INVESTIGATION OF THE MODELLING METHOD OF THE DAM - FOUNDATION - RESERVOIR SYSTEM

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SUMMARY

In the dynamic FEM or FDM analysis of dam - foundation - reservoir system, it has been found that the solution may diverge even the traditional stability condition is satisfied in the dam, foundation, reservoir domain respectively. And it is usually difficult to evaluate the accuracy of the solutions. Such problems are investigated in the study. First, the discretization method of the system with FE or FD is investigated theoretically. It has been pointed out that the parameters such as the maximum mesh size, the time interval should be determined according to the material properties and the interested minimum natural period of the system. The stability condition defined with the mentioned parameters has been given which should be satisfied besides the traditional one introduced from uniform medium. Moreover, a method has been given out with it the mesh sizes of the zones near the interface of dam - reservoir and that of dam - foundation could be adjusted according to the relation of their material properties in order for vibrating waves to pass through the interfaces without distortion. Based on such investigation a systems approach for proceeding dynamic analysis is proposed for any given computing environment, accuracy requirement, and other conditions. Then, a numerical experiment has been done with an ideal 3D elastic model for testing the above proposals, and the experiment verified the theoretical results. It is expected that this paper is informative to the dynamic analysis of the dam – foundation – reservoir system.

INTRODUCTION

A program called "UNIVERSE" has been developed for the dynamic linear and nonlinear analysis of coupled dam – foundation – reservoir system with a hybrid method of FEM and FDM. However, in several cases of the real dam analyses, it has been found that the analytic solutions diverged even the convergence requirements on the mesh sizes of the finite elements and the time intervals had been satisfied. For example, for time domain analysis, some times the satisfaction of the requirements $\Delta t < V/\Delta n$ and $\Delta t < T_{min}/\pi$ (Here, Δt, Δn is the time interval and mesh size respectively. V, T_{min} is the velocity of elastic wave and the minimum natural period of the structure respectively) can not ensure a stable analytic solution. And some times an implicit form of the finite difference method, or $\beta = 1/4$ in Newmark's direct integration method is still insufficient for getting a stable solution. It has been well known that even an analytic solution converges, it usually fluctuates with the variation of the mesh size or time interval. Hence, it is still not completely clear that what effects the mesh size and time interval exacts on the analytic solutions, and in what conditions, to what degree the analytic solutions are reliable. In order to get an understanding on such problems, and further, for getting reliable solutions in ordinary dynamic analyses, it is necessary to clarify the effects of the modeling conditions on the analytic solutions.

In this study, a theoretical investigation is done on the variation of the hydrodynamic pressure with the modification of the modeling conditions. A constrain condition which should be satisfied among the parameters used in the dynamic analysis is discussed. Furthermore, the condition for the elastic wave transmitting through the interface between dam and reservoir and that between dam and foundation is elucidate, and some proposals have been made on the FEM analysis of the dam – reservoir – foundation system.

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COUPLED DAM – FOUNDATION – RESERVOIR SYSTEM

In the program "UNIVERSE", the coupled dam – foundation – reservoir system is expressed with following equations

$$
\begin{cases}
\dfrac{\partial^2 \Phi}{C_0^2 \partial t^2} = \dfrac{\partial^2 \Phi}{\partial x^2} + \dfrac{\partial^2 \Phi}{\partial y^2} + \dfrac{\partial^2 \Phi}{\partial z^2} & (1.1) \\
\begin{bmatrix} M_d & M_{df} \\ M_{fd} & M_f \end{bmatrix} \begin{Bmatrix} \ddot{u}_d \\ \ddot{u}_f \end{Bmatrix} + \begin{bmatrix} C_d & C_{df} \\ C_{fd} & C_f^* \end{bmatrix} \begin{Bmatrix} \dot{u}_d \\ \dot{u}_f \end{Bmatrix} + \begin{bmatrix} K_d & K_{df} \\ K_{fd} & K_f \end{bmatrix} \begin{Bmatrix} u_d \\ u_f \end{Bmatrix} = \begin{Bmatrix} F_w \\ T_f \end{Bmatrix} & (1.2) \\
[M_g]\{\ddot{u}_g\} + [C_g]\{\dot{u}_g\} + [K_g]\{u_g\} = \{T_b\} & (1.3)
\end{cases}
$$

Formula (1.1), (1.2), (1.3), in turn, is the equation concerning the wave motion of the reservoir water, the equation concerning the vibration of the dam and foundation, and the equation concerning the vibration of the free field. The variables and matrices of the above equation group are explained in the following list.

