EVALUATION OF THE RADIATION DAMPING IN DAM-FOUNDATION SYSTEM

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SUMMARY

The radiation damping of foundation in the dynamic analysis of dam - foundation system, have been investigated with a FE-FD hybrid analytic method which has a excellent energy absorbing function at the artificial FE boundaries. Examination of frequency response functions of acceleration shows that the energy embarrassed in the vibrating system, when radiation damping is not considered, decreases with the increment of foundation stiffness and increases with the increment of foundation density. An estimation method of the equivalent radiation damping factor has been given, with which the dynamic responses of dams could be predicated with considerable accuracy.

INTRODUCTION

It is well known that in the dynamic analysis of finite element method, radiation damping has remarkable effects on the responses of dam – foundation system. In order to eliminate the effect of energy reflection at FE boundaries, radiation damping is usually taken into account by introducing a damping parameter as that of the material damping. However, it is not clear that how to determine such parameter, what relationship exist between this parameter and the material properties such as stiffness, density etc.

Recently, for solving the problem in the dynamic analysis of infinite media, viscous boundaries which absorbs energy at boundaries [Watanabe and Cao 1998, Miura and Okinaka 1989], and boundary element method etc.[Wolf and Song 1996] have been developed and applied in numerical analysis of the coupled dam – foundation system. It has been reported that with such methods reasonable solutions have been got. But for practice, especially in the primary phase of dam design, replacing the mentioned methods a simple and applicable way for taking the effect of the radiation damping into account is preferable. Replying to such need, the behavior of the radiation damping in the dam – foundation system is investigated, and a simple predication method for evaluating the effect of radiation damping is presented in the study.

ANALYTIC METHOD

A 3-D dynamic and static analytic program “UNIVERSE” [EPDC 1999]is used in the study. With this program, the energy radiating form the inner part of the analyzed domain can be absorbed fully at the boundaries around the foundation and the effects of the free field vibration on the response of the dam – foundation system can be taken into account at the same time. This coupled system is expressed in the following equation

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$$
\begin{cases}
\begin{bmatrix} M_d & M_{df} \\ M_{fd} & M_f \end{bmatrix} \begin{Bmatrix} \ddot{u}_d \\ \ddot{u}_f \end{Bmatrix} + \begin{bmatrix} C_d & C_{df} \\ C_{fd} & C_f^* \end{bmatrix} \begin{Bmatrix} \dot{u}_d \\ \dot{u}_f \end{Bmatrix} + \\
\qquad \begin{bmatrix} K_d & K_{df} \\ K_{fd} & K_f \end{bmatrix} \begin{Bmatrix} u_d \\ u_f \end{Bmatrix} = \begin{Bmatrix} F_w \\ T_f \end{Bmatrix} & (1.1) \\
[M_g]\{\ddot{u}_g\} + [C_g]\{\dot{u}_g\} + [K_g]\{u_g\} = \{T_b\} & (1.2)
\end{cases}
$$

where Eq. (1.1) demonstrates the motion of the dam - foundation system and Eq. (1.2) expresses the motion of the free field around the foundation. The variables of the equations are:

M, C, K : Mass, damping, stiffness matrix, and the footnotes d, df (fd), f and g mean that the matrix belongs to the dam, the junction of dam and foundation, foundation, and free field respectively.

C_f^* : Damping matrix (including the components of the material damping and that of the viscous boundaries, please refer equation 2)

$u, \ \dot{u}, \ \ddot{u}$: Displacement, velocity and acceleration vectors

F_w : Hydrodynamic pressure acting on the upstream surface of the dam

T_f : Earthquake load and the inflow of energy resulted from the motion of the free field

T_b : Earthquake input of the free field

The detail of the above matrices is explained in the reference [EPDC 1999, and Miura 1989]. The matrix C_f^* of Eq. (1.2) is

$$C_f^* = C_f + C_v \qquad (2)$$

where C_f is the component of the material damping of the foundation and C_v is the damping component of viscous boundaries around the foundation. The viscous boundary matrix C_v is introduced based on the principle of virtual work. It has been proved that in such boundary energy propagating from the inner part of the foundation will be absorbed fully. At same time, the effect of the free field motion to the response of inner domain (dam–foundation) is taken into account with the vector of T_f . Therefore the energy giving and receiving between inner domain and outer domain (free field) is considered in such system.

