

## IDENTIFICATION OF PATH AND LOCAL SITE EFFECTS ON PHASE SPECTRUM OF SEISMIC GROUND MOTION

Sumio SAWADA<sup>1</sup>, Hitoshi MORIKAWA<sup>2</sup>, Kenzo TOKI<sup>3</sup> And Keijyu YOKOYAMA<sup>4</sup>

### SUMMARY

The seismic wave is characterized by source, path and local site effects. Many researchers have analyzed those effects using observed data, but have treated only Fourier amplitude spectra. There are few reports which discuss those effects on Fourier phase spectra.

Path and local site effects on Fourier phase spectra are identified using ground motions due to small events. Group delay time (Tgr), defined as the gradient of the phase spectrum, is used to make clear the relationship between phase characteristics and time histories. In order to decrease the number of data for representing the characteristics of seismic motion, the average Tgr spectrum is defined as the smoothed Tgr. The variance spectrum of Tgr is defined as the averaged square residual of the Tgr and the average Tgr spectrum. The former corresponds to the mean arrival time of the wave group of the frequency, and the latter the duration.

The analysis is based on the following assumptions. The observed variance Tgr spectrum can be estimated by summing variance spectra of the source, path and local site effects. Identification of the variance spectrum of path and site effects was done by the generalized inverse matrix method using seismic records from 3 stations in Osaka region, Japan. We consider that the variance spectrum of source effects must be zero because only the records of 5 small earthquakes with magnitudes less than 5.0 were used. Conclusion derived by the present study is that the variance spectrum of path effects takes a large value when  $1/Q$  is large and that one of site effects is large when amplification factor is large.

### INTRODUCTION

Seismic ground motions are characterized by source, path and local site effects. Researchers have made many efforts to identify these effects on the basis of observed data. Iwata and Irikura (1988), for example, identified the source, path and local site effects on Fourier amplitude spectra using the generalized inverse matrix method.

It is notable, however, that almost all those who discuss these effects deal only with the Fourier amplitude spectra, very few studies have been concerned with the characteristics of the phase spectra.

The response spectra are widely used to define the design seismic motion for many types of infra-structures. Although the response spectra are very useful for the super-structures, it is not suitable for defining the input ground motion for underground structure because the maximum displacement of the seismic motion on the ground surface is necessary for the design. Fourier spectrum is promising for this purpose, if its phase characteristics are defined.

Ohsaki (1979) showed that the histogram of the phase difference is similar to the envelop of the original time history. Izumi and Katsukura (1983) and Katsukura et al. (1984) also showed this using the group delay time (Tgr) introduced by Papoulis(1962). Soda (1986) discussed the spectral characteristics of Tgr from the

<sup>1</sup> Disaster Prevention Research Institute, Kyoto University, Uji, Japan, Email : sawada@catfish.dpri.kyoto-u.ac.jp

<sup>2</sup> Department of Civil Engineering, Tottori University, Tottori, Japan, Email : morika@men.ne.jp

<sup>3</sup> Dept of Civil Engineering Systems, Kyoto University, Kyoto, Japan, Email: toki@quake.kuciv.kyoto-u.ac.jp

<sup>4</sup> Mitsubishi Corporation, Tokyo, Japan.

stochastic point of view. He raised several important arguments about the relationship between a time history and its phase spectrum.

We have discuss the source, path and local site effects on Fourier phase spectra of seismic ground motion records. We introduce the average and variance spectra of group delay time to represent the characteristics of the Fourier phase spectrum. The spectra of the observed seismic ground motions are given as the summation of the source, path and local site effects. Path and local site effects in the Fourier phase spectrum are identified from the observed ground motions of small events.

