

THE NONLINEAR RESPONSE EMULATION ANALYSIS OF THE STOCHASTIC STRUCTURE SUBJECTED TO THE EARTHQUAKE EXCITATION

G Y DING¹ And J LI²

SUMMARY

The earthquake response of the stochastic structures are analysed and studied by means of the emulation technology in this paper. A kind of non-linear emulation analysis algorithm of the stochastic structure is suggested. Also, a new kind of duplicate stochastic simulation concept is put forward. On this basis, combining the delamination sampling method, two examples of the RC structure are used to demonstrate the application and validity of the proposed emulation algorithm.

INTRODUCTION

In civil engineering, the uncertainties of system parameters affect the mechanics behaviours to a great extent. An increasing interest has been shown in treating the uncertainties involved in the modelling of the structure characteristics. These uncertainties are more obvious in the modelling course of the structure such as the system parameter variability resulted from the geometry, the constitutive behaviour of the materials and the boundary conditions. When system parameter variability are very remarkable, the corresponding mechanics behaviours of engineering structures also embody certain randomness and consequently conduce the great change of the structural dynamic response and the reliability. Therefore, when considering a structural model, such as a finite element model, using the stochastic structure analysis model is necessary and practical^{[1]-[5]}.

In actual engineering, the most key phase is the non-linear one. So, the non-linear analysis of stochastic structure is greatly important. As to the non-linear analysis of stochastic structure, in the one hand, all the kinds of uncertainties have made the analysis very complex. In the other hand, those parameters related to the uncertainties show that the different non-linear properties make the analysis work more complex, especially the non-linear dynamic analysis of stochastic structure. Many researchers have made efforts to study by means of such as the stochastic perturbation expansion technique, the stochastic differential equations, the spectral approach combined statistical linearization and the orthogonal polynomial expansion in conjunction with a variational method and so on^{[6]-[10]}. However, with the remarkable enlarging of the memory and the increasing operational speed of the modern computer, some obstacles of high computational quantities and difficulties in statistical stability have been increasingly overcome in utilising the traditional simulation method. Consequently, the traditional simulation method will acquire more widely and practical applied foreground.

On the background mentioned above, the non-linear earthquake response emulation analysis algorithm based on the modified delamination sampling method, the duplicate stochastic simulation technique and deterministic dynamic non-linear analysis method is proposed. Two examples show that the simulation efficiency can be greatly improved by the suggested method in this paper. Moreover, the random behaviours and varieties of the structural non-linear response can be revealed and accordingly more objective and true reference can be supplied to the engineering practice.

¹ Department of civil engineering, Tongji University, 200092, Shanghai, China. e-mail: ddgy@public7.sta.net.cn

² Department of civil engineering, Tongji University, Shanghai

DETERMINISTIC DANYMIC NONLINEAR ANALYSIS METHOD

The danymic equilibrium increment differential equations of structure under the earthquake excitation

$$[M]\Delta\{\ddot{\delta}_{(t)}\}+[C]\Delta\{\dot{\delta}_{(t)}\}+[K]\Delta\{\delta_{(t)}\}=-[M]\Delta\{\ddot{\delta}_{g(t)}\} \quad (2-1)$$

In this paper, the danymic non-linear analysis is based on the increment modified stiffness technology and wilson- θ time-history analysis method.