In order to investigate the energy radiation behavior of the dam – foundation system, the energy confined in the system is examined by comparing the responses of the following two cases. One is that of the viscous boundary and the other is that of no such boundary (please refer Fig.1). In the two cases the energy inflow from the free field is considered, and the earthquake is inputted from the rock surface (basal boundary of the foundation).

The following method is used for investigating the variation of the energy confined in the system. As well known, frequency response function is a kind of expression of the energy existing in the vibrating system. Hence, the comparison of the area (please refer Fig.2) formed by the coordinate axes and the response curve is to be done between the two cases mentioned in the above paragraph. The area ratio μ expressed by the Eq. (3) is taken as a measurement for the comparison. It is thought that a larger value of the μ means that, if radiation damping is not considered, the effect of the energy reflection problem on the response of dam – foundation system is larger.

$$\mu = \frac{S_0}{S_r} = \int_0^F A_0(f)\,df \Big/ \int_0^F A_r(f)\,df \qquad (3)$$

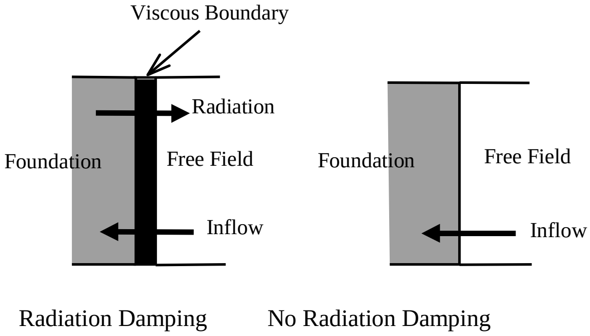


Fig.1 Model for Evaluation of Radiation Damping of Dam – Foundation System

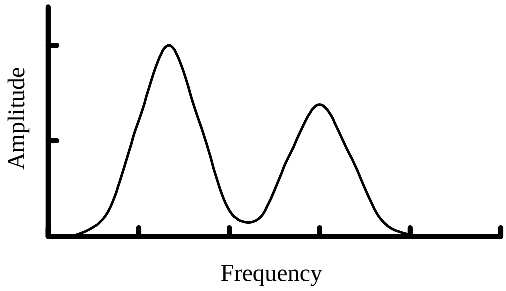


Fig.2 Frequency Response Function is Examined for the Evaluation of Radiation Damping

In Eq.(3), A_r, A_0 is the frequency response function of the case where radiation damping is taken into account and that of the case without this consideration respectively. F is the maximum interested frequency.

It is also known that radiation damping is dependent on natural vibration modes. But for the dynamic analysis in time domain, which mode should be selected as the representative one to give the estimation on the radiation damping is difficult to determine. In the way proposed in the paper, since the evaluation on the radiation damping is based on the total dimension of the frequency response functions, it could be thought that the evaluation result will be independent on any special mode, and convenient to be applied in time domain analysis.

A concrete gravity dam coupled with foundation and reservoir is taken as the object model. The maximum height of the dam is 60 m, and the maximum length of its top is 160 m. Since free field is enclosed in the analytic model, the foundation is modeled in a relatively small domain where the interaction between dam and foundation occurs (Fig.3).

In this model the free field in the stream direction is modeled in the same shape as the valley, and in the direction of dam axis is of the stratiform shape. The reservoir is of a depth 60 m, and a length 200 m. The dam - foundation system is dispersed into 406 elements with total 1132 nodes. The reservoir is dispersed with 312 FD grids.

