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A GLOBAL MODEL FOR AFTERSHOCK BEHAVIOUR

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SUMMARY

This paper considers the distribution of aftershocks in space, abundance, magnitude and time. Investigations to date all show that aftershocks have simple statistical behaviour. A previously established, homogeneous, global earthquake catalogue has been used to build a database of aftershock sequences. The catalogue has been searched for related events by a simple window in space and time. Spatial analysis of this database of aftershock sequences has lead to the definition of aftershock area A (km^2) as a function of mainshock magnitude M:

$$\log_{10} A = M - (3.34 \pm 0.03).$$

The area includes approximately 90% of events that are close in space and time to the mainshock. It may include background seismicity. The number of aftershocks within the model area, the abundance, has been analysed as a function of mainshock magnitude. The frequency distribution of abundance has an exponential behaviour. Work on modelling this behaviour is in progress. The model for abundance of aftershocks will be combined with the established magnitude-frequency distribution of earthquakes to probabilistically predict the number of large earthquakes following a mainshock.

INTRODUCTION

This paper represents the foundation of a PhD project, which seeks to determine the probability of a damaging earthquake following a damaging earthquake. The main question is how aftershocks are distributed in space, abundance, magnitude and time. There is surprisingly little in the literature on this subject. Reasenberg and Jones [1989] studied 62 Californian sequences and established a generic Californian model for the probability of a large earthquake subsequent to a large earthquake. Our study is global, and requires a large homogenous set of aftershock sequences. An ideal earthquake catalogue would report all earthquakes above a certain size, and the magnitudes of all earthquakes would be determined consistently. Christophersen [1999] compared different magnitudes from various international agencies, and defined a 'best magnitude'. We look at the results of this comparison, and how they lead on to establish a global earthquake catalogue. Utsu and Seki [1954] modelled the size of an aftershock area as a function of mainshock magnitude. Their formula has been used to define a search region for each earthquake of magnitude 5 or greater. A refined area versus magnitude relation has been developed. This gives a consistent definition of 'aftershock', and leads on to determining the distribution of numbers of aftershocks ('abundance') as a function of mainshock magnitude. The magnitude-frequency distribution of earthquakes is well known to follow a power law, the Gutenberg-Richter relation. The aftershock decay in time can be described by another power law, Omori's law [e.g. Utsu, 1995]. At this stage the time analysis of the global set of aftershock sequences is yet to be done.

A 'BEST MAGNITUDE' SCALE

Magnitude and seismic moment are the common measures of earthquake size. The seismic moment describes the overall deformation of the source. It is determined from the modelling of earthquake waveforms. Because fault geometry and observer azimuth are taken into account, the seismic moment is the most consistent measure of earthquake size. The Harvard group has regularly reported moments for moderate to large earthquakes since

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1977 [e.g. Dziewonsky et al., 1998]. Hanks and Kanamori [1979] related moment to magnitude. Christophersen [1999] used their formula to calculate the moment magnitude M_W from the Harvard seismic moment and compared it with other magnitudes. Magnitudes are generally determined from the amplitude of seismic waves at a particular frequency, corrected for the distance between source and recorder. The British International Earthquake Information Centre (ISC) and the US National Earthquake Information Center (NEIC) report body and surface wave magnitude, m_b and M_S, in their global catalogues. As ISC and NEIC use similar procedures to determine m_b and M_S, their magnitude reports vary insignificantly.

The body wave magnitude m_b is derived from short period instruments of the world-wide standardised seismic network [WWSSN]. It has never been calibrated against any other magnitude, and underestimates the event size. However, data truncation, neglect of station amplitude terms, and errors in the amplitude-distance correction factor, bias m_b towards a bigger magnitude. For large earthquakes, m_b is systematically too small as it is determined from a period too small to represent the whole rupture process. Over all sizes of earthquakes, m_b by itself is not suitable for seismological studies.

The surface wave magnitude M_S generally underestimates the earthquake size in comparison to M_W for events below $M_S = 6.1$. It is not completely reported for even larger events. For very large events M_S is too small again because the period at which it is determined does not represent the whole rupture process.