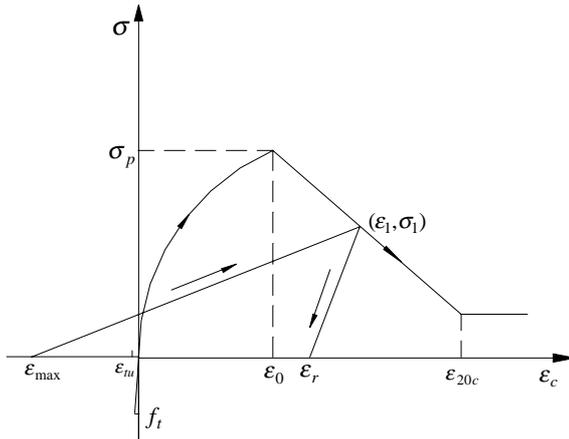


Figure1. Hysteretic rule of concrete

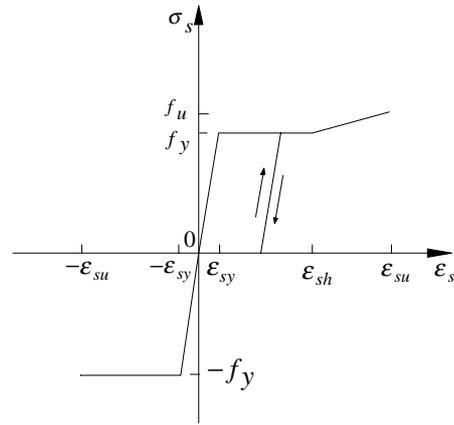


Figure2. Hysteretic rule of feinforcement

The constitutive hysteretic models of the concrete and the reinforcement

The study object in this paper is the reinforced concrete frame where the cyclic load is the earthquake wave. In the constitutive hysteretic model of concrete, the framework curve is used according to the concrete stress-strain curve proposed by Kent and Park. To uncracked concrete, the tension stress-strain relationship is treated as elastic and the tension effect must be considered if the tensile limit or compressive limit has not reached in history. Also, when the structure is under applied or reversing loads, the principal stress-strain relationship is assumed beeline. When the reversing loads point $\varepsilon_1 \leq \varepsilon_0$, the beeline slope is the Young' modulus E_c ; when $\varepsilon_1 > \varepsilon_0$, the beeline slope is regarded as $k \cdot E_c$ where k is the reduced coefficient of concrete degradation stiffness. But the absolute value of stress don't exceed the corresponding point of the framework curve when the structure is under applied loads again; if the applied loads is lasting after reaching at framework, the direction of applied loads will be along the framework of stress-strain curve, where $\sigma_p, \varepsilon_0, f_t$, are the peak stress, the peak strain and the tensile limit strength.

To cracked concrete, the effects of the crack-surface must be included when the structure need to be applied the loads. When $\varepsilon \leq \varepsilon_{\min}$, the curve of the applied loads again are represented by equations (2-2),(2-3)

$$\sigma = \begin{cases} 0 & \varepsilon_B \geq \varepsilon \\ \frac{\sigma_{\min}}{\varepsilon_{\min} - \varepsilon_B} \cdot (\varepsilon - \varepsilon_B) & \varepsilon_B < \varepsilon \leq \varepsilon_{\min} \end{cases} \quad (2-2)$$

$$\varepsilon_B = 0.283 \times \left[0.1 + 0.9 \cdot \frac{\varepsilon_0}{(\varepsilon_0 + |\varepsilon_{\max}|)} \right] \cdot |\varepsilon_{\max}| \quad (2-3)$$

where ε_{\min} , σ_{\min} and ε_{\max} are the maximum compressive strain, the corresponding stress and the maximum tensile strain which has reached in history.

To the constitutive hysteretic model of the reinforcement, the hardening effect of the reinforcement isn't considered when the reinforcement is compressive in order to be convenient for analysis. But the absolute value of the reinforcement stress don't exceed the corresponding point of the framework curve when being under applied loads again; if the applied loads is lasting after reaching at framework, the direction of applied loads will be along the framework of stress-strain curve, where $f_y, f_u, \varepsilon_{sy}, \varepsilon_{su}$, are the yielding stress, the tensile limit strength, the yielding strain and the tensile limit strain.

Forming the element stiffness

To the members of RC frame, from the internal force equilibrium of the element sections the element internal force increment can be represented equation.(2-4) from the internal force equilibrium of the element sections,

$$\begin{cases} dN = \int_A d\sigma \cdot dA \\ dM = \int_A d(\sigma \cdot y) \cdot dA \end{cases} \quad (2-4)$$

where d_N, d_M, d_σ, d_A are the element internal force increment, moment increment and stress increment and area increment. By means of the section strain relationship ,the element increment physical equation can be written as equation (2-5),

$$\begin{Bmatrix} dN \\ dM \end{Bmatrix} = [D_T] \cdot \begin{Bmatrix} d\varepsilon_{01} \\ d\Phi \end{Bmatrix} \quad (2-5)$$

where $d_{\varepsilon_{01}}, d_\phi, D_T$ are the element axle centre strain increment, the element section curvature increment and the material stiffness matrix.