It is supposed that the dam concrete and foundation rock are of linear elastic properties. The properties of the foundation are adjusted in order to investigate the effect of material properties on the radiation damping behavior. But for all of the cases, the material properties of the dam are fixed at the following values:

Yang's Modulus E=16000 N/mm2

Density ρ=2.4 g/cm3

Poison Ratio ν=0.167

Damping Factor h=5%

The material properties of foundation and free field are adjusted in the following way. When the density is fixed to be 2.4 g/cm3, the Yonng's modulus is adjusted with the values shown in Table 1. And when the Young's modulus is fixed to be 24000 N/mm2, the density is adjusted to be the values shown in Table 2. For all of the cases the Poison ratio is set to be 0.15. In order to eliminate the effect of the material damping on the response of the system the material damping factor of the foundation and free field is supposed to be 0.0%.

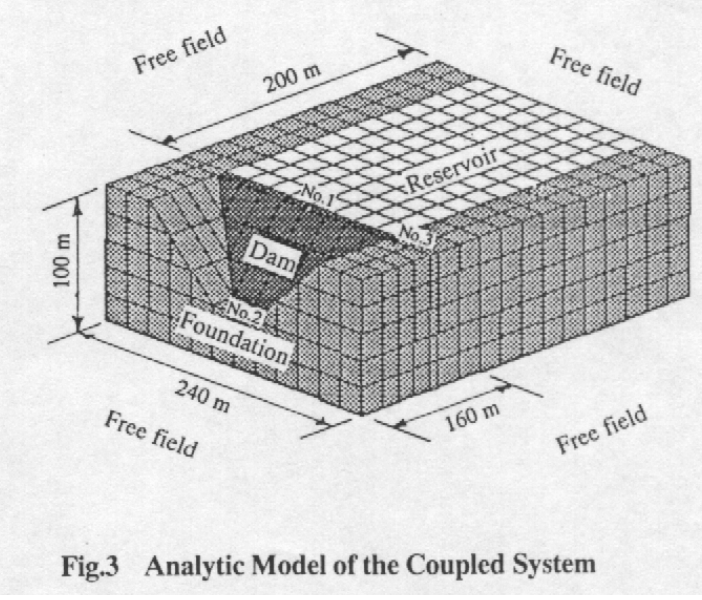


Fig.3 Analytic Model of the Coupled System

Table 1 Shear Modulus of Foundation & Free Field for Investigating the Relationship of Stiffness - Radiation Damping (N/mm^2)

Case1	Case2	Case3	Case4	Case5	Case6	Case7	Case8
8000.0	16000.0	24000.0	32000.0	48000.0	64000.0	80000.0	160000.

Table 2 Density of Foundation & Free Field for Investigating the Relationship of Density - Radiation Damping (g/cm^3)

Case1	Case2	Case3	Case4	Case5	Case6
0.0	1.0	2.0	2.4	3.0	4.0

Gauss Pulse of the maximum amplitude 1 m/s^2 is applied as the earthquake motion. As shown in Fig.4, in the frequency domain until 16 Hz (around the third natural frequency of the model shown if Fig.3), the Fourier spectrum of the wave is almost flat. It is expected that within this frequency domain the response of the system can be analyzed with high accuracy and the result will be independent on frequency. The length of the pulse is 5.12 sec., and the time interval for the time history calculation is 0.01 sec.

