

SEISMIC RELIABILITY ASSESSMENT OF MULTI-STOREY R/C FRAMES

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SUMMARY

The reliability of a regular multi-storey R/C frame designed to the Eurocodes for the medium ductility class is assessed. The adopted methodology considers random member capacity and interstorey drift, different spatial distribution scenarios and several failure criteria for both the serviceability and ultimate limit states. The uncertainty in the seismic input is accounted for by means of a small set of real accelerograms covering a wide range of seismic parameters, verified to give very similar results as those obtained using larger sets. The vulnerability curves obtained describe the probability of attaining a particular limit state given an earthquake of a particular intensity. These are consequently convoluted with appropriate seismic hazard curves for the derivation of the unconditional probability of failure for a given design lifetime. Adequate safety margins are found to exist for the ULS, whereas for the SLS these vary significantly depending on the criterion used to define the attainment of this state. Furthermore, the ULS results are used for an assessment of the EC8 behaviour factor.

INTRODUCTION

The primary scope of this study is to assess probabilistically a reinforced concrete (R/C) structural system subjected to seismic forces, taking into account what are believed to be the most important uncertainties associated with both the seismic input and structural modelling. As previous studies [e.g. Singhal & Kiremidjian (1996), Arede & Pinto (1996)] have given much greater emphasis to the uncertainty in the former aspect, the procedure adopted herein focuses on the uncertainties in aspects of structural modelling, and especially considers uncertain failure criteria.

Another consideration is the probabilistic evaluation of code provisions and particularly that of the behaviour (or response modification) factor. This has previously been considered in both deterministic [e.g. Salvitti & Elnashai (1996)] and probabilistic [e.g. Bento & Azevedo (1998)] studies, but its explicit evaluation in probabilistic terms has so far been neglected. In order that the results may be related to code specifications, failure probabilities, both conditional and unconditional, are derived for serviceability and ultimate limit state criteria. Those for the ULS are consequently used for obtaining characteristic values of the behaviour factor which can be compared with code-specified values.

ADOPTED METHODOLOGY

The methodology adopted for this study has been developed by the authors and is described in more detail elsewhere [Dymiotis et al. (1999)]. It takes into account the uncertainty in failure criteria, such as member capacity and the critical interstorey drift as well as random variable uncertainty in material properties. The main steps involved in this process are summarised in Figure 1. The random variables consist of the cylinder concrete strength (f_c), the steel yield and ultimate strengths (f_y and f_u , respectively), the ultimate steel strain (ε_{su}) and a model uncertainty factor for the estimation of the ultimate strain of confined concrete ($X_{m,\varepsilon_{cu}}$) [Kappos et al.,

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Figure 1. Flowchart summarising adopted methodology

(1998)]. All of these parameters are assumed to follow normal distributions with statistics as listed in Table 1. It is further noted that full positive correlation is assumed between the yield and ultimate steel strengths, whereas full negative correlation is assumed between the yield strength and the ultimate strain of steel. An additional random variable is the critical interstorey drift (δ_{cr}) which is assumed to follow a lognormal distribution with a mean of 6.6% and a coefficient of variation (COV) of 31%, as derived in [Dymiotis (1999)] using a large databank of available experimental results.

The use of randomly simulated combinations of the random material parameters obtained using Latin Hypercube Sampling (LHS) as described in [Ayyub & Lai (1989)], in conjunction with equations derived using the Response Surface Methodology (RSM) for the prediction of member strength and ductility [Kappos et al. (1998)] enable the estimation of damage indices given by

$$I_{d,el} = \max\left\{\frac{\left(\theta_p\right)_{req}}{\left(\theta_p\right)_{avail}}, \frac{\left(V_{R2}\right)_{req}}{\left(V_{R2}\right)_{avail}}, \frac{\left(V_{R3}\right)_{req}}{\left(V_{R3}\right)_{avail}}\right\}$$
(1)

In the above equation θ_p is the plastic rotation and V_{R2} and V_{R3} are the shear resistance to web crushing and the shear resistance to diagonal compression failure contributed by the concrete, axial load and transverse reinforcement, respectively. The subscripts "req" refer to the values of these parameters required during the dynamic, inelastic, time-history analyses whereas the subscripts "avail" refer to the section capacities calculated as described in [Penelis and Kappos (1997)]. In order to achieve the desirable level of accuracy, sufficient combinations of the random parameters must be simulated. This sample size depends on the number of random variables, any assumed correlation between these, and also on any assumed spatial distribution of the random variables. For the current study a set of 100 Latin hypercube samples for each random variable are shuffled for producing the random frames to be analysed [Dymiotis (1999)].