Φ : Velocity potential function of water

x, y, z : Cartesian coordinates

t : Time

C_0 : Sound velocity in water (1440 *m/s*

M, C, K : Mass, damping, stiffness matrix, and the footnote d, df, f and g denote that the matrices belong to the dam, the junction of the dam and foundation, the foundation, and the free field respectively.

C_f^* : Damping matrix (including the components of the material damping and that of the viscous boundaries)

$u, \dot{u}, \ddot{u}$: Displacement, velocity and acceleration vectors

F_w : Hydrodynamic pressure acting on the upstream surface of the dam

T_f : Earthquake load and the inflow of energy resulted from the motion of the free field

T_b : Earthquake input of the free field

The details of the above matrices are explained in the reference papers [EPDC: 1999, MIURA: 1989].

The connecting condition between dam and reservoir is

$$
\begin{cases}
\dfrac{\partial \Phi}{\partial n} = V_d \\
F_s = P_w
\end{cases}
\qquad (2)
$$

It means that on the interface of the dam and reservoir, the velocity of the dam in the normal direction equals that of the adjoining water particle. Inversely, the water pressure generated on the interface is treated as surface load on the dam.

On the reservoir bottom, following boundary condition is applied which is impedance dependent

$$\frac{\partial \Phi}{\partial t} - C_0 \beta \frac{\partial \Phi}{\partial n_r} = 0 \qquad (3)$$

where β is the impedance ratio between the sediment of the reservoir bed and water. n_r denotes the normal direction of the reservoir bed. And the statement of the other boundary conditions of non-direct relationship with this study is omitted here.

In the program “UNIVERSE”, the Eq. (1.1) is formulated with FDM, and the Eq. (1.2), (1.3) are formulated with FEM. It is thought that it can make good use of the conciseness of FDM and the applicability of FEM in this way. Half infinite reservoir and free field are comprised in the system, hence, the energy giving and receiving between the dam – foundation – reservoir system and outer media can be considered.

DEPENDANCE OF WAVE PROPAGATING CHARACTERISTICS ON THE MESH SIZE

The transmitting wave from dam to reservoir and its propagating characteristics in the reservoir water are investigated here. Since the viscosity of water is neglected, only the motion in the normal direction of the interface between dam and reservoir can be transmitted. For conciseness, 1 dimensional problem shown in Fig.1 is taken for explanation.

It is supposed that the dam vibrates with harmonic wave of circular frequency ω, and velocity amplitude V_0. Therefore the motion of the dam can be expressed as

$$V_d(t) = V_0 e^{i\omega t} \qquad (4)$$

The velocity of water particle at position j is designated as V_j, then the velocity potential function can be written as:

$$\Phi_j(t) = \phi(V_j) e^{i\omega t} \qquad (5)$$

Here, $\phi(V_j)$ is the amplitude of potential function Φ_j.

It is assumed that the mesh size in the normal direction n is uniform. By mid–differencing spatially, Eq. (1.1) can be approximately formulated as

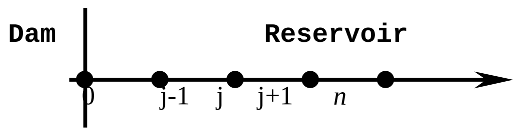


FIG.1 MODEL OF WAVE PROPAGATION

$$\frac{\partial^2 \Phi_j}{C_0^2 \partial t^2} = \frac{1}{\Delta n^2}\left(\Phi_{j+1} - 2\Phi_j + \Phi_{j+1}\right) \qquad j = 1, 2, \cdots\cdots \qquad (6)$$

The above form is then submitted into Eq. (5), then following form can be got

$$-\frac{1}{C_0^2}\omega^2 \phi(V_j) = \frac{1}{\Delta n^2}\left[\phi(V_{j+1}) - 2\phi(V_j) + \phi(V_{j-1})\right] \qquad (7)$$

By introducing a transfer function λ, the amplitude function $\phi(V_j)$ can be rewritten as

$$\phi(V_j) = \lambda_j \phi(V_0) \qquad j = 1, 2, \cdots\cdots \qquad (8)$$

Furthermore, let

$$\omega_c = 2\frac{C_0}{\Delta n} \qquad (9)$$

From Eqs. (7), (8) and Eq. (9), there is

$$\lambda_{j+1} + 2\left[2\left(\frac{\omega}{\omega_c}\right)^2 - 1\right]\lambda_j + \lambda_{j-1} = 0 \qquad (10)$$