## 2. REPRESENTATION OF THE FOURIER PHASE SPECTRUM

Let  $f(t)$  be the time history of seismic ground motion and  $F(\omega)$  the Fourier transform of  $f(t)$ ;

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = A(\omega) e^{-i\phi(\omega)}, \quad (1)$$

where,  $i = \sqrt{-1}$ .  $A(\omega)$  is the Fourier amplitude spectrum and  $\phi(\omega)$  the Fourier phase spectrum.  $A(\omega)$  and  $\phi(\omega)$  then are written

$$A(\omega) = \sqrt{F_I^2(\omega) + F_R^2(\omega)}, \quad (2)$$

$$\phi(\omega) = \tan^{-1} \frac{F_I(\omega)}{F_R(\omega)}, \quad (3)$$

respectively where  $F_R(\omega)$  and  $F_I(\omega)$  are the real and imaginary part of  $F(\omega)$ .

The Fourier amplitude spectrum of observed ground motion,  $A^O(\omega)$ , is represented as the product of the source, path and local site effects,

$$A^O(\omega) = A^S(\omega) A^P(\omega) A^L(\omega), \quad (4)$$

where the superscripts O, S, P and L stand for the observed, source, path and local site effects. The observed Fourier phase spectrum,  $\phi^O(\omega)$ , is represented by the summation of the Fourier phase spectra of these three effects;

$$\phi^O(\omega) = \phi^S(\omega) + \phi^P(\omega) + \phi^L(\omega). \quad (5)$$

The Fourier phase spectrum, however, has no direct relationship to its time history.

Izumi and Katsukura (1983) and Katsukura et al.(1984) showed that the group delay time ( $T_{gr}$ ) of a record is related to the envelope of its time history.  $T_{gr}$  is defined by the gradient of the phase spectrum with respect to the frequency,  $\omega$ , (Papoulis(1962), Cohen(1995));

$$T_{gr}(\omega) = \frac{d\phi(\omega)}{d\omega} \quad (6)$$

If  $f(t)$  is defined in the finite range of  $[0, T]$ , then the value of  $T_{gr}(\omega)$  is  $[0, T]$ . Note that  $T_{gr}(\omega)$  of an impulse in the time domain has a constant value in frequency domain, which value coincides with the time when the impulse is placed. The dots in Figure 1 (b) show an example of  $T_{gr}(\omega)$  calculated from the observed seismic

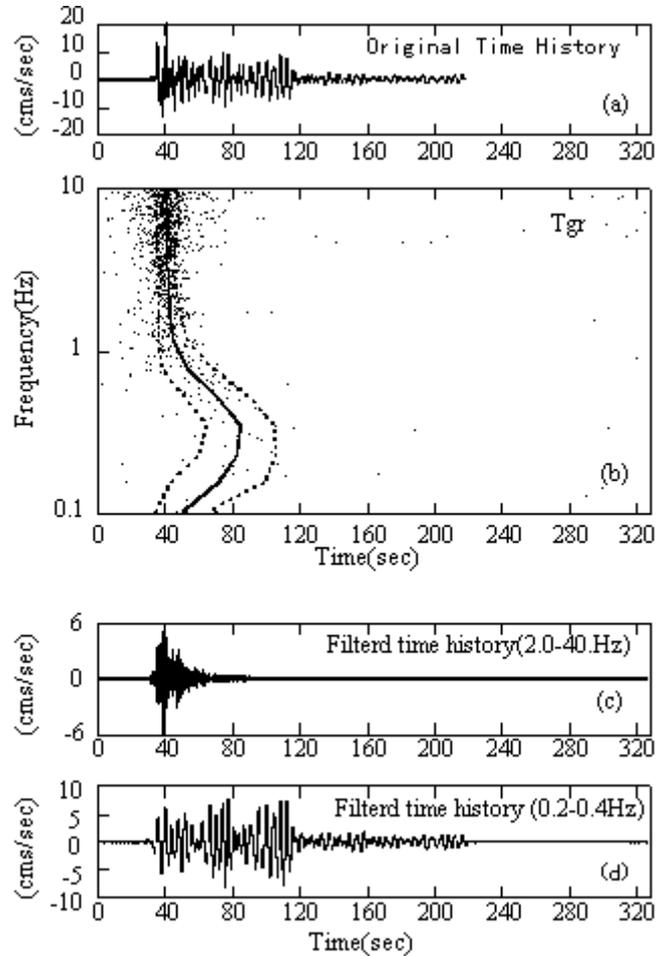


Figure 1: Example of  $T_{gr}$  calculated from a seismic record

ground motion shown in Figure 1 (a). Izumi and Katsukura(1983) pointed out that the shape of the distribution of  $T_{gr}(\omega)$  is similar to the envelop of the time history. Consequently the average and standard deviation of  $T_{gr}(\omega)$  correspond to those of the envelop function. The standard deviation of  $T_{gr}(\omega)$  therefore is directly related to the duration of the original time history.