Based on assumption of the isoparametric element and the suitable element displacement mode, the element increment geometry equation can be given as following equation ,

$$d\{\varepsilon\} = ([B_l] + [B_{nl}]) \cdot d\{r\}^e \quad (2-6)$$

where $[B_l], [B_{nl}], d\{r\}^e$ are the element material strain matrix, the element geometry strain strain and the node displacement increment matrix.

After further derivation combining the above equations we can obtain equation(2-7)

$$d\{P\} = \left(\int_l d[B_{nl}]^T [D_T] ([B_l] + [B_{nl}]) dl + \int_l [B_l'] [D_T] d[B_l'] dl \right) \cdot d\{r\}^e \quad (2-7)$$

where $d\{P\}$ is the element node force increment matrix. By the similar deduction, equation(2-7) is rewritten as

$$d\{P\} = ([K_S] + [K_l] + [K_{nl}]) d\{r\}^e \quad (2-8)$$

in which $[K_l], [K_S], [K_{nl}]$ are the material non-linear stiffness matrix, the axle force quadratic moment element stiffness and the geometry non-linear element stiffness matrix.

After being rearranged further, the specific expression of the element stiffness matrix can be obtained.

THE DUPLICATE STOCHASTIC SIMULATION METHOD AND THE DELAMINATION SAMPLING TECHNIQUE

The duplicate stochastic simulation method

The non-linear behaviours of the stochastic structure are very complex especially the stochastic non-linear dynamic analysis. The main reason is that people knew few details about some parameters of the restoring force or the hysteretic model particularly when these parameters are random process or random fields. Because of this reason, the non-linear dynamic problems can't be solved from mechanism. In fact, due to the modelling course of structure analysis, their parameters are commonly considered deterministic parameters and accordingly the true condition can't be reflected. On this background, several main hysteretic parameters in the constitutive relationship are regarded as random parameters in order to conduct the duplicate stochastic simulation. In the first place, the first step of the duplicate stochastic simulation is carried through. Firstly, the random parameters samples are generated according to the definite distribution. Then in term of the definite sampling technique the sampling courses are conducted. Because one member of RC frame can be modelled as a few units and in order not to make the parameters dispersion very large, the samples which have been randomly sampled are arranged and grouped according to the definite rule and then allotted randomly these samples to the different members. In the second place, in order to simulate the parameters variance of the different units of the same member, the samples which have allotted to the one member are randomly allotted to different units. Consequently the duplicate simulation is finished and the stochastic structure sample which is regarded as object of the non-linear dynamic analysis are generated. On this basis, the different stochastic structure samples are generated repeatedly in terms of the given distribution in order to conduct the non-linear dynamic analysis. At last, the simulation is terminated according to the given convergence rule.

The delamination sampling technique

The estimated variance can be lowered by means of the delamination sampling technique. Furthermore, if we divide the sampling district to very small and distribute the sampling times, the estimated variance can be remarkably lowered. Firstly, the sample space D is divided to the little space D_1, \dots, D_m and $\bigcup D_i = D$. Then, the sampling times in each space are decided in terms of their contribution. To illustrate this point, we define $p_i = \int_{D_i} f(x)dx$ and the times in D_i should be proportional to p_i . Thus, the sampling efficiency can be improved greatly.

Considering integral $\theta = \int_0^1 f(x)dx$, we divided $[0,1]$ to m parts which can be shown as $0=a_0 < a_1 < \dots < a_m=1$. Then the following equation is obtained.

$$\theta = \int_0^1 f(x)dx = \sum_{i=1}^m \int_{a_{i-1}}^{a_i} f(x)dx \cong \sum_{i=1}^m I_i \quad (3-1)$$

here $l_i = a_i - a_{i-1}$ as the length of the i -th space ($i=1,2,\dots,m$), the integral I_i can be obtained by use of the sample average method and then an estimate of θ can be calculated by as following steps.