As far as the earthquake input is concerned, a selection of real accelerograms as listed in Table 2 is used to account for the variability in the ground motion. These are chosen so as to cover a wide range of seismic parameters, as can be seen in the table. These are then scaled to the various desired levels of intensity, herein described by the ratio A'/A_d . The values for A' are in fact fractions of the design peak ground acceleration (A_d) derived on the basis of spectral intensities. Vulnerability (also known as fragility) curves are therefore obtained for each input motion, each scenario and each limit state considered. These give the probability of failure at a certain intensity level, and are hence conditional on the occurrence of the earthquake. The curve which best describes the structural vulnerability from a design assessment point of view is that obtained by taking the average of those corresponding to the various input motions.

STRUCTURAL MODELLING

The structural system studied herein is the 10-storey, 3-bay regular bare frame shown in Figure 2 designed according to Eurocode 8 (EC8) [CEN (1995)] for medium ductility class (DC "M"), rock or stiff soil (soil class A), importance category II and A_d =0.25g. A behaviour (q) factor of 3.75 has therefore been applied during

 Table 1. Probabilistic distribution parameters for uncertainty modelling of material properties (subscripts "k" and "m" denote code-specified and mean values, respectively)

Property	COV (%)	Mean
f _c	18	$f_{ck} + 8 = 28 \text{ MPa}$
f_y	6	$f_{yk}+\ 40=440\ MPa$
$\mathbf{f}_{\mathbf{u}}$	6	$1.15 f_{ym} = 506 \text{ MPa}$
$\epsilon_{ m su}$	9	9%
Х _{m. єси}	39	1.0

Earthquake / Site / Component	Legend	Site	Magnitude	R	A (g)	A/V	Duration	A (g) for
		conditions		(km)		(g.s/m)	used (s)	A'=A _d
Aegion 15/6/95, N80E	AEGL	Alluvium	6.0 (M _s)	26	0.508	1.155	9.0	0.457
Alkyonides (Corinth) 24/2/81, N55W	CORW	Alluvium	6.7 (M _s)	20	0.286	1.163	17.8	0.378
Imperial Valley (El Centro)18/5/40, S00E	ELC	Alluvium	7.1 (M _s)	9	0.348	1.039	26.5	0.324
Kalamata 13/9/86, N10W	KALW	Alluvium	6.2 (M _s)	12	0.273	1.152	8.5	0.339
Loma Prieta (Presidio)17/10/89, N90W	LPRL	Rock	7.1 (M _s)	98	0.201	0.618	25.0	0.227
San Fernando (Old Ridge Road) 9/2/71, N69W	SFERT	Rock	6.5 (M _L)	29	0.274	1.054	25.0	0.362
Volvi (Thessaloniki) 20/6/78, N30E	THNS	Alluvium	6.5 (M _s)	27	0.142	1.118	12.0	0.308

Table 2. Input motions considered

design, and the concrete and steel grades used are C20/25 and S400, respectively. The frame geometry and dimensions of the cross-sections are presented elsewhere [Kappos (1997)], where deterministic assessment of the frame subjected to the serviceability (A=0.10g), design and survival (A=0.50g) earthquakes for a set of input motions have shown satisfactory performance under all three intensities.

The frame is modelled using DRAIN-2D/90 [Kappos (1996)], a modified and enhanced version of the original DRAIN-2D program [Kanaan & Powell (1973)]. Element types are selected from a list of available lumped plasticity models and the Direct Stiffness Method is employed for analysing the generated two-dimensional models. External columns are modelled as beam-column elements exhibiting elasto-plastic response through a combination of elastic and elasto-plastic components acting in parallel, and may account for varying axial loads. Internal columns and beams are modelled as beam elements, consisting of a constant stiffness (quasi-elastic) element with point hinges at its ends which follow the bilinear version of the Takeda moment-rotation $(M-\theta)$ model [Otani & Sozen (1972)]. Second order (P- Δ) effects are approximated by considering the axial forces in columns due to the static loads. Furthermore, DRAIN-2D/90 has been enhanced especially for the purpose of probabilistically assessing R/C frames. The equations derived using RSM [Kappos et al. (1998)] have been introduced to the program for the estimation of member strength and ductility and, as an option, the fully cracked stiffness. Damage indices defined by equation (1) are calculated at every time step and for each element, so that local member failure is assumed to take place once $I_{d,el}$ exceeds unity. Moreover, beam failures are taken into account for the remaining time-history analysis by assuming that a failed beam loses its flexural capacity at its ends whilst continuing to support the shear forces and span moments due to the initial static loads, in line with previous studies [Priestley & Calvi (1991)].