By expressing λ_j of Eq. (8) in the form

$$\lambda_j = \lambda^{j\Delta n} \qquad (11)$$

and submitting it into Eq. (10), there is

$$\lambda + 2\left[2\left(\frac{\omega}{\omega_c}\right)^2 - 1\right]\lambda + 1 = 0 \qquad (12)$$

And the roots of the equation are:

$$\lambda_{\frac{1}{2}} = 1 - 2\left(\frac{\omega}{\omega_c}\right)^2 \pm 2\left(\frac{\omega}{\omega_c}\right)\sqrt{\left(\frac{\omega}{\omega_c}\right)^2 - 1} \qquad (13)$$

In Fig.2 the variation of the transfer function λ when $\omega/\omega_c = 1$ and $\omega/\omega_c > 1$ (when $\omega/\omega_c > 1$, there are two kinds of possibilities: $|\lambda| < 1$ and $|\lambda| > 1$) is shown. From Fig.2, it can be found that when $\omega/\omega_c > 1$, the transfer function λ will attenuates or amplifies sharply as the distance from dam face gets longer. That means, in the case of $\omega/\omega_c > 1$, the wave generated from the interface of dam – reservoir will disappear or diverge. The condition for the wave to propagate normally should be that λ is in a stable state, i.e., the following form must be satisfied.

$$\omega \le \omega_c \qquad (14)$$

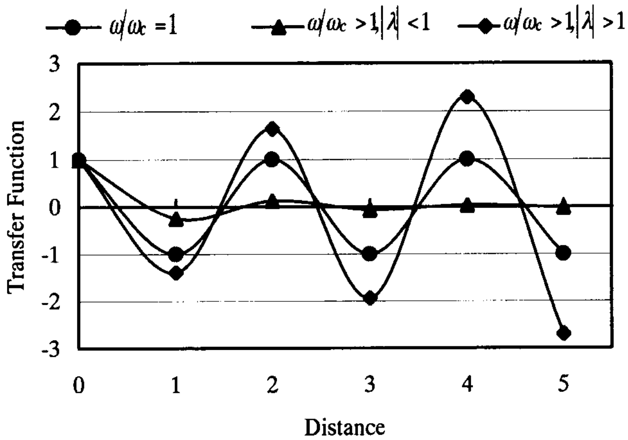


Fig.2 Relationship of Distance from Dam and Transfer Function

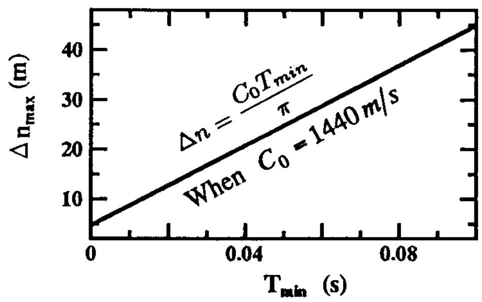


Fig. 3 Relationship of the Maximum Mesh Size With the Minimum Period and Wave Velocity

After all, when reservoir is modeled with finite difference method, among the waves generated from dam face only the wave components whose frequencies are lower than ω_c can propagate normally. For this reason, here ω_c is called critical circular frequency.In numerical analysis, in order to let the waves of the minimum natural period T_{min} of the analytic model to propagate normally, in other words, in order to obtain the solution containing the components of frequency $1/T_{min}$, the maximum mesh size should be no larger than a certain limit. By submitting equation (9) into equation (14), and replacing ω with $2\pi/T_{min}$, the maximum mesh size can be determined. That is, the following constraining condition about the parameters T_{min} , the maximum mesh size Δn and the wave velocity C_0 have to be satisfied.

$$\frac{\Delta n}{C_0} \le \frac{T_{min}}{\pi} \qquad (15)$$

In fact, T_{min} plays the tool of accuracy control. When material properties are known, the maximum mesh size will be determined according to the requirement on accuracy. Fig.3 shows the relationship between the maximum mesh size and T_{min} for the hydrodynamic pressure analysis of reservoir. The domain under the line is the stable range of the propagating condition.

It should be noticed that, Eq.(15) is independent of time. It means that, even in the case of implicit difference, or in the case where Newmark's direct integration method of a parameter $\beta = 1/4$ is used, so called, in the absolutely stable condition, the Eq. (15) must be kept.