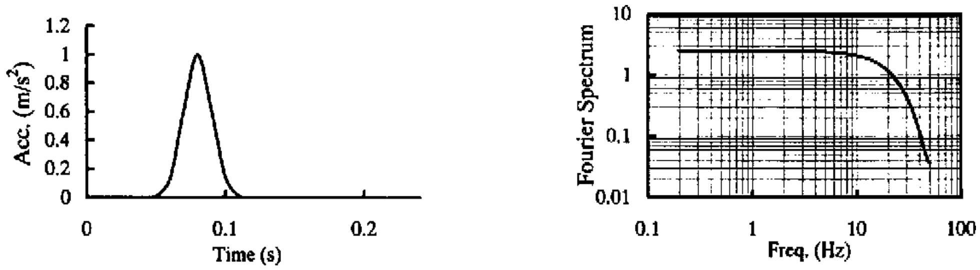


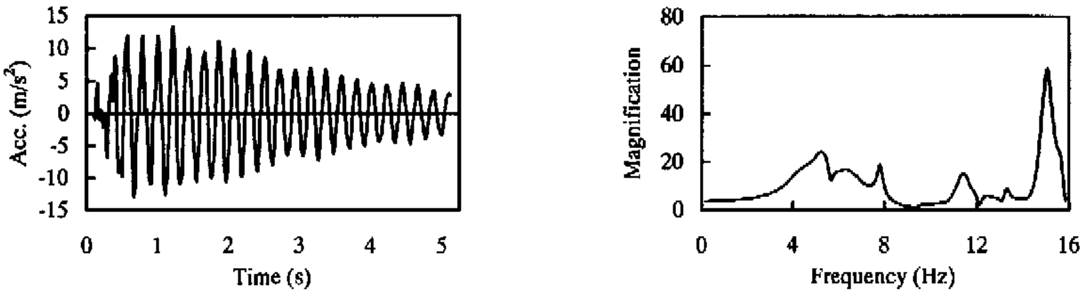
Fig.4 Input Wave (Gauss Pulse)

RELATIONSHIP OF STIFFNESS, DENSITY AND RADIATION DAMPING

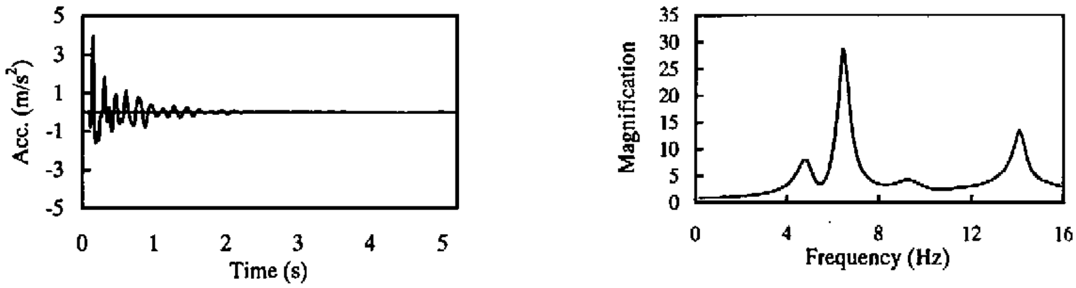
The energy confined in the dam - foundation system will vary with the modification of the stiffness and density of the foundation. This phenomenon is examined at the selected positions (shown in Fig.3). The acceleration responses and the acceleration transfer functions from the dam toe to the top are examined. Fig.5 shows the comparison between the case where radiation damping is considered by applying the viscous boundary condition and the case where the radiation damping is not considered.

It is clear that the responses of the system are absolutely different depending on the condition where the radiation damping is considered or not. The relationships between the ratio μ (S_0/S_r , S_r and S_0 denote the area of the frequency response function when radiation damping is considered and that when this damping is not considered respectively) and the material properties shown in Table 1 and Table 2 are plotted in Fig.6. From Fig.6 followings can be found.

1). As the stiffness of the foundation increases, the energy confined in the system decreases. Except the cases where the foundation stiffness is extremely low, the energy confined in the system (assumed to be Ω') can be expressed as a function of Young's modulus as below.