For the present study various levels of intensity need to be considered, hence a combined stiffness criterion involving moderately and/or fully cracked sections at different accelerations is adopted. Analysis is therefore carried out with the effective rigidity for the quasi-elastic elements modelled by

$$EI_{ef} = \begin{cases} 0.4 \sim 0.8EI_g \text{, for A'/A}_d \le 1.0\\ \text{average response from } 0.4 \sim 0.8EI_g \text{ and } M_y / \varphi_y \text{, for } 1.0 < \text{A'/A}_d \le 2.0\\ M_y / \varphi_y \text{, for A'/A}_d > 2.0 \end{cases}$$
(2)

where EI_g is the gross section rigidity, and the rigidity of fully cracked sections is given by the ratio of yield moment to yield curvature. The estimation of effective stiffness for moderately cracked sections by assuming



Figure 2. Geometry of frame considered and dimensions of member cross-sections (in mm)

40% and 80% of EI_g for beams and columns, respectively [Kappos (1991)], is in good agreement with the values recommended by the [New Zealand Concrete Standard (1995)].

VULNERABILITY AT SERVICEABILITY AND ULTIMATE LIMIT STATES

The quantification of two damage scenarios, defined through the "ultimate" and "serviceability" limit states (ULS and SLS, respectively) is attempted through collapse criteria appropriate for member (element) or storey (system) failure modes. The ULS is defined using a dual criterion, namely column failure or failure due to interstorey drift. This limit state generally corresponds to the "collapse prevention" requirement, particularly when relatively high drift thresholds are adopted. The first criterion is therefore determined, as already discussed, from a damage index $I_{d,el} \ge 1.0$, whereas the second criterion is the critical interstorey drift which as mentioned earlier may be treated as a random parameter rather than adopting the traditionally used deterministic value of 3%. The SLS is more difficult to define in a quantitative manner as different parameters may govern depending on the performance level sought for the structural as well as for the non-structural members. A limiting interstorey drift of 0.5% is considered, based on the average value of those suggested in EC8 for the SLS (0.4% for buildings with brittle non-structural elements attached to the structure and 0.6% for buildings with non-structural elements fixed so as not to interfere with structural deformations), and also on recent US recommendations [Wen et al. (1996)]. A critical consideration with regard to the SLS is the amount of damage sustained by the structural members, hence the criterion $\theta > \alpha \theta_v$ is adopted, where $\alpha \ge 1$ is a coefficient describing the amount by which the yield rotation (θ_v) is allowed to be exceeded. In the absence of detailed information, θ =1 and θ =2 are tentatively considered.

Using the input motions listed in Table 2, two sub-groups of accelerograms are identified, again covering the ranges of seismic parameters given by the seven motions. These consist of (AEGL, ELC, LPRL, KALW, SFERT) and (AEGL, ELC, LPRL). As Figure 3(a) shows, very little accuracy in the average vulnerability curves for the ULS is lost by using the smallest group consisting of just three accelerograms, which is hence adopted for this limit state.

Another consideration relates to the scenarios of spatial correlation of the random variables. Considering that in a realistic situation the reinforcement would normally be ordered from the same producer, only the effect of uncorrelated concrete strengths at different storeys is investigated. The four scenarios considered herein are therefore the following:



Figure 3. Average vulnerability curves for (a) three sets of input motions and (b) four scenarios of spatial correlation of f_c

• Scenario A: all members in all storeys follow a single distribution of f_c,

• Scenario B: the frame is separated into two zones, so that the f_c distribution of members in the first five storeys is independent from that in the upper five storeys,

• Scenario C: as scenario B, but the demarcation is now placed after the first storey, and

• Scenario D: the frame is separated into three zones with demarcations after the third and sixth storeys.