Step1. Generating $U(0,1)$ the random numbers $\{u_{ij}, j = 1, \dots, n_i, i = 1, \dots, m\}$.

Step2. Calculating $x_{ij} = a_{i-1} + l_i u_{ij}, j = 1, \dots, n_i, i = 1, \dots, m$

Step3. Calculating $\hat{I}_i = \frac{l_i}{n_i} \sum_{j=1}^{n_i} f(x_{ij})$

Then the estimated value of θ can be given as follows.

$$\hat{\theta}_3 = \sum_{i=1}^m \hat{I}_i \quad (3-2)$$

From the sample average value method we can obtain $E\hat{I}_i = I_i$, so $\hat{\theta}_3$ is the agonic estimate of θ and its variance is given as follows.

$$Var(\hat{\theta}_3) = Var\left\{ \sum_{i=1}^m \frac{l_i}{n_i} \sum_{j=1}^{n_i} f(X_{ij}) \right\} = \sum_{i=1}^m \frac{l_i^2}{n_i} \sigma_i^2 \quad (3-3)$$

in which

$$\sigma_i^2 = \int_{a_{i-1}}^{a_i} \frac{f^2(x)}{l_i} dx - \left(\frac{I_i}{l_i}\right)^2 \quad (3-4)$$

If σ_i^2 and l_i are known,
$$n_i = nl_i \sigma_i / \left(\sum_{i=1}^m l_i \sigma_i \right) \quad (3-5)$$

we can easily prove that the variance of estimate according to the above equation is minimum value which is $\frac{1}{n} \left(\sum_{i=1}^m l_i \sigma_i \right)^2$.

If σ_i^2 are unknown, though the most simply distribution $n_i = nl_i / \sum l_i = nl_i / (b-a)$ we have this equation $Var(\hat{\theta}_3) \leq Var(\hat{\theta}_2)$, in which $Var(\hat{\theta}_2)$ is the estimate variance based on sample average value method.

In fact, substituting $nl_i / (b-a)$ into (3-3) gives

$$Var(\hat{\theta}_3) = \frac{b-a}{n} \sum_{i=1}^m l_i \sigma_i^2 \quad (3-6)$$

According to Cauchy-Schwarz inequation, we can obtain

$$\begin{aligned} \theta^2 &= \left(\sum_{i=1}^m I_i \right)^2 = \left[\sum_{i=1}^m \frac{I_i}{\sqrt{l_i}} \sqrt{l_i} \right]^2 \\ &\leq \sum_{i=1}^m \frac{I_i^2}{l_i} \sum_{i=1}^m l_i = (b-a) \sum_{i=1}^m \frac{I_i^2}{l_i} \end{aligned} \quad (3-7)$$

on this basis, multiplying l_i with the equation (3-4) and adding up gives

$$\begin{aligned} \sum_{i=1}^m l_i \sigma_i^2 &= \int_a^b f^2(x) dx - \sum_{i=1}^m \frac{I_i^2}{l_i} \\ &\leq \int_a^b f^2(x) dx - \theta^2 / (b-a) \end{aligned} \quad (3-8)$$

we can obtain this inequation $Var(\hat{\theta}_3) \leq Var(\hat{\theta}_2)$

THE STOCHASTIC STRUCTURE NONLINEAR DYNAMIC ANALYSIS EMULATION ALGORITHM

The algorithm of the non-linear dynamic analysis for stochastic structure can be summarised as follows.
Step 1: Inputting the structural parameters and necessary dates. Then dealing with the boundary conditions and the earthquake wave.

Step 2: Sorting the structural random parameters and generating uniform distribution random dates. Then transferring these dates into object random samples and sampling according to the delamination sampling technique.

Step 3: Arranging and grouping the samples which have been randomly sampled according to the definite rule and then allotting randomly these samples to the different members. Then, to the different units of the same member, randomly allotting the samples which have allotted to this member to different units.

Step 4: Calculating the beginning stiffness matrix $[K]$, the beginning damping matrix $[C]$ and the mass matrix $[M]$. Then defining the beginning displacement vector, the beginning velocity vector and the beginning acceleration vector.

Step 5: Calculating the equivalent stiffness matrix.

