedom columns were designed then analysed to assess their ductility demands. The design procedure followed Kowalsky, Priestley and McRae's proposed procedure [1994] and the details are explained fully in Judi [1999]. The columns all supported a mass of 500 tonnes, varied in height from 2.50m to 20.00m, had initial periods in the range 0.2 to 4.0 seconds and were designed for both elasto-plastic and stiffness degrading behaviour.

The displacement spectra used for the designs were developed using the method described by Priestley [1997] and based upon the ductility one seismic hazard spectrum for intermediate soils in the New Zealand loadings code. These are shown in Fig. (4). The results of analyses of some of the designs using the normalised version of the El Centro ground motion are presented in Figs. (5), (6), (7) and (8). In these figures, the evaluated ductility demands are plotted against the design ductility, and the straight line represents the "ideal", where ductility demand equals the design value. From Fig. (5) it can be observed that the equivalent damping relationship for elasto-plastic systems provides largely nonconservative designs, whereas this form of damping proves to be adequate for the design of stiffness degrading systems as seen in Fig. (6). The results plotted in Figs. (6) and (8) show that designs based upon Priestley's damping recommendations and substitute damping values perform equally well. A summary set of results for designs analysed with other earthquake records are in agreement with the plotted results and are presented in Tables 3 and 4.

CONCLUSIONS

The conclusions listed below can be drawn from this study.

1. For a selected hysteretic form, substitute damping values are highly dependent upon the ductility of the systems, and far less on the initial period of the system.
2. For a specific hysteretic form, period range and ductility value, substitute damping values are similar for different earthquake ground motions.
3. Substitute damping values calculated from averages over eight earthquakes are similar to the damping values recommended by Priestley for DDBD.
4. The seismic performance of simple column structures designed using the DDBD method with the three different damping approaches were compared. It was concluded that although equivalent damping may be suitable in the DDBD procedure for the design of stiffness degrading structures, as demonstrated by Kowalsky et al [1994], it is not suitable for the design of elasto-plastic systems. Both elasto-plastic and stiffness degrading structures designed using substitute damping values had demand ductility values similar to their design ductility. Consequently the authors recommend that this form of damping is the most suitable for use in the direct displacement based design procedure.

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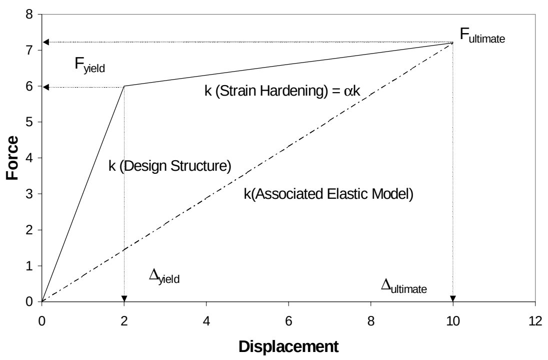