Generally, m_b and M_S tend to be smaller than M_W. However, there is some scatter in the ISC and NEIC data. Choosing the largest of ISC/NEIC m_b or M_S reports corresponds reasonably well with M_W. This leads to the definition of a 'best magnitude' for the combined ISC/NEIC and Harvard data. That is, choose M_W if available, otherwise take the largest ISC/NEIC m_b or M_S report.

EARTHQUAKE CATALOGUE

The ISC provides the most complete global earthquake catalogue. It gathers information from other agencies around the world for approximately two years after an event before calculating its own hypocentre and magnitude. The ISC reports its own estimates as well as other agencies' hypocentre and magnitude determinations, including NEIC and Harvard. The ISC catalogue dates back to the beginning of the WWSSN in the early sixties. The data are published on CD-ROM, the latest comprising the years 1964 to 1995. This study uses the ISC earthquake time and location, m_b and M_S, the NEIC m_b and M_S, a local magnitude M_L (if reported), and for the years 1993 to 1995, the Harvard moment. For the years 1977 to 1992 the moment reports have been taken from Harvard directly [http://tempo.harvard.edu/~smith/CMT-ISC.html]. The body wave magnitude has been calculated consistently since the beginning of the catalogue. The surface wave magnitude M_S has experienced some adjustment following the Zürich recommendations in 1967 [Båth, 1981]. Before 1979, many moderate to large earthquakes have no M_S reported. It is generally possible to extend the 'best magnitude' to the beginning of the catalogue, but it is important to be aware that in the early years magnitudes may underestimate the size of moderate to large events. Figure 1a shows the 1964 Alaskan earthquake sequence. The 'best magnitude' for the mainshock is the NEIC body wave report of magnitude 8.5. A moment magnitude of 9.2 was subsequently calculated [Kanamori, 1977], and added by hand to the catalogue.

In this study, a maximum likelihood method has been developed to find the cut-off magnitude above which the catalogue can be assumed to be complete. For the whole data set this magnitude is 5.0. Over the 32 year period of the catalogue the global seismicity has been very stable with about 120 shallow (depth < 70 km) events per year of magnitude 6 and larger [Christophersen, 1999].

A DATABASE OF AFTERSHOCK SEQUENCES

All earthquakes shallower than 70 km have been sorted into 50 commonly used seismic regions to get smaller, more manageable subsets of data. The seismic regions can be grouped into different tectonic settings for regional comparisons. Each region has been searched with a simple window in time and space to find related events. Utsu and Seki [1954]) analysed the aftershock area *A*, as a function of mainshock magnitude *M*, for 39 Japanese earthquakes and found the relation

$$\log_{10} A = (1.02 \pm 0.08)\, M - (4.01 \pm 0.57)$$

This formula has been used to calculate a generous magnitude-dependent search radius by taking four times the radius of the equivalent circular area. Table 1 displays the search radius for some example magnitude. Each magnitude above the 5.0 qualifies as a potential mainshock. In time, the search window is extended by 30 days after each event of magnitude 5 or larger.

Table 1: The search radius for related events for some example mainshock magnitudes