As well known, for wave propagating problem, the following stability criterion exists:

$$\Delta t < \frac{\Delta n}{C_0} \tag{16}$$

And for vibration problem, from Von Neumann criterion, Watanabe [Watanabe 1972] has introduced that

$$\Delta t < \frac{T_{min}}{\pi} \tag{17}$$

And it should also be noticed that equation (16) is independent of vibrating frequency, and equation (17) is independent of mesh size. Therefore, in the coupled dam – foundation – reservoir system a criterion satisfying all of the above condition is

$$\Delta t < \frac{\Delta n}{C_0} \le \frac{T_{min}}{\pi} \tag{18}$$

In dynamic analysis, for obtaining a reliable solution, the time interval, the maximum mesh size, the wave velocity, and the interested minimum period etc. must satisfy the above criterion. To put it concretely, in the dynamic analysis of dam – foundation – reservoir system, the maximum mesh size should be determined according to Eq. (15), and then, the time interval should be determined according to Eq. (16).

A PROPOSAL FOR MODELING THE DAM – FOUNDATION – RESERVOIR SYSTEM

In the last section the constrained conditions on the parameters concerning the numerical analysis have been discussed based on the investigation of the propagating characteristics of water waves in the reservoir. Such conditions resulted from the statement of the spatial differencing of the wave motion equation, but it is considered that a result of similar physical meaning could be got even the wave motion equation is formulated with finite element method.

Previously, in the dynamic analysis of dam – foundation – reservoir system there was seldom discussion on the mesh sizes adjacent to the interface between dam and foundation and that between dam and reservoir. From the statement of the last section, it is necessary to adjust the mesh sizes of the both side media in order to let the waves to pass through the interface normally like it propagates in the uniform medium.

From Eq.(15), the condition for waves (here P wave is taken as example) propagate normally in the dam or foundation should be

$$\frac{\Delta n^s}{V_p} \le \frac{T^s_{min}}{\pi} \tag{19}$$

where $\Delta n^s, V_p, T^s_{min}$ are, in turn, the mesh size, the velocity of P wave, and the minimum natural period of the dam or foundation to which order the response should contain.

On the interface between dam and reservoir, for waves generated form dam and those of the hydrodynamic pressure pass through the interface without distortion, the critical circular frequencies of the two kinds of different media should be same, i.e., the following relationship should be kept

$$\omega_c = \omega_{cd} \tag{20}$$

where ω_{cd} is the critical circular frequency of the dam and which can be defined from Eq. (9)

$$\omega_{cd} = 2\frac{V_P}{\Delta n^s} \tag{21}$$

From Eq. (20), the following relationship can be got.

$$\frac{V_P}{C_0} = \frac{\Delta n^s}{\Delta n} \tag{22}$$

From above equation it should be noticed that the ratio of the optimum mesh sizes of the dam and reservoir adjacent to the interface should equal to the ratio of the wave velocities of the both sides. Similarly, the mesh sizes adjacent to the interface of dam and foundation should keep the same relationship.

Generally, the wave velocities of dam, foundation and reservoir are different, therefore near the interfaces the mesh sizes should be adjusted. At lest, the mesh sizes in the normal directions of the interfaces should be modified.

NUMERICAL INVESTIGATION

Accompanying the theoretic discussion of last section, a numerical investigation on the same problem is done. A gravity dam of a height of 64 m and a top length of 150 m is taken as analytic object. As for the dimension of the coupled foundation, since free field around the foundation is also included in the analytic model, the foundation domain contains only the main area where the interaction with the dam would occur. The coupled reservoir is of a depth of 60 m. and a length of 200 m. Fig.4 (a) shows the analytic model of the coupled system where the mesh figure demonstrates one of the various kinds of discretization cases). And Fig.4 (b) shows the maximum section of the dam and the section of the reservoir (same shape as the upstream surface of the dam).

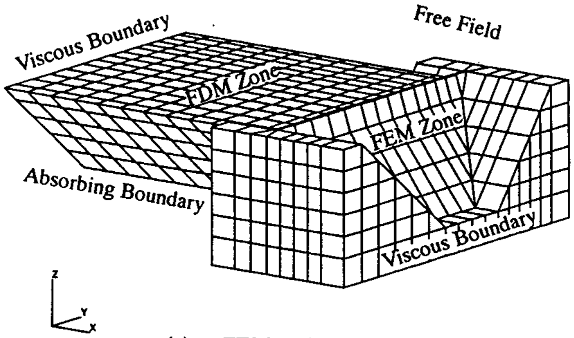
It is supposed that the dam, the foundation and the free field have same material properties shown in the Table 1 where an reference case of a rigid dam is expressed with the shear modulus $10^{30}\ N/mm^2$. At the reservoir bed, the impedance ratio of the sediment is 2.0. At the upstream end of the reservoir a viscous boundary developed by the authors [Watanabe and Cao 1998] is applied.