The average and variance spectrum of  $T_{gr}(\omega)$  are used to discuss the non-stationary characteristics of seismic motion. We define the average spectrum of  $T_{gr}(\omega)$ ,  $\mu_{T_{gr}}(\omega)$ , as the smoothed  $T_{gr}(\omega)$  using the frequency window function,  $W(\omega;\omega_0)$ ;

$$\mu_{T_{gr}}(\omega_0) = \frac{1}{E} \int_0^{\infty} W(\omega;\omega_0) |A(\omega)|^2 T_{gr}(\omega) d\omega \quad , \quad (7)$$

where

$$E = \int_0^{\infty} W(\omega;\omega_0) |A(\omega)|^2 d\omega \quad , \quad (8)$$

Various functions can be adopted as the frequency window function  $W(\omega;\omega_0)$ . We adopt the rectangle function with octave width. The variance spectrum of  $T_{gr}(\omega)$ ,  $\sigma_{T_{gr}}^2(\omega)$ , is defined as the second central moment with respect to  $\mu_{T_{gr}}(\omega)$  within the same frequency window;

$$\sigma_{T_{gr}}^2(\omega_0) = \frac{1}{E} \int_0^{\infty} W(\omega;\omega_0) |A(\omega)|^2 (T_{gr}(\omega) - \mu_{T_{gr}}(\omega_0))^2 d\omega \quad . \quad (9)$$

$\mu_{T_{gr}}(\omega)$  corresponds to the mean arrival time of the wave group at frequency  $\omega$ , and  $\sigma_{T_{gr}}^2(\omega)$  the duration. The time history shown in Figure 1 (a) is recognized as having a short duration in the high frequency range and a long one in the low frequency range. These characteristics are clearly seen in Figure 1 (c) and (d) which show the filtered time histories. The  $\mu_{T_{gr}}(\omega)$  and  $\mu_{T_{gr}}(\omega) \pm \sigma_{T_{gr}}(\omega)$  of the time history are shown in Figure 1 (b) respectively by solid and dash lines.  $\sigma_{T_{gr}}(\omega)$  is small in the frequency range higher than 2 Hz and takes a large value in a lower frequency range.

$\mu_{T_{gr}}(\omega)$  and  $\sigma_{T_{gr}}^2(\omega)$  satisfy the principle of linear operation shown for  $A(\omega)$  and  $\phi(\omega)$  by Eqs.(4) and (5). The  $T_{gr}(\omega)$  of an observed seismic ground motion can be estimated by summing those of source, path and local site effects because of the linearity of the differential operator. The differentiation of Eq.(5) therefore gives

$$T_{gr}^O(\omega) = T_{gr}^S(\omega) + T_{gr}^P(\omega) + T_{gr}^L(\omega) \quad . \quad (10)$$

From Eq.(10), the following relation is expected;