Mainshock magnitude	5.0	6.0	7.0	8.0	9.0
Search radius (km)	8	25	83	271	877

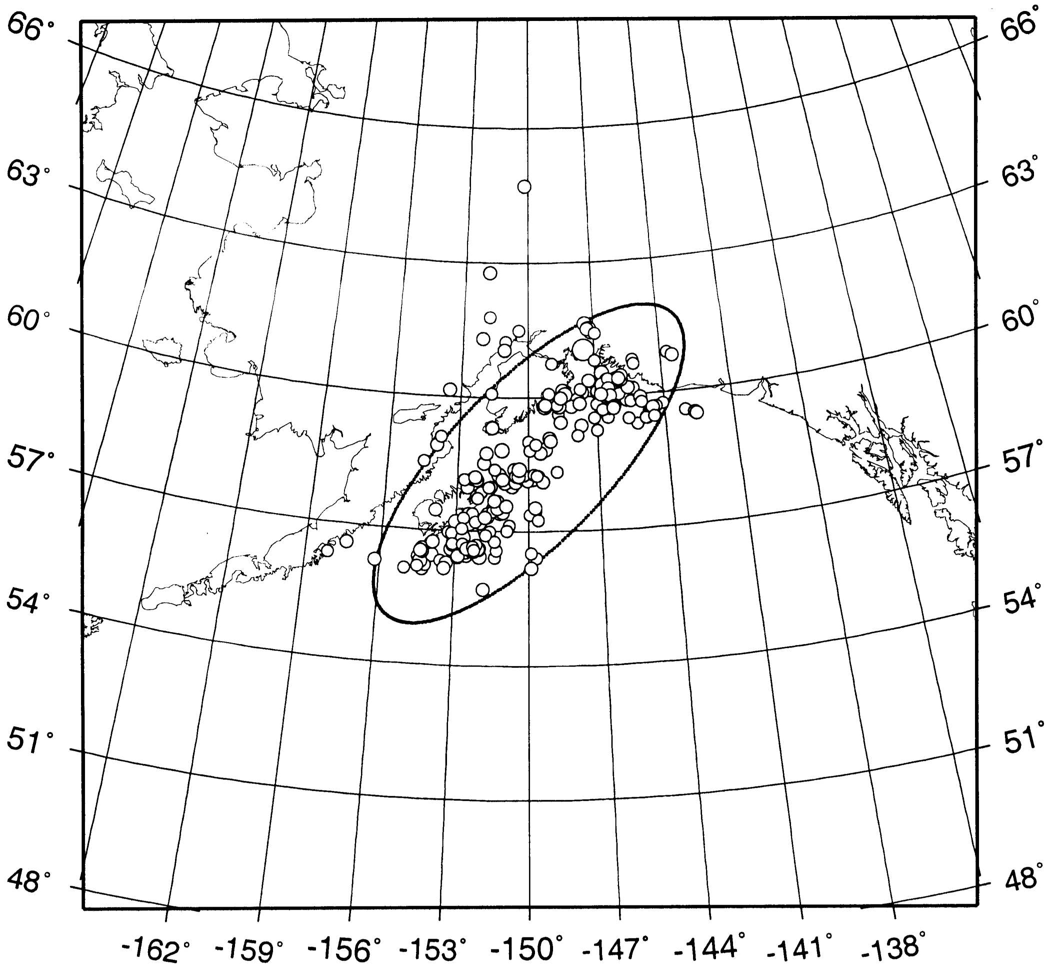


Figure 1: Examples of model ellipses fitted to aftershock sequences. The ellipse contains approximately 90% of the plotted events. Figure 1a: The 1964, M_W = 9.2, Alaskan earthquake and aftershocks. Epicentres of 243 earthquakes larger than magnitude 5.0 within 1103 km of the mainshock and 10 months after the mainshock are plotted.

AFTERSHOCK DISTRIBUTION IN SPACE

The spatial analysis of the first database of aftershocks has resulted in a clear and objective definition for the area of an aftershock sequence. An orthogonal least squares method has been used to find the best fitting elliptical distribution for the epicentres of each aftershock sequence. Figure 1 gives two examples, the 1964 M_W = 9.2 Alaskan earthquake, and the 1987 M_L = 6.3 Edgecumbe, New Zealand earthquake. A minimum of three epicentres is required to fit an ellipse. For a small number of data, the area of the ellipse depends largely on the location of each individual event, and the area is uncertainly determined. For a larger number of epicentres, the area stabilises. There are 85 sequences with 20 or more events, including a total of 4561 earthquakes. Figure 2 shows the cumulative frequency of the squares of the distances of these earthquakes from the centre of the aftershock distribution, scaled by the area. The plotted line is a cumulative chi-square distribution with two degrees of freedom. The difference between the data and the line is significant, and is caused by aftershocks being slightly more densely clustered at the centre of the ellipse than chi-square. However, the model is close enough to the data to use the chi-square 90th percentile to scale the model area. The chi-square 90th percentile equals 4.61. If the squared distance of an epicentre scaled by the aftershock area is equal or smaller than 4.61, the event is counted as part of the sequence, otherwise it is treated as triggered event or background seismicity, and is analysed separately. Figure 3 displays the logarithm of the 90th percentile areas in km^2 versus the mainshock magnitude for all 85 sequences with 20 or more events. Theoretical studies [Kanamori and Anderson, 1975] suggest that the logarithm of the aftershock area scales with the mainshock magnitude. Under this assumption the data fit the model