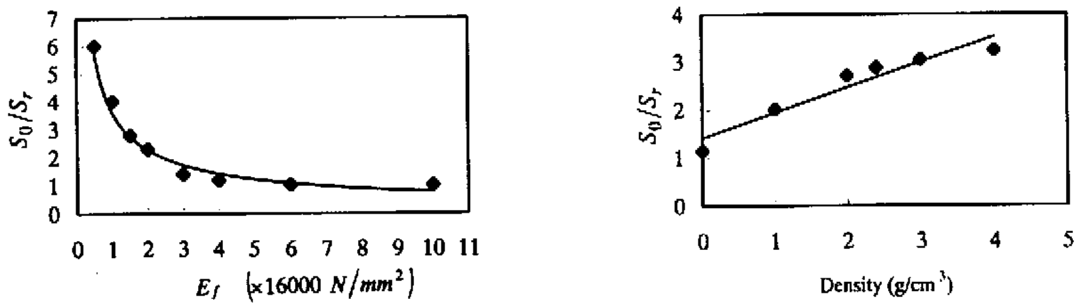


(a) When radiation damping is not considered

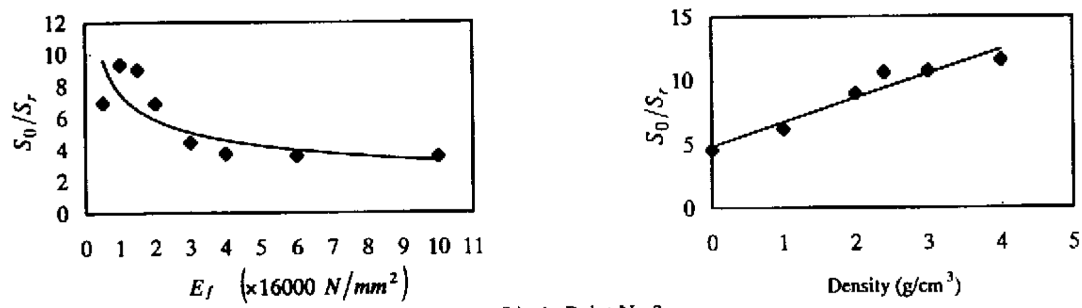


(b) When radiation damping is considered

Fig.5 Comparison of the Dam Responses



(a) At Point No.2



(b) At Point No.3

Fig.6 Relationship of Stiffness - S_0/S_r and That of Density - S_0/S_r

E_f : Young's modulus of foundation

S_0/S_r : (S_r and S_0 are the dimension of the frequency response function when radiation damping is considered and that when this is not considered respectively)

$$\Omega' \propto C_1' \cdot e^{-C_2' \cdot E_f} \tag{4}$$

where C_1', C_2' are the unknown parameters which could been determined by experiment. Eq. (4) means that the energy which should be disembarrassed to the half infinite free field gets less exponentially with the increment of Young's modulus of the foundation.

2). The larger the density of the foundation, the more the energy confined in the system. The relationship between the amount Ω'' and the foundation density ρ can be expressed approximately with a linear form.

$$\Omega'' \propto C_3'' \cdot \rho \tag{5}$$

where C_3'' is an unknown parameter and which also could be determined by experiment .

Synthesizing Eq. (4) and Eq. (5), we can get a general expression on the relationship between the material properties of the foundation and the energy which should been disembarrassed to the half infinite free field in the following form.

$$\Omega \propto \left(\mathrm{C}_1 \cdot e^{-C_2 \cdot E_f} + C_3 \cdot \rho \right) \tag{6}$$

ESTIMATION OF RADIATION DAMPING

In the ordinary dynamic FE analysis where no viscous boundary or other energy absorbing condition is available, a radiation damping factor h_r can be introduced in order to disembarrass the energy Ω through the artificial boundaries. Therefore, h_r should directly proportionate to the Ω expressed in the Eq. (6).

It is thought that h_r should be, except the dependence on the material properties, a function of ground motion intensity, and have an uneven distribution in the direction of depth. But for conciseness, here it is assumed that h_r depends on material properties only, and has a linear relationship with Ω. Then h_r can be expressed as

$$h_r = \alpha_1 \cdot e^{-\alpha_2 \cdot E_f} + \alpha_3 \cdot \rho + \alpha_4 \tag{7}$$

where $\alpha_i \ (i = 1, \cdots, 4)$ are unknown parameters, and they could been determined by experiments.