Step 6: To each discrete time, calculating the following steps: (1) Calculating the equivalent loads increments; (2) calculating the displacement increment and accumulative total at $t+\theta t$ time; (3) Calculating the internal force increment based on the displacement increment; (4) Calculating the element strain increment matrix and then defining the next stiffness according to the material constitutive hysteretic rule; (5) Calculating the damping matrix $[C]$ renewably.

Step 7: repeating step 2~step 7 until meeting the requirement of convergence.

NUMERICAL EXAMPLE

The single variable and multiply variables emulation analysis

The analysis is a double-floor and double-span RC frame structure where the span is 2.4m and the floor highth is 1.4m. The structure is divided to fifty units and the beginning elastic modulus of the concrete constitutive hysteretic model is considering as the random parameter. The input excitation is EL-Centro NS wave and the maximum acceleration peak value is 196gal. The objective of this analysis is to clarify the characteristics of the displacement mean-square deviation and the random emergence of the plastic hinges. The results of the single variable and the multiply variables emulation analysis are shown as following figure3~figure4 and figure5~figure6.

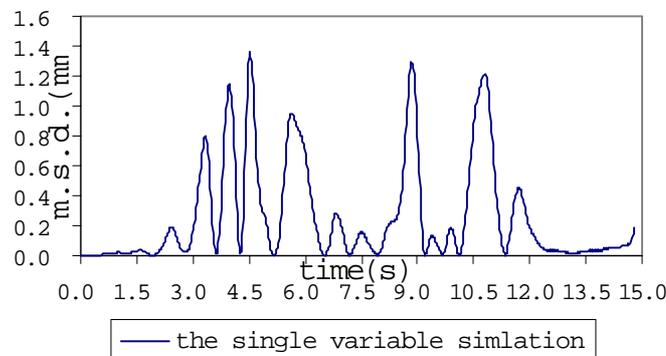


Figure3: The displacement mean-square deviation

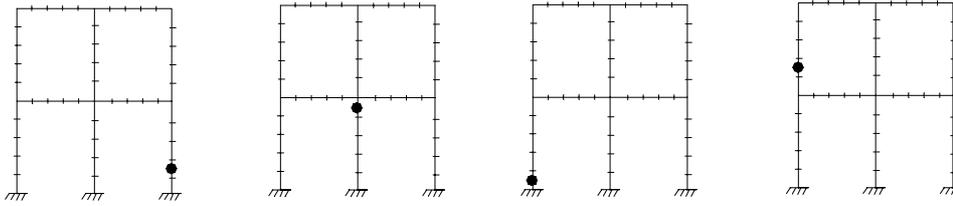


Figure4:Random emergence figure of the plastic hinges (the single variable simulation)

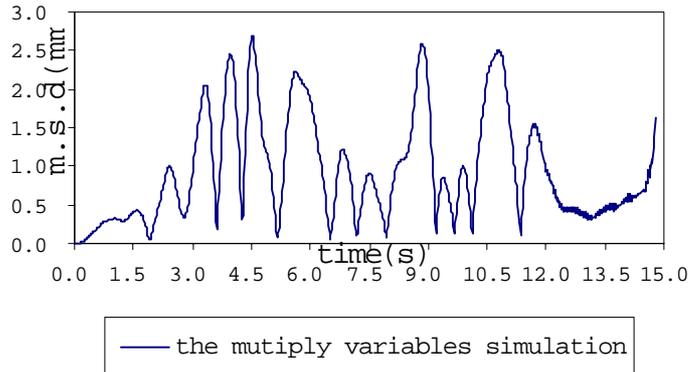


Figure5:The displacement mean-square deviation (considering the beginning elastic modulus, the peak stress and the peak strain of the concrete constitutive hysteretic model as the random parameters).