TABLE 1: DYNAMIC PROPERTIES OF DAM AND FOUNDATION

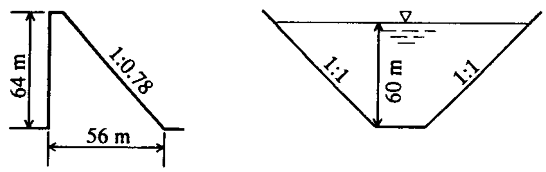
Shear Modulus G (N/mm^2)	Density ρ (g/cm^3)	Poisson's Ratio ν	Dissipation Factor h (%)
2.45×10^4	2.4	0.2	5
1.0×10^{30} (rigid body)			0

Gauss Pulse is taken as earthquake wave and its maximum amplitude is normalized to be 196 cm/s^2. The wave shape and its Fourier spectrum are shown in Fig.5. This wave has the advantages of short time, rich frequency components, and the frequency range contained in the wave can be modified according to necessity.
First, the variation of the hydrodynamic pressure with the alteration of mesh size is examined. For that purpose, the dam and foundation are assumed to be rigid body. When the time interval of the ground motion history is set at 1/1000 sec., the minimum vibration period included automatically in the response will be 0.002 sec. Among these, for getting a response of the components of the minimum period 0.025 sec., the maximum mesh size of the reservoir should be 11.46 m. according to equation (15)(notice that the P wave velocity of water is about 1440 m/s^2).

The above ground motion is input in the upstream – downstream direction, and the solution stability of the hydrodynamic pressure is examined as the mesh size in this direction is adjusted. Fig.6 shows the variation of the maximum hydrodynamic pressure (the position is around the 1/3 -- 1/4 of the depth above the reservoir bottom) with this



(a) FEM and FDM Mesh



(b) The Maximum Sections of Dam & Reservoir

Fig. 4 Dam–Foundation–Reservoir System

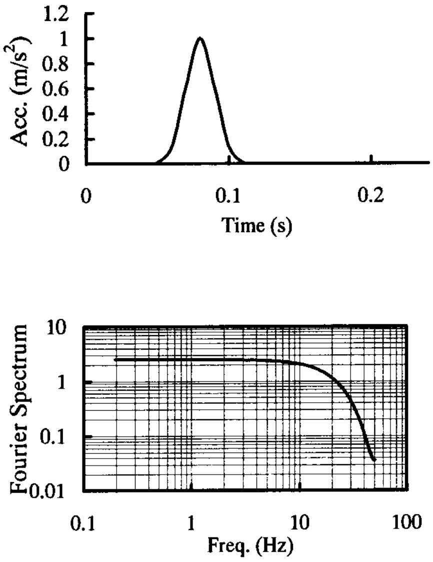


Fig. 5 Input Wave (Gauss Pulse)

adjustment. As reference, the maximum hydrodynamic pressure got from the Westergaard's experience equation [Westergaard 1933] is also printed in the same figure. It is clear that as the mesh size gets smaller the solution gets stable. And it should approaches the right solution if the mesh size is smaller than the lower limit shown in the figure, i.e., the solution got from a numerical model tends to the theoretic solution. But, it should be no great variation at the solution even the mesh size is made smaller than the lower limit of the Fig.6. Therefore, an investigation on the characteristics of the solution when the mesh size is in the normal range of numerical analysis is of more realistic importance. It should be also noticed that the solution approaching the right one is smaller than that got from the Westergaard's experience equation, it is mainly due to the effect of sediment at the reservoir bed. From the traditional stability condition of wave propagation problem, i.e., Eq. (16), for any model of the mesh size of Fig.6 the solution should be stable if the time interval $\Delta t = 0.001$ sec. But obviously, there is great variation. It is considered that this phenomenon is due to that the Eq. (18) is not satisfied. From Eq. (9) and Eq. (14), it can be found that as the mesh size increases, the frequency domain of the waves which can propagate normally gets smaller, and the effects of the distorted waves become larger, and hence the error contained in the solution gets greater and greater. On the other hand, in numerical analysis, the frequency components automatically being integrated in the final solution depend on the selected time interval Δt, it is very difficult to get a solution of all the stimulated frequency components without any error. For example, when $\Delta t = 0.001$ sec., the minimum period of the stimulated wave will be $T_{\min} = 0.002$ sec. For getting a solution containing such frequency component, the minimum mesh size should be larger than 1.44 m. from Eq. (16). But from Eq. (18) it is known that the maximum mesh size should be less than 0.92 m. For this reason, generally numerical analysis is only assured accuracy to a certain extent.