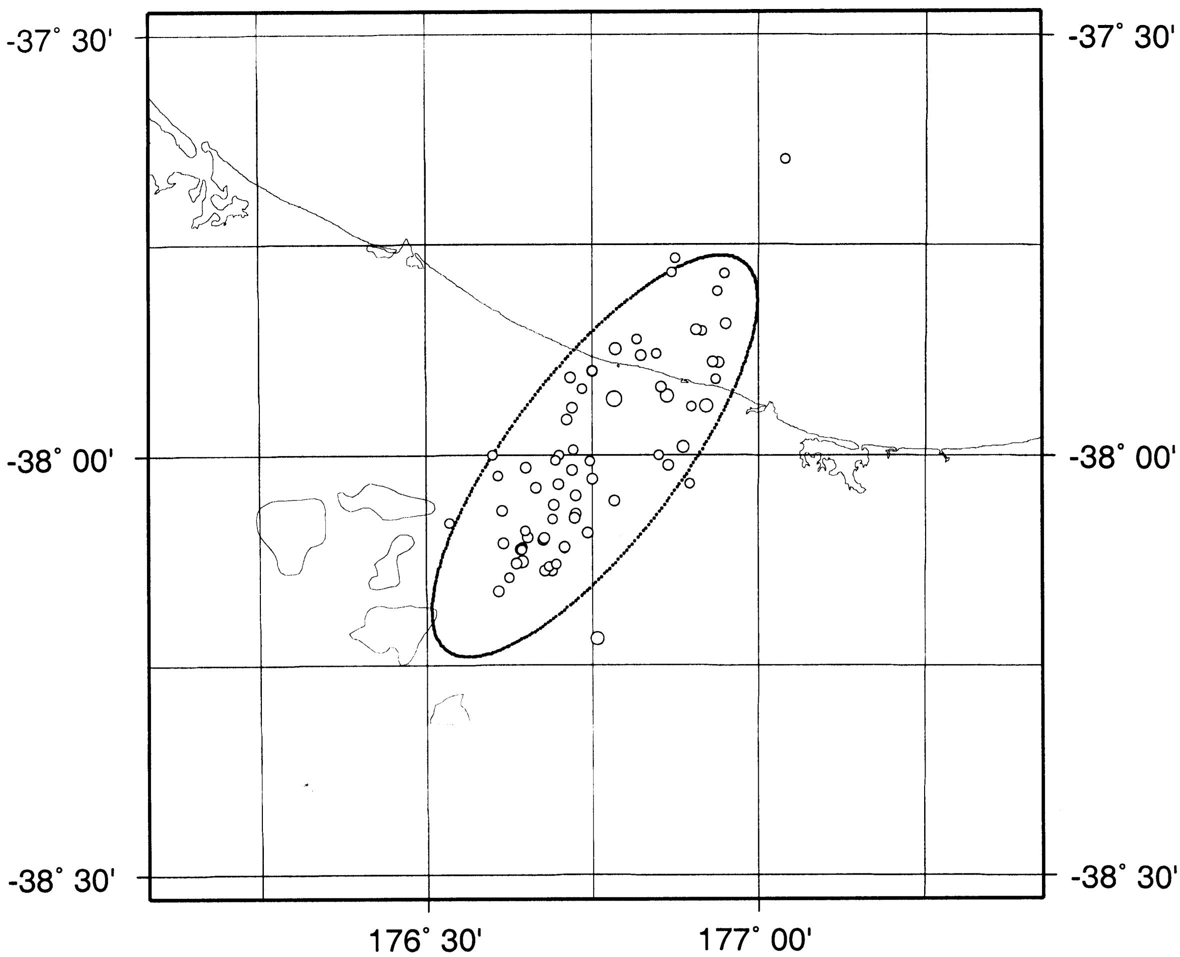


Figure 1b: The 1987, M_L = 6.3, Edgecumbe, New Zealand earthquake. Epicentres of 72 earthquakes larger than M_L = 4.0, within 39 km of the mainshock, and 1 month after the mainshock are plotted. A foreshock that occurred 7 minutes before the mainshock is included.