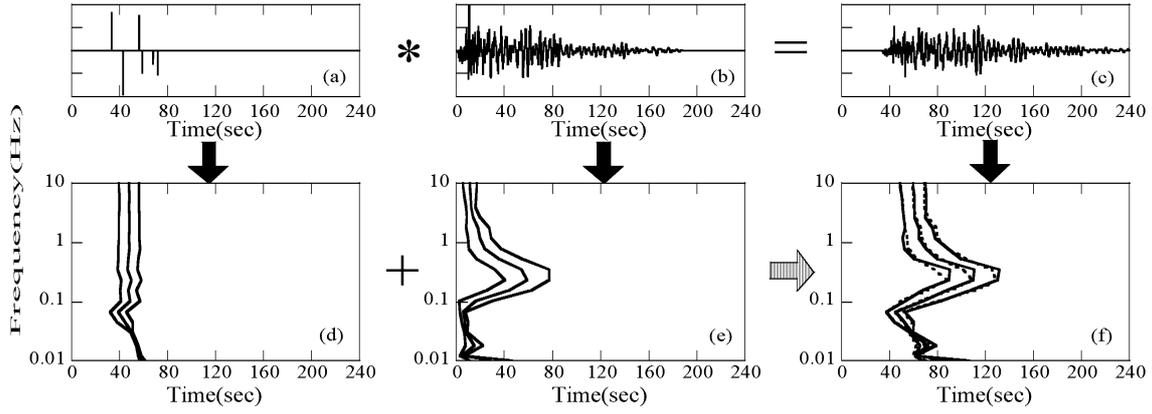
$$E[\mu_{T_{gr}}^O(\omega)] = E[\mu_{T_{gr}}^S(\omega) + \mu_{T_{gr}}^P(\omega) + \mu_{T_{gr}}^L(\omega)] \quad . \quad (11)$$

where,  $E[ ]$  means the expected value of ensemble average. Furthermore, when the source, path and local site effects are independent, the observed variance spectrum of  $T_{gr}(\omega)$ ,  $\sigma_{T_{gr}}^2(\omega)$ , will be

$$E[\sigma_{T_{gr}}^2(\omega)] = E[\sigma_{T_{gr}}^2(\omega)^S + \sigma_{T_{gr}}^2(\omega)^P + \sigma_{T_{gr}}^2(\omega)^L] \quad . \quad (12)$$

Eqs.(11) and (12) are verified by an numerical example as shown in Figure 2. For simple representation, the first two terms of the right hand side of Eqs.(11) and (12) are considered. The panels (a) and (b) in Figure 2 are the modeled time histories of source and path effects. The panel (c) shows the observed time history which is obtained from convolution of (a) and (b). Observed  $\mu_{T_{gr}}^O$  and  $\sigma_{T_{gr}}^O$  are calculated by Eqs. (7) and (9), as shown by the solid lines in the panel (f). On the other hand, the observed  $\mu_{T_{gr}}^O$  and  $\sigma_{T_{gr}}^O$  are also obtained by Eqs. (11) and (12), respectively, as shown by the dash lines in (f);  $\mu_{T_{gr}}^S$  and  $\sigma_{T_{gr}}^S$  (panel (d)) are calculated from the modeled time history of panel(a),  $\mu_{T_{gr}}^P$  and  $\sigma_{T_{gr}}^P$  (panel (e)) from panel (b). It is observed that the solid and dash lines are compared well. This results shows that the Eqs. (11) and (12) are approximately satisfied.

Eqs.(11) and (12) provide the basic relationship by which to represent the Fourier phase spectrum of observed seismic ground motion.



**Figure 2: Verification of the principle of linear operation on  $\mu_{Tgr}(\omega)$  and  $\sigma_{Tgr}^2(\omega)$**

### 3. INVERSION ANALYSIS TO IDENTIFY THE PATH AND LOCAL SITE EFFECTS ON THE FOURIER PHASE SPECTRUM

Iwata and Irikura(1988) carried out the inversion analysis to identify amplitude spectra of source, path and local site effects. They assumed the relationship;

$$\log A[O]_{jk} = \log A[S]_j + \log A[P]_{jk} + \log A[L]_k , \quad (13)$$

where subscript j stands for events 1,2,..,m, and k for sites 1,2,..,n. [O], [S], [P] and [L] respectively denote the observed, source, path and local site. “(ω)” representation are omitted.