Fig. (1) The Design Structure & The Associated Elastic Model in DDBD

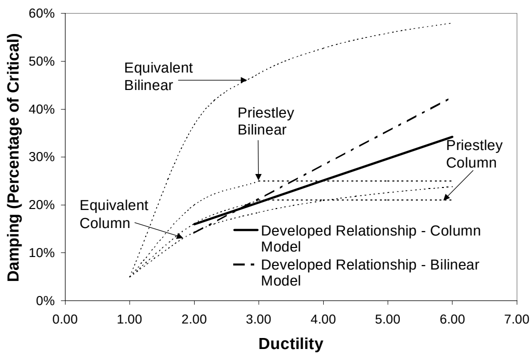

Fig. (2) Damping vs. Ductility

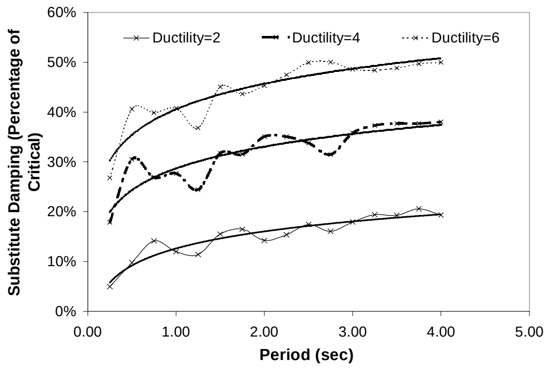

Fig. (3) Substitute Damping Elastoplastic Model – Normalised El Centro

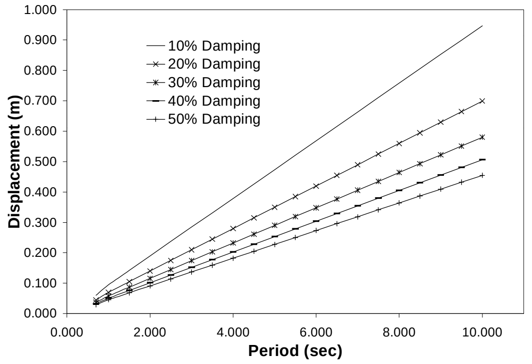

Fig. (4) NZS4203: 1992 Elastic Design Displacement Spectra

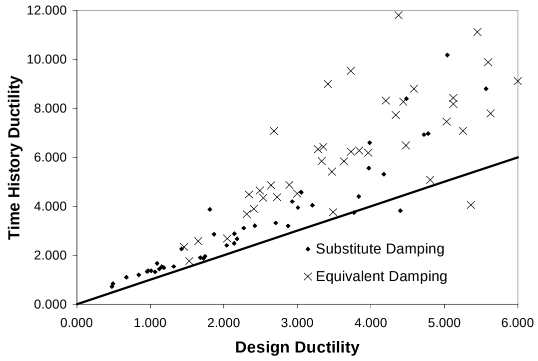

Fig. (5) Comparison of Design Results (Elastoplastic Model)

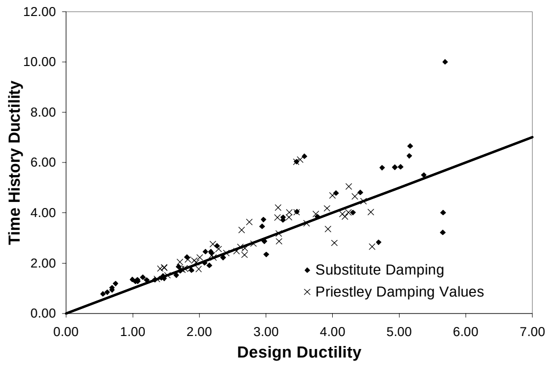

Fig. (6) Comparison of Design Results (Elastoplastic Model)

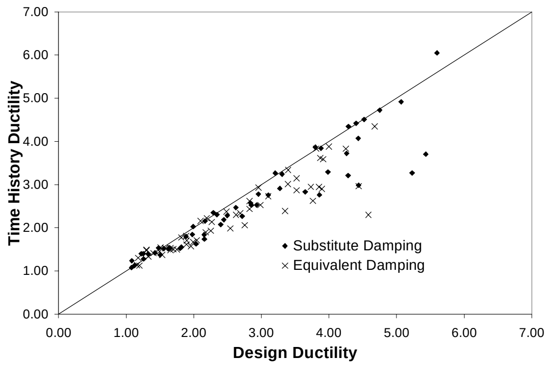

Fig. (7) Comparison of Design Results (Stiffness Degrading Column Model)

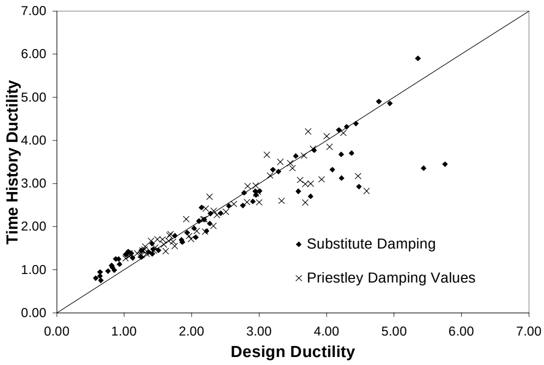

Fig. (8) Comparison of Design Results (Stiffness Degrading Column Model)