With the material properties shown in Table 1 and Table 2, comparison analyses are done between the following two conditions. One is that where viscous boundaries are applied, and the other is that where replacing the viscous boundary, the radiation damping factor h_r is introduced and its value is adjusted in order to get equivalent result as that of the former condition. In this way, a series of data concerning the material properties and the factor h_r has been obtained. Based on such data the parameters $\alpha_i \ (i = 1, \cdots, 4)$ of the Eq. (7) have been determined tentatively. Then the Eq. (7) can be rewritten as

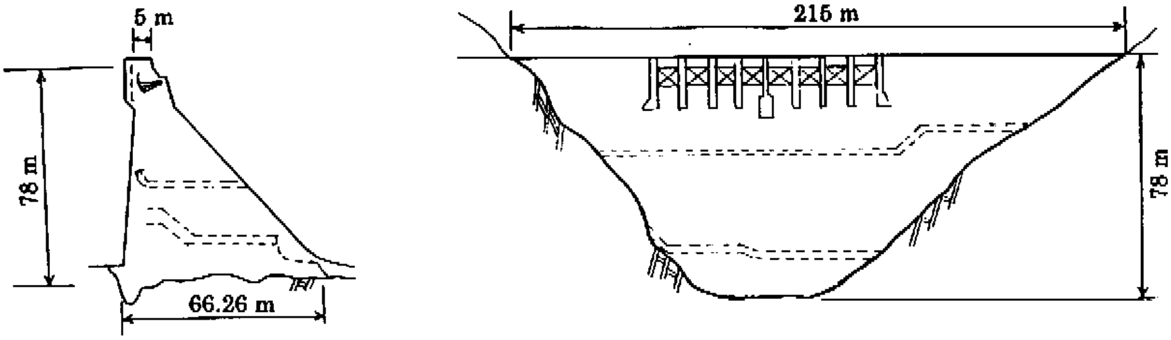
$$h_r = 67.2 e^{-12.0 \times 10^{-5} E_f} + 2.3\rho + 3.8 \tag{8}$$

where the units of E_f and ρ are N / mm^2 and g / cm^3 respectively, and h_r is expressed in percentage.

It should be noticed that Eq. (8) is introduced from the analysis of concrete dams. And hence attention should be paid on its application.

APPLICATION ON THE EARTHQUAKE RESPONSE ANALYSIS OF A REAL DAM

In order to test the validity of Eq. (8) introduced in the above paragraph, it is applied to the analysis of a real concrete gravity dam [Hatano et al. 1958]. It is thought that if Eq. (8) is adequate, the analytic result got with the radiation damping factor h_r should be consistent with that got with the viscous boundary conditions.



(a) Maximum Section (b) Upstream Surfacc

Fig.7 The Maximum Section and Upstream Surface of a Real Dam

The maximum section and the upstream surface of the dam are shown in Fig.7. The sizes of the foundation in the directions of stream, depth and width are separately 1.0, 0.5, 0.5 times of the dam height.

Table 4 Material Properties of Dam and Foundation

	G (*N/mm*^{2})	ρ (*g/cm*^{3})	ν	h
Dam	28000	2.4	0.167	5%
Foundation, Free Field	50000	2.6	0.2	3%

The material properties of the dam and foundation (shown in Table 4) are obtained from the dam site experiment. The earthquake wave of EL-Centro NS is used as the ground motion, but its maximum amplitude is adjusted to be 1 m/s^2. The ground motion lasts 10.24 sec., and the time interval of the history data is 0.01 sec. As mentioned above, two cases are analyzed. One is that of viscous boundary, and the other is that of no viscous boundary but the radiation damping factor determined from Eq. (8) is applied.

The acceleration response and the Fourier spectrum of the selected points (the middle of the crest of the dam and the bottom of the dam) are shown in Fig.8.

By comparing the acceleration response histories, it can be found that the maximum differences are no more than 10% and 5% at the middle of the crest and the bottom of the dam respectively. Generally, the time histories got in the two ways are consistent with each other. On the other hand, the spectrum ratio between the middle of the crest and the dam bottom, and that between the dam bottom and the rock surface are also generally similar to each other between the mentioned two cases.