$\log A = M - (3.34 \pm 0.03)$.

The data scatter about the model with a standard deviation of 0.29. This new model finds aftershock areas to be about four times larger than Utsu and Seki's model. The increase in the aftershock area may be due to the inclusion of a larger percentage of events, or to studying a longer period in time, during which the aftershock area expands. In a later model Utsu [1969] defined aftershock areas by drawing ellipses that included 95% of the events that he identified. He updated his earlier model to

$\log A = M - 3.7$,

which corresponds to about double the area of his old model, and half the area of our newly defined model.

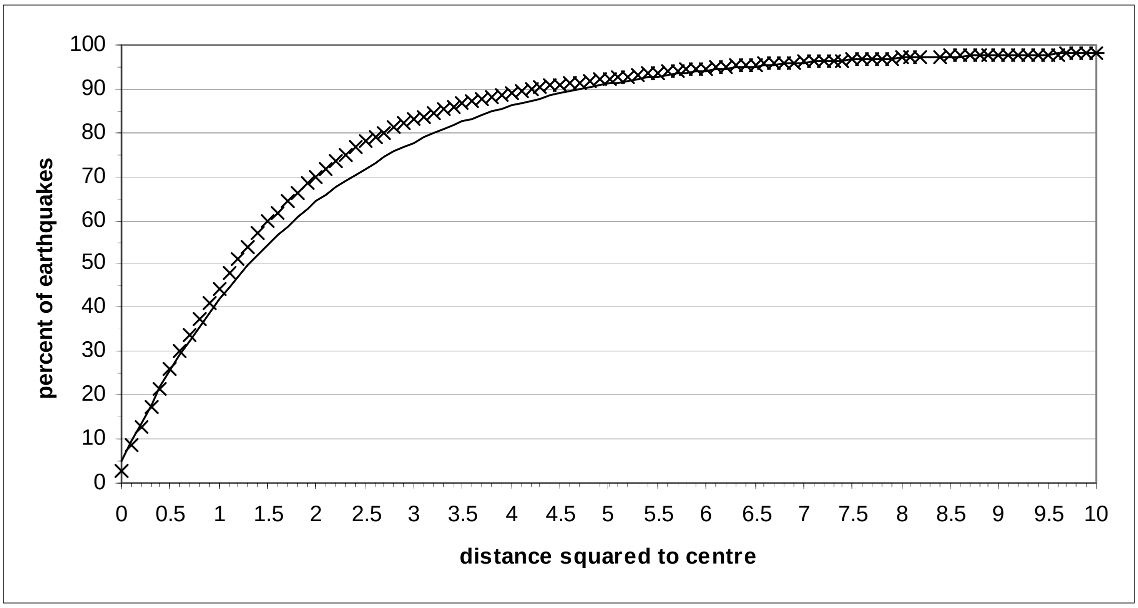


Figure 2: The square of the distance from the centre of the aftershock area, scaled by the size of the aftershock area, for 4561 earthquakes from 85 sequences with 20 or more events (crosses). The line is a chi-square distribution with two degrees of freedom.