$A[P]_{jk}$  was modeled as

$$A[P]_{jk} = \frac{1}{R_{jk}} \exp\left(\frac{-\omega R_{jk}}{2QV_s}\right) , \quad (14)$$

where  $R_{jk}$  is the distance from the hypocenter of event j to site k,  $V_s$  the shear wave velocity of the crust and the term  $1/R$  and the exponential function correspond respectively to geometric and internal damping: the latter being parameterized by the Q value. Using the matrix form, Eqs.(13) and (14) are rewritten

$$\left\{ \begin{array}{l} \log A[O]_{11} + \log R_{11} \\ \vdots \\ \log A[O]_{1n} + \log R_{1n} \\ \log A[O]_{21} + \log R_{21} \\ \vdots \\ \log A[O]_{2n} + \log R_{2n} \\ \vdots \\ \log A[O]_{m1} + \log R_{m1} \\ \vdots \\ \log A[O]_{mn} + \log R_{mn} \end{array} \right\} = \left[ \begin{array}{cccccccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & R_{11}\omega \log e/2V_s \\ \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 & \vdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & R_{1n}\omega \log e/2V_s \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & R_{21}\omega \log e/2V_s \\ \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 & \vdots \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & R_{2n}\omega \log e/2V_s \\ \dots & \dots \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & R_{m1}\omega \log e/2V_s \\ \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 & \vdots \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & R_{mn}\omega \log e/2V_s \end{array} \right] \left\{ \begin{array}{l} \log A[S]_1 \\ \log A[S]_2 \\ \vdots \\ \log A[S]_m \\ \log A[L]_1 \\ \vdots \\ \log A[L]_n \\ 1/Q \end{array} \right\} . \quad (15)$$

The above equation can be solved with respect to  $A[S]_j$ ,  $A[L]_k$ , and Q using the generalized inverse matrix analysis ( Lawson and Hansen (1974) ) under two constraints:  $1/Q > 0$  and  $A[L]_k > 2$ , which are determined from the amplification factor of the free surface.

The same representation is obtained for the phase spectra;

$$\sigma_{Tgr}^2[O]_{jk} = \sigma_{Tgr}^2[S]_j + \sigma_{Tgr}^2[P]_{jk} + \sigma_{Tgr}^2[L]_k . \quad (16)$$

If only small events are involved, the source time function can be modeled by an impulse; therefore,  $\sigma_{Tgr}^2[S]_j$  can be assumed to be zero.  $\sigma_{Tgr}^2[P]_{jk}$  is modeled as

$$\sigma_{Tgr}^2[P]_{jk} = \alpha^2 R_{jk}^2 , \quad (17)$$

where  $\alpha$  is a constant. Eq.(17) presupposes that the standard deviation  $\sigma_{Tgr}^2[P]_{jk}$ , which corresponds to the duration of the ground motion, is directly proportional to the hypocentral distance  $R$ .

The inversion analysis is based on the following equation which is the result of the above assumption;

$$\sigma_{Tgr}^2[O]_{jk} = \alpha^2 R_{jk}^2 + \sigma_{Tgr}^2[L]_k . \quad (18)$$

Eq.(18) is rewritten using matrix form as

$$\begin{Bmatrix} \sigma_{Tgr}^2[O]_{11} \\ \vdots \\ \sigma_{Tgr}^2[O]_{1n} \\ \sigma_{Tgr}^2[O]_{21} \\ \vdots \\ \sigma_{Tgr}^2[O]_{2n} \\ \vdots \\ \sigma_{Tgr}^2[O]_{m1} \\ \vdots \\ \sigma_{Tgr}^2[O]_{mn} \end{Bmatrix} = \begin{Bmatrix} 1 & 0 & 0 & R_{11}^2 \\ \vdots & \ddots & \vdots & \\ 0 & 0 & 1 & R_{1n}^2 \\ 1 & 0 & 0 & R_{21}^2 \\ \vdots & \ddots & \vdots & \\ 0 & 0 & 1 & R_{2n}^2 \\ \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & R_{m1}^2 \\ \vdots & \ddots & \vdots & \\ 0 & 0 & 1 & R_{mn}^2 \end{Bmatrix} \begin{Bmatrix} \sigma_{Tgr}^2[L]_1 \\ \vdots \\ \sigma_{Tgr}^2[L]_n \\ \alpha^2 \end{Bmatrix} \quad (19)$$

The same technique can be used to solve Eq.(19) for  $\sigma_{Tgr}^2[L]_k$  and  $\alpha^2$  under the constraints:  $\alpha^2 > 0$  and  $\sigma_{Tgr}^2[L]_k > 0$ .