From above test it has been known that Eq. (8) can give a good predication on the radiation damping factor, and with this factor the dynamic analysis can be done with an acceptable error even though viscous boundary is not applied. As a further research topic, the validity of the presupposition for introducing Eq. (8) should be examined with more sampling analysis and dam site experiments. It is necessary to modify Eq. (8) by applying it to the analyses of more real dams in order to estimate the radiation damping factor, and further more for give a predication on the dam response with higher accuracy.

CONCLUSIONS

With the method proposed in the study, i.e., by examining the variation of the frequency response function, the radiation

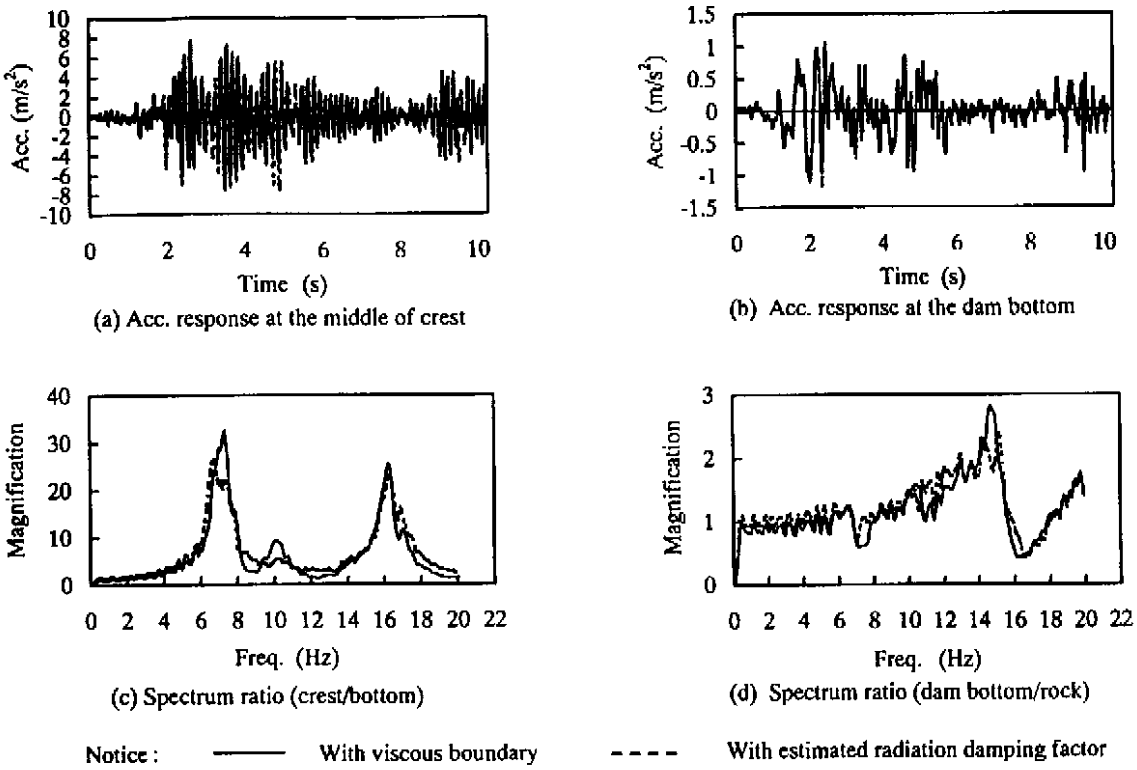


Fig.8 Comparison between the Results Got with Viscous Boundary and Those Got with Radiation Damping Factor

damping behavior of the concrete dam - foundation system has been generally understood. When radiation damping is not considered in a dynamic analysis, the energy confined in the system decreases as the foundation stiffness gets greater. And this amount increases as the density of the foundation gets larger. By introducing a radiation damping factor determined with the proposed Eq. (8) in the study it is possible to get an accurate predication on the earthquake response of the dam - foundation system even in the case where no viscous boundary or other energy disembarrassing condition is available. As a further research topic, the dependence of the radiation damping factor on the ground motion intensity, the radiation behavior in the soft foundation like that of fill dams should be investigated. And it is necessary to modify the predication equation proposed in the study with more data of real dams.

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