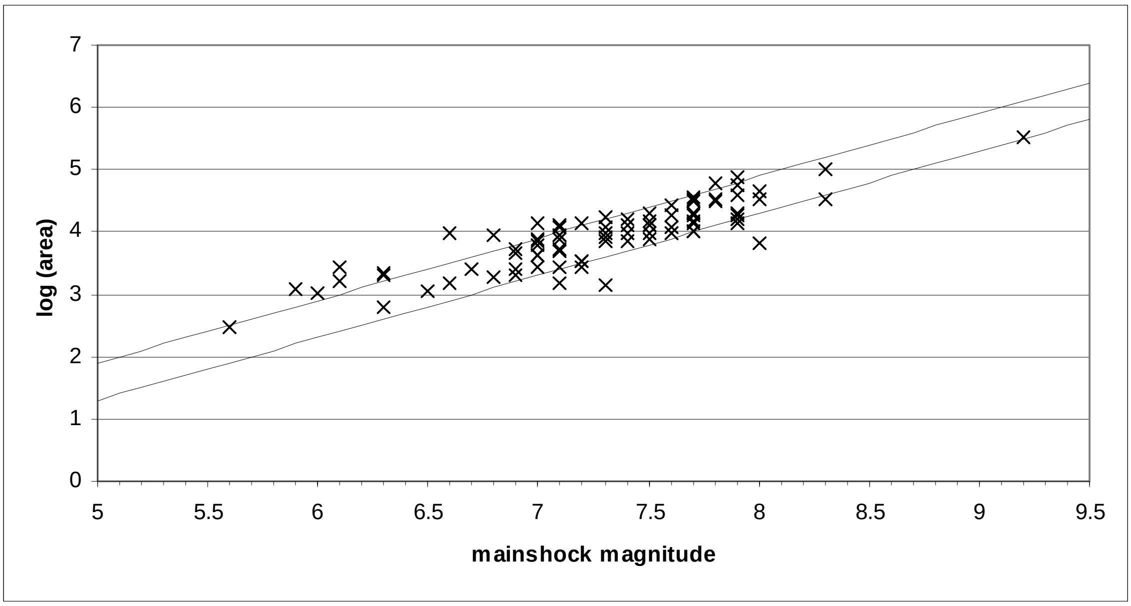
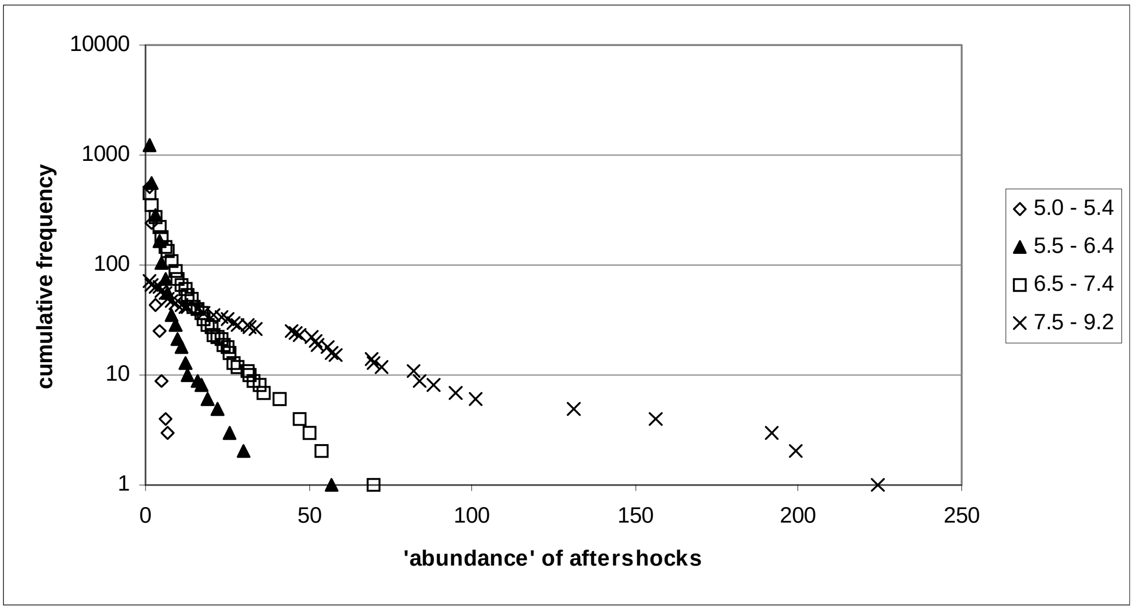


Figure 3: The logarithm of area (in km^2) versus magnitude for 85 mainshocks with 20 or more foreshocks and aftershocks. The two lines have a slope of one, and intercepts plus and minus one standard deviation, of the data scatter, from the best fitting line.