#### 4. RESULTS AND DISCUSSIONS

Identification of the path and local site effects on the phase spectra was made using the seismic records observed at three stations in Osaka, Japan. The records of five small events with magnitudes less than 5.0 were used. Information about the stations and events used in this analysis is given in Tables 1 and 2. Figure 3 shows the locations of the sites and events. The sites are operated by CEORKA (Toki et al. (1995)) and are located near the area damaged by the 1995 Kobe earthquake. CHY and KBU are considered to be rock sites, and ABN a soil site. The ground motions were recorded by velocity seismograms of the over-damped type.

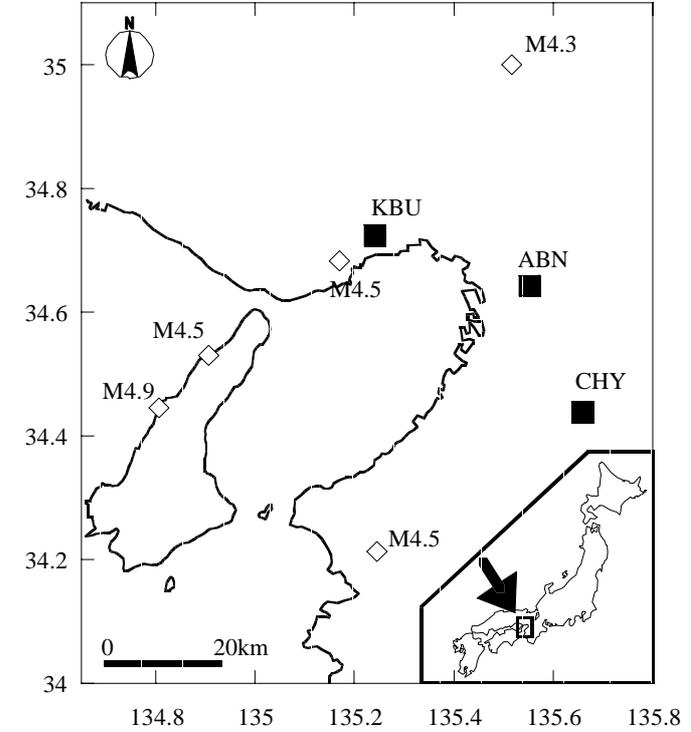


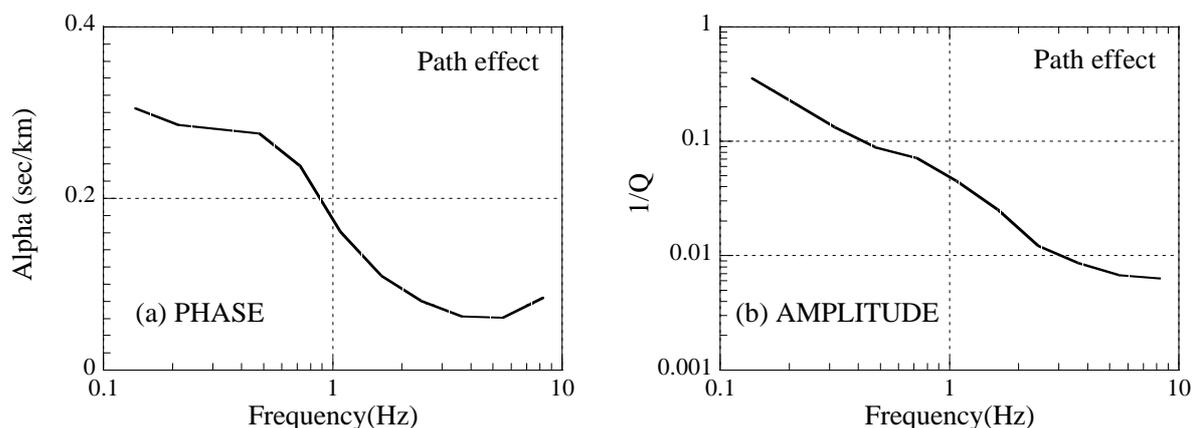
Figure 3: Location of events and sites used in the analysis