Table 1. Substitute Damping (Fraction of Critical) - Mean Values for each earthquake (Elastoplastic Model)

	μ	El Centro		Matahina		Hachinohe		Taft		Mean	COV
		N	O	N	O	N	O	N	O		
T<1sec	2	.102	.113	.099	.105	.100	.082	.095	.098	.099	.089
T<1sec	4	.257	.212	.234	.269	.192	.175	.205	.189	.217	.155
T<1sec	6	.370	.317	.290	.355	.308	.288	.305	.303	.317	.094
1<T<4sec	2	.169	.136	.142	.133	.147	.149	.125	.120	.140	.110
1<T<4sec	4	.341	.249	.319	.279	.299	.303	.267	.244	.288	.118
1<T<4sec	6	.470	.329	.451	.382	.411	.398	.398	.366	.400	.114

N = Normalised ground motion, O = Original ground motion

Table 2. Substitute Damping (Fraction of Critical) - Mean Values for each earthquake (Column Model)

	μ	El Centro		Matahina		Hachinohe		Taft		Mean	COV
		N	O	N	O	N	O	N	O		
T<1sec	2	.147	.141	.128	.131	.125	.112	.145	.138	.133	.088
T<1sec	4	.261	.240	.244	.248	.267	.228	.235	.245	.246	.052
T<1sec	6	.327	.293	.306	.276	.334	.295	.302	.296	.304	.062
1<T<4sec	2	.171	.146	.152	.172	.161	.151	.141	.141	.154	.080
1<T<4sec	4	.291	.253	.263	.278	.270	.238	.266	.245	.263	.066
1<T<4sec	6	.358	.251	.330	.278	.338	.267	.332	.311	.308	.124

N = Normalised ground motion, O = Original ground motion

Table 3 . Ductility Demand/Design Values (Elastoplastic Model)

Normalized Ground Motions		El Centro		Matahina		Hachinohe		Taft	
		E	S	E	S	E	S	E	S
$\mu<4$	Mean	1.740	1.354	1.504	1.358	1.688	1.390	1.769	1.542
	COV	.224	.163	.217	.226	.423	.243	.206	.242
$4<\mu<6$	Mean	1.671	1.505	1.220	1.160	1.493	1.350	1.454	1.332
	COV	.265	.253	.203	.128	.267	.321	.238	.246
Original Ground Motions		El Centro		Matahina		Hachinohe		Taft	
		P	S	P	S	P	S	P	S
$\mu<4$	Mean	1.111	1.169	1.116	1.163	1.220	1.151	1.282	1.232
	COV	.171	.194	.213	.183	.313	.240	.238	.178
$4<\mu<6$	Mean	.932	1.071	.817	.886	.927	.951	.922	.984
	COV	.205	.298	.290	.182	.376	.314	.302	.299

E = Equivalent damping analysis, S = Substitute damping analysis, P = Analysis based on Priestley's damping values.

Table 4 . Ductility Demand/Design Values (Column Model)

Normalized Ground Motions		El Centro		Matahina		Hachinohe		Taft	
		E	S	E	S	E	S	E	S
$\mu<4$	Mean	0.908	0.939	0.869	0.908	0.882	0.925	0.986	1.026
	COV	.110	.102	.176	.158	.164	.150	.177	.167
$4<\mu<6$	Mean	0.750	0.882	0.647	0.710	0.705	0.797	0.801	.858
	COV	.271	.179	.071	.078	.173	.222	.228	.131
Original Ground Motions		El Centro		Matahina		Hachinohe		Taft	
		P	S	P	S	P	S	P	S
$\mu<4$	Mean	1.009	1.072	0.959	1.038	0.956	1.038	1.092	1.131
	COV	.129	.163	.226	.226	.182	.187	.226	.186
$4<\mu<6$	Mean	0.815	0.866	0.625	0.716	0.745	0.772	0.784	0.835
	COV	.222	.197	.141	.077	.216	.202	.287	.153

E = Equivalent damping analysis, S = Substitute damping analysis, P = Analysis based on Priestley's damping values.