DISTRIBUTION OF NUMBERS OF AFTERSHOCKS ('ABUNDANCE')

All earthquakes within the model area of a mainshock have been counted. This number of aftershocks, the 'abundance', has been analysed as a function of mainshock magnitude. Figure 4a shows the cumulative frequency of abundance for four different mainshock magnitude intervals. The first interval includes mainshocks of magnitude 5.0 – 5.4. The abundance of aftershocks in these sequences is strongly affected by the proximity of the mainshock magnitude to the cut-off magnitude 5.0. The next two intervals of mainshock magnitude include one magnitude unit of mainshock each, and the last interval presents the rest of the events. Table 2 shows the total number of sequences that have an abundance of at least one; that is, the mainshock is followed by at least one event larger than magnitude 5 within 30 days. Figure 4b presents the cumulative relative frequency of abundance. Each data point in figure 4a was divided by the total number of sequences in the respective interval. The data are plotted in a linear-log diagram, where a straight line indicates exponential behaviour. The modelling of this behaviour is still in progress.



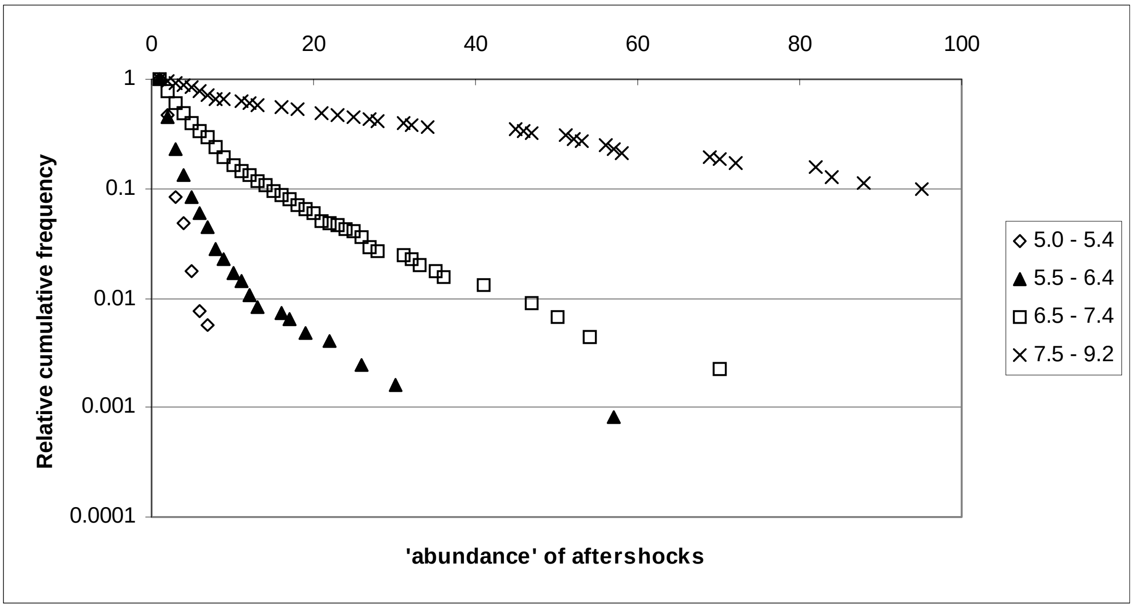


Figure 4: Cumulative distribution of number of aftershocks ('abundance') in aftershock sequences that have at least one aftershock larger than magnitude 5.0. Figure 4a: Cumulative frequency of abundance. Figure 4b: Relative cumulative frequency normalized by the total number of sequences (see table 2).

Table 2: The total number of sequences with at least one aftershock for different mainshock magnitude intervals

Mainshock magnitude interval	5.0 – 5.4	5.5 – 6.4	6.5 – 7.4	7.5 – 9.2
Total number of sequences	520	1233	449	71

Figure 5 shows the percentage of mainshocks for each magnitude that were not followed by another event within the search criteria. Such 'aftershock free' mainshocks need to be considered when forecasting (probabilistically) the number of aftershocks following a large earthquake.