As may be observed in Figure 3(b) showing the average vulnerability curves from the group of three input motions, significantly higher probabilities of failure arise as a result of varying f_c within the frame. It is also evident from the near coincidence between the curves for scenarios B and C, that the position of the demarcation plays a very small role compared to that of the number of zones. Similarly, the introduction of a third zone makes little difference to the curves obtained for scenarios B and C. This agrees with observations from deterministic studies [Kappos (1997)] where two critical regions were identified around the 1st-2nd and 7th-8th storeys. The introduction of an independent distribution for f_c for storeys falling outside these regions is therefore not influential on the overall results.

In view of the results just described, scenario B is conservatively chosen for the remaining analyses. Figure 4 compares the average vulnerability obtained for the ULS assuming both deterministic and random critical drift. The small distinction between the two curves is simply explained by the fact that a very small number of structures initially failed due to exceedance of the deterministic critical interstorey drift of 3%. For example, for A'/A_d=3.0, $\delta_{cr}>3\%$ was encountered in only 5% of the failed structures. For scenario A, however, these structures accounted for 31% of those that failed and the introduction of random δ_{cr} is therefore expected to have a larger influence on the corresponding vulnerability curve. In most cases this particular frame is found to fail mainly due to inadequate rotational capacity in ground storey columns for low intensities where moderately cracked sections are assumed (refer to equation (2)), whereas for larger intensities and fully cracked sections a similar failure mode occurring at the seventh or eighth level is most frequent.

Average vulnerability curves are similarly obtained for the SLS corresponding to the three failure criteria identified above. Due to the fact that these analyses require less computational effort than their ULS counterparts, as failure is expected to be encountered much sooner, the average values are taken from the set of five input motions. This helps to avoid any loss of accuracy due to the SLS criteria possibly being more sensitive to seismic parameters. The resulting curves are therefore plotted in Figure 4, alongside the ULS curves for comparison purposes. It is found that the curves obtained for a critical interstorey drift of 0.5% are significantly less onerous in terms of the implied P_f than those obtained for $\theta > \alpha \theta_y$. In fact, the pair of curves corresponding to $\theta > \alpha \theta_y$ show that the exact value of α in the range $1.0 \le \alpha \le 2.0$ has a relatively small effect on the vulnerability curves. Since $\theta > \alpha \theta_y$ is a local damage criterion (member rotation), it is possible that the SLS is violated at a single beam (as beams yield before columns in capacity-designed frames), while for the remaining members $\theta < \alpha \theta_y$. Focusing on the intensity of the serviceability earthquake which for EC8 is 0.10g (A'/A_d=0.4), the local damage criterion clearly implies a high likelihood of violation of the SLS (P_f=0.76 and 0.38 for $\alpha = 1.0$ and 2.0, respectively). Consequently, the New Zealand Standard serviceability earthquake of intensity 0.04g (A'/A_d=0.17) appears to be a more reasonable assumption for $\theta < \alpha \theta_y$.



Figure 4. Average ULS and SLS vulnerability curves (scenario B)

SEISMIC RISK AND BEHAVIOUR FACTOR

In order to assess the actual reliability of the structure unconditional on the occurrence of the earthquake, a probabilistic representation of the seismic hazard must be taken into account. Appropriate hazard curves, quantifying the probability of occurrence of an earthquake of a given maximum intensity, need to be selected so as to adequately represent an area of seismicity similar to that for which the structure has been designed. In a recent hazard study [Crespellani et al. (1997)] considering Città di Castello, a site in Central Italy, it was found that the "mean plus one standard deviation" acceleration associated with zero fundamental period on the response spectra and a probability of exceedance of 10% in 50 years was 0.27g. The hazard curves that matched this finding were therefore selected for use in the present study together with additional cases corresponding to a larger number of design lifetimes (t_d). The latter are obtained by assuming that the annual occurrence rate (v) relates to probability of exceedance (P) through the following expression

$$P = 1 - e^{-t_d V}$$

Figure 5(a) shows all the hazard curves used herein corresponding to the mean values of acceleration at zero fundamental period.