Table 1: Observation sites

Station	Latitude	Longitude	Installation level	Condition
KBU	34.7250	135.240	In underground tunnel	Rock
CHY	34.4390	135.659	Ground surface	Rock
ABN	34.6431	135.552	Ground surface	Soil

**Table 2: The events used in the analysis**

Epicenter location	Date and Origin time	Latitude	Longitude	Mag. (JMA)
NW Wakayama Pref.	Oct. 16, 1994, 08:21:07	34.2130	135.245	4.5
Kyoto-Osaka Border Reg.	Oct. 24, 1994, 11:51:06	35.0000	135.517	4.3
SE Hyogo Pref.	Jan. 18, 1995, 06:50:19	34.6830	135.170	4.5
Awaji Island Reg.	Jan. 23, 1995, 06:02:28	34.5300	134.907	4.5
Awaji Island Reg.	Feb. 18, 1995, 21:37:34	34.4450	134.807	4.9



**Figure 4: Identified  $\alpha$  and  $1/Q$  in the path effects**

Figure 4 (a) shows the identified value of  $\alpha$  defined by Eq. (17). The identified  $1/Q$ , obtained from the analysis of the Fourier amplitude spectra by Eq. (13), is shown in Figure 4 (b). These two figures show similar tendencies; both  $1/Q$  and  $\alpha$  decrease as frequency increases. This finding is understood from the following:

1. a large  $1/Q$  means large damping,
2. large damping is caused by strong wave scattering,
3. strong scattering lengthens the duration,
4. a long duration means a large  $\sigma_{Tgr}[P]$ ,
5. a large  $\sigma_{Tgr}[P]$  results in a large  $\alpha$ .

Figures 5, 6 and 7 show the identified local site effects. In these figures, (a) shows the  $\sigma_{Tgr}[L]_k$  of the phase spectra and (b) the amplification factor  $A[L]_k$  of the amplitude spectra. At the rock sites, a large  $A[L]_k$  is seen in the high frequency range, as shown in Figures 5 (b) and 6 (b). This is explained by assuming there is a soft and shallow surface layer over the bedrock at these sites. Interestingly, large  $\sigma_{Tgr}[L]_k$  values in the phase spectra are seen in the high frequency range in Figures 5(a) and 6(a). In the contrast, a large  $A[L]_k$  and large  $\sigma_{Tgr}^2[L]_k$  are present in the low frequency range at the soil site (Figure 7). This site is considered to have a thick soil layer more than 600 m deep. These finding can be understood from the following facts:

1. a large  $A[L]$  is caused by superposition of the multiple reflections of the waves in the layer,
2. strong multiple reflection corresponds to strong scattering,
3. strong scattering leads to a long duration,
4. a long duration results in a large  $\sigma_{Tgr}[L]$ .

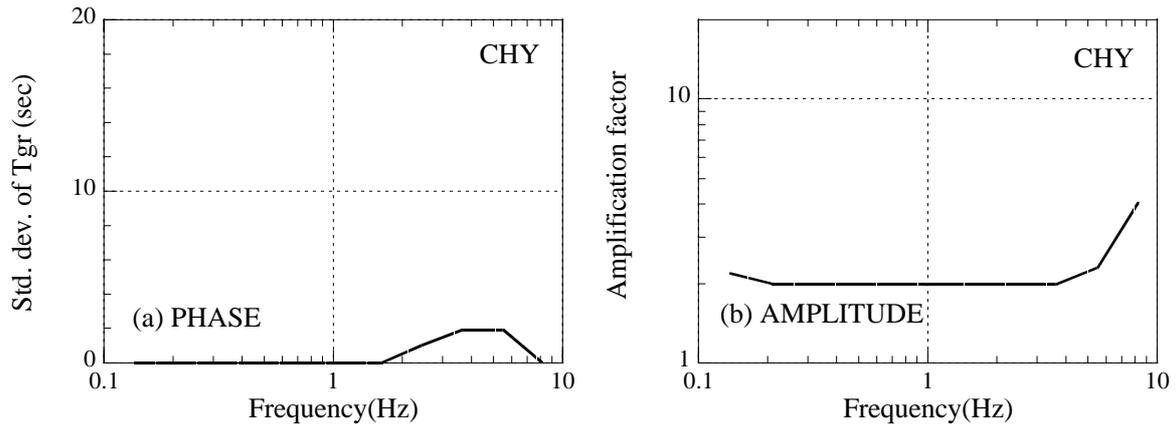


Figure 5: Identified  $\sigma_{Tgr}[L]$  and  $A[L]$  at CHY (rock site)