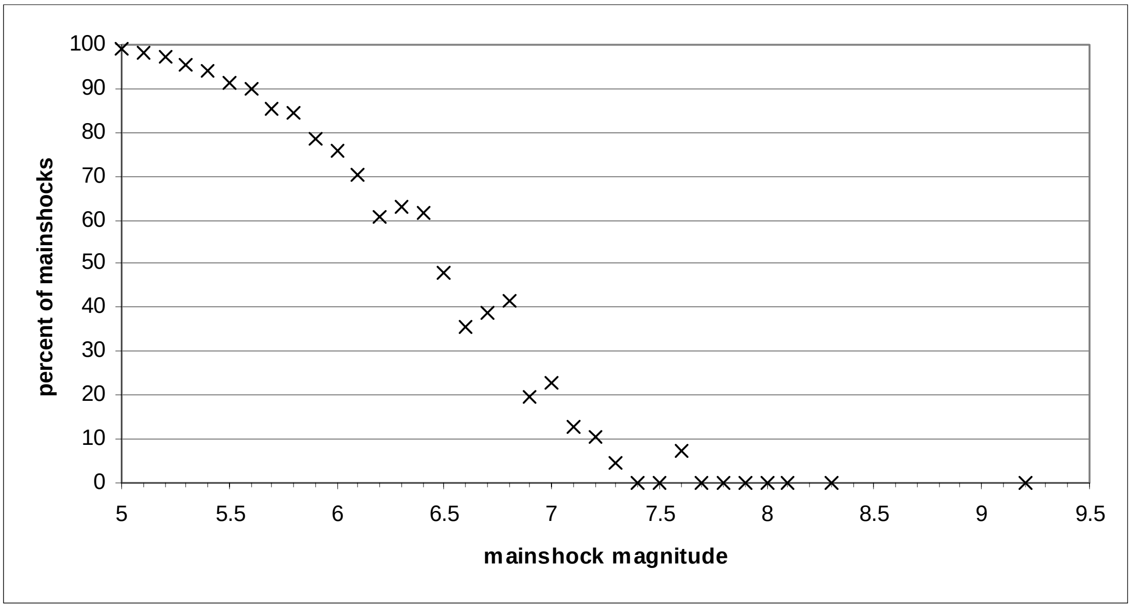


Figure 5: Percentage of mainshocks without aftershocks above magnitude 5.0 within 30 days.

DISTRIBUTION OF AFTERSHOCK MAGNITUDES

The magnitude-frequency distribution of earthquakes is well established to follow a power law, the Gutenberg-Richter relation,

$$\log N(M) = a - b^*M,$$

where $N(M)$ is the number of earthquakes with magnitude M, a describes the rate of seismic activity, and b presents the proportion of small to large events. The magnitude study [Christophersen, 1999] has confirmed Kagan's result [1997]: The *b*-value varies insignificantly from 1, except for mid-ocean ridges, where b is about 1.3, and in complex tectonic regions such as New Zealand.

AFTERSHOCK DISTRIBUTION IN TIME

At this stage the analysis of the aftershock sequences in time remains to be done. There are two related aspects that need to be studied: The duration of an aftershock sequence and the decay of aftershock activity. The later is described by the well-known Omori's law [e.g. Utsu, 1995].

DISCUSSION

- An area model has been derived that defines an aftershock sequence in space as a function of mainshock magnitude. The duration of each aftershock sequence was established by extending the search time interval by 30 days from each magnitude 5 or larger event. The duration of an aftershock sequence is thus defined to be the time interval from the mainshock to the last event in the sequence, after which no event occurs near the mainshock for at least 30 days.

- The probability of a large aftershock following a mainshock can be derived by combining the frequency distribution of abundance with the magnitude-frequency law of earthquakes. The parameters of the abundance distribution still have to be modelled.

- The well established behaviour for aftershock decay in time, Omori's law, has to be analysed for the global data. Combined with the abundance and magnitude-frequency distribution, this will enable the calculation of the probability of occurrence of an aftershock of any magnitude at any time following the mainshock.

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