As demonstrated by, e.g. [Thoft-Christensen & Baker (1982)], the probability of failure (P_f) of a structural system of resistance R (i.e. effective acceleration A') subjected to a load S (i.e. earthquake with effective acceleration of A') may be given by

$$P_f = \int_{-\infty}^{\infty} f_R(A') (1 - F_S(A')) dA'$$
(4)

where $f_X(x)$ and $F_X(x)$ are the probability and cumulative density functions (pdf and cdf), respectively. Figure 5(b) shows the trend with which P_f increases with t_d , obtained using all hazard curves of Figure 5(a) and the ULS vulnerability curve of Figure 4 for random δ_{cr} . Of particular interest is the P_f of 0.82% corresponding to the scenario assumed in EC8 for this limit state (return period T_r =475 years). This value is within the range estimated in [Arede & Pinto (1996)] and may also be compared with the range of $0.6 \le P_f \le 1.8\%$ calculated in [Wen et al. (1996)] for the ULS of a 7-storey R/C frame designed according to NEHRP criteria.

Regarding the SLS, values of P_f are obtained using the hazard curve for T_r =50 years, the return period which would normally be associated with the serviceability earthquake. The three values corresponding to the different criteria considered in the preceding section are also plotted in Figure 5(b) at an abscissa t_d =5.2 years. The probabilities that correspond to the criteria concerning member yielding are found to be an order of magnitude larger than that corresponding to the drift criterion. In fact, the latter is comparable to the value obtained for the ULS and T_r =475 years.

The probability density function describing the maximum acceleration sustained by structures which fail (R', a subset of R) may be obtained from [Thoft-Christensen & Baker (1982)]

$$f_{R'}(A') = \frac{f_R(A').(1 - F_S(A'))}{P_f}$$
(5)

Through numerical integration using the actual histograms of both seismic hazard and structural vulnerability, the resulting histogram of R' corresponding the 475 years return period hazard curve is used for the estimation of

(3)



Figure 5. (a) Hazard curves used for derivation of (b) seismic risk

the q-factor. As has been suggested in [Kappos (1991)], the actual q-factor that a R/C frame can develop may be estimated from

$$q = q_d \frac{A_{cr}}{A_d} = 3.75 \left(\frac{A_{cr}}{0.25}\right) = 15A_{cr}$$
(6)

The inclusion of the critical acceleration (A_{cr}) leading to structural failure implies that the actual q-factor may only be evaluated for cases where collapse (i.e. violation of the ULS) is encountered. It is therefore suggested that the probability distribution describing the resistance of structures that fail, R', should be used. Using the linear transformation of equation (6) the cdf of the q-factor is presented in Figure 6.

The design q-factor of 3.75 is associated with a probability of non-exceedance of 75%. Even though this is a high probability, the probability of non-exceedance for the whole population of structures is obtained from

$$P(q \le q_d) = P(q \le q_d | failure) P_f = 0.75 \times 0.0082 = 0.00615$$
(7)

Hence, only 0.6% of regular 10-storey 3-bay frames designed to EC8 for DC "M" in accordance with the procedures adopted in this study [Kappos (1997)] are expected to fail whilst developing a q-factor less than or equal to the code value due to uncertainties relating to material variability, failure criteria and the seismic excitation.



Figure 6. Derivation of q-factor from conditional probability curve (failed structures)

CONCLUSIONS

The developed methodology has been successfully applied for assessing the reliability of a multi-storey frame designed to EC8. This took into account the uncertainty in seismic forces and especially those in several structural parameters, including damage indicators. The outcomes clearly indicate the difficulty in quantifying target performance criteria, especially for the SLS. As discussed in [Bertero et al. (1991)], there are significant differences in the ductility and drift limits adopted by various codes. The deterministic drift limits applied herein for the ULS and SLS fall in the upper ranges of currently proposed values. However, although the deterministic ULS drift limit of 3% was found to be highly conservative compared with experimental values of maximum drift

at failure (but not to the analytical values at which column failure is usually encountered), the corresponding SLS limit of 0.5% is unconservative if yielding of beams is to be avoided at the SLS. As a matter of fact, even the lower limit of 0.3% recommended in the aforementioned study would in most cases be marginally preceded by $\theta > \theta_y$ or even $\theta > 2\theta_y$ in certain beams of the frame. These remarks should be seen in the context of defining the SLS earthquake, wherein current code provisions are far from uniform.

Investigating the unconditional failure probability for a relevant site, it has been concluded that the safety margins associated with the ULS of R/C frames designed to EC8 are satisfactory and comparable to those achieved in other modern seismic codes. The range encompassed by the values corresponding to the SLS, however, indicates the need of calibration for both local and displacement-based criteria. With regard to the q-factor, while the code-specified value which underpins the deterministic design was found to have been exceeded in 75% of the failed structures, these accounted for just 0.6% of the overall population of frames.

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