Next, the variation of the dynamic response of the elastic dam whose properties are shown in the Table 1 is examined when it is coupled with the reservoir mentioned above. The velocity of P waves in the dam and foundation is 3368 m/sec. In the case of the maximum mesh size 14 m., the critical circular frequency will be 481.14 rad./sec. Therefore the components of the vibrating period greater than $T_{\min} = 0.013$ sec. could be calculated. Here, the meshes of the dam and foundation are assumed to be certain, and the variation of the dam response with the modification of the reservoir mesh is examined. In the discussion of last section, the dissipation factor is assumed to be 0, but here for the investigation on real cases a dissipation factor 5% is assumed.

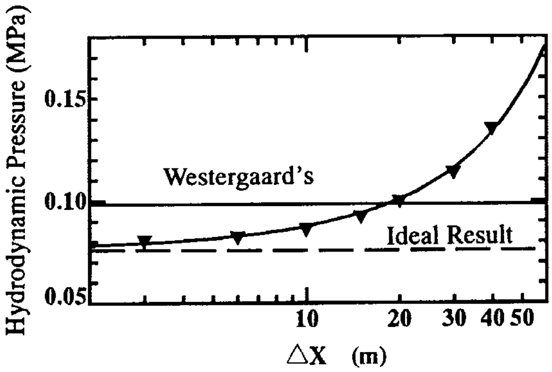


Fig. 6 Variation of Hydrodynamic Pressure Result from the Modification of Mesh Sizes

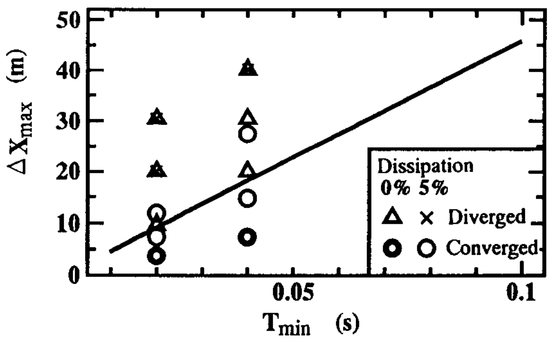


Fig. 7 The Stability of the Dam Response Effected by the Maximum Mesh Size and Minimum Period

Fig.7 demonstrates the stability of the dam response when the mesh of the reservoir is adjusted to be of various sizes and the time interval is set at 1/100 and 1/50 sec. respectively. When time interval is fixed, the frequency components stimulated in the model are certain. But the values of the frequency components contained in the solution will vary as the mesh size of the reservoir is adjusted. From Eq. (9), Eq. (15) and Fig.7, the followings can be found. In the case where the dissipation factor is 0%, if the critical circular frequency of the reservoir model is greater than that of the dam model, which means that the waves generated from the dam can propagate normally in the reservoir model, the response of the dam is stable. It can also be noticed that the stability of the response in the case of the elastic dam is more sensitive to the variation of mesh size than that in the case of rigid dam. This is due to that the Eq. (20) should be satisfied, and there is the interaction effect between the dam and reservoir. On a whole, in the coupled dam – foundation – reservoir system, the dependency of the stability of the solution on the vibration period, the maximum mesh size, the wave velocity is of a similar tendency as that of the reservoir model only (the line in Fig.7). In the case of dissipation factor 5%, the stability condition is alleviated somewhat. It is due to that the distorted part of the response is constrained.

CONCLUSIONS

In the dynamic analysis of dam – foundation – reservoir system with FEM, or FDM, or the hybrid method of them, the relationship presented in the paper should be satisfied. That is, the parameters such as the mesh sizes of the model, the material properties (Velocity of P wave or S wave) and the minimum vibration period until which the solution contains should obey the constrain condition, except the traditional stability condition. Near the interface between dam and foundation and that between dam and reservoir, it is necessary to adjust the mesh sizes of the both sides according to their material properties. The theoretic and numerical investigations are done on 1D and 3D problems respectively, but a similar result has been obtained. As a further research topic, the theoretic study on the 3D problem is necessary.

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