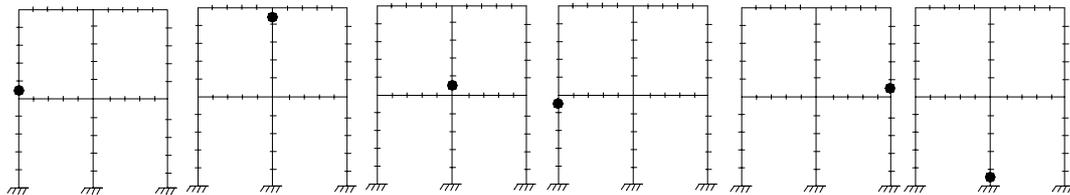


Figure6:Random emergence figure of the plastic hinges (the multiply variable simulation)

Figure3 and figure5 after variance analysing show that the structural response variation can reach 25%~35% when conducting the single variable non-linear dynamic emulation. However, the structural response variation can reach 30%~60% when the multiply variables emulation and the displacement response mean-square deviation of the multiply variables emulation is close to the one of the single variable emulation.

The emulation results statistics show that the plastic hinge unit of the first appearance which are shown in figure4 indicate some extent dispersion, The plastic hinge unit of the first appearance according to the multiply variables emulation take on more randomness as are shown in figure6.

CONCLUSION

A dynamic non-linear emulation analysis algorithm of stochastic structure which make use of the modified delamination sampling method, the duplicate stochastic simulation technique and deterministic danymic non-linear analysis method is proposed. Two examples show that the simulation efficiency can be greatly improved by the suggested method in this paper. The obtained conclusions are as follows.

(1) The random impacts of the material parameters uncertainties on the structural applied load properties can be truly reflected by the proposed stochastic dynamic non-linear analysis algorithm.

(2) When adopting the duplicate stochastic simulation technique suggested in this paper the true conditions of structure can be objectively reflected. Thus, it will be propitious to the reliability of analysing structure.

(3) The structural mechanism behaviours show the remarkable dispersion when the beginning elastic modulus, the peak stress and the peak strain in the constitutive hysteretic model are considered respectively or simultaneously as the random parameters especially as far as the multiply variables stochastic non-linear simulation.

(4) The study in this paper show that by means of the stochastic non-linear dynamic emulation the structural random mechanism behaviours not only can be described but some foundations can also be supplied to know about the random probability distribution of the structural mechanism behaviours in order to control the structural behaviours.

REFERENCES

1. Ding GuangYing and Li Jie. The study about the impacts on the HRC-long column mechanism behaviours of the concrete material dispersion properties. Proc.5th China Conf. on RC structure theory and practice. 1998.
2. LiJie. Stochastic structural system. China science publishing house.1996.
3. Shinozuka, M. and Deodatis,G. Response variability of stochastic finite system. Journal of engineering Mechanics, ASCE, 114(3),499-519.1988.
4. I. Elishiakoff, Y.J. Ren, and M.Shinozuka ,Fellow. Variational principles developed for and applied to analysis of stochastic beams. J. Engng Mech.,ASCE, Vol.122,No.6.
5. Shinozuda, M. and Yamazaki, F. Stochastic Finite Element Analysis: An introduction in stochastic Structural dynamics, eds. S.T. Ariaratnam,G.I. Schueller and I. Elishakoff. Elsevier Applied Science Publishers Ltd.
6. S.W.Zhang ,B.Ellingwood and R.Corotis . Direct integration method for stochastic finite element analysis of non-linear dynamic response.Structural Engineering and Mechanics. Vol.3. No.3, 273-287.
7. H.U.Koyluoglu, S.R.K.Nielsen and A.S.Cakmak. Stochastic dynamics of geometrically non-linear random structures subject to stationary random excitation. Structural Reliability Theory. Paper No.116, ISSN 0902-7513 R9418,1994, University of Aalborg. Denmark. J. Sound Vibrat.
8. Liu,P.L. and Der Kiureghian, A., Finite element reliability of geometrically non-linear uncertain structures. Journal of Engineering Mechanics,117(8),1806-1825.
9. H.Grundmann and H.Waubke. Non-linear stochastic dynamics of systems with random properties: a spectral approach combined with statistical linearization. Int.J. Nonlinear Mechanics,Vol.31.No.5. 619-630. 1996
- 10.Wilfred D. Iwan and Ching-Tung Huang. On the dynamic response of non-linear with parameter uncertainties. Int.J. Nonlinear Mechanics,Vol.31.No.5. 631-645. 1996.