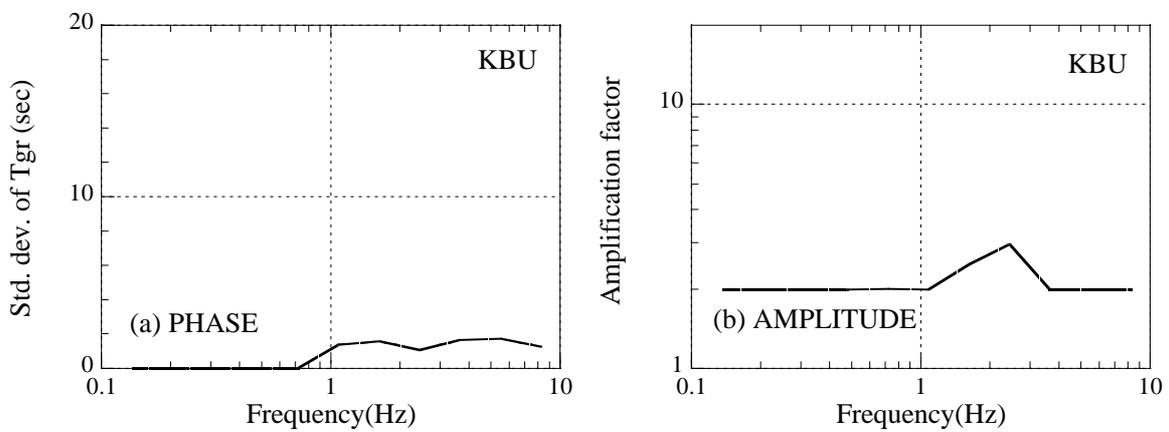


Figure 6: Identified  $\sigma_{Tgr}[L]$  and  $A[L]$  at KBU (rock site)

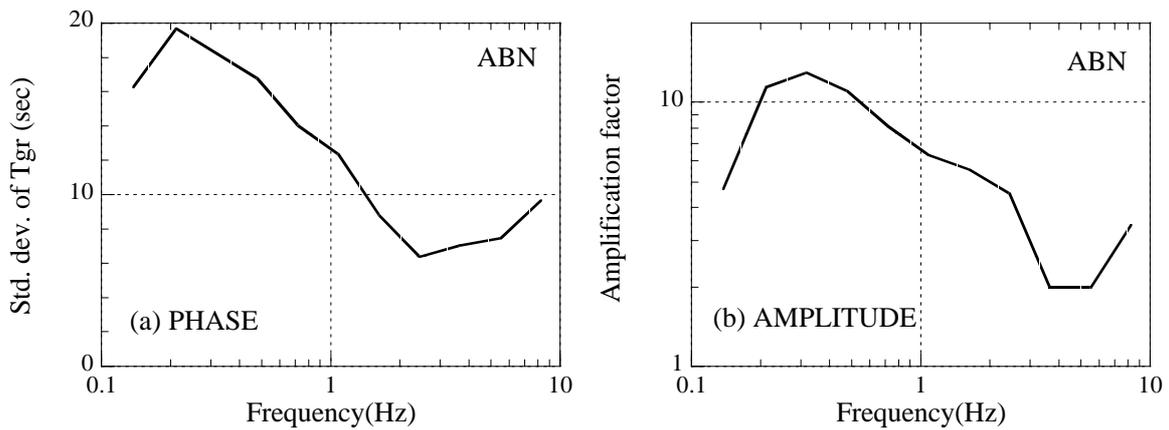


Figure 7: Identified  $\sigma_{Tgr}[L]$  and  $A[L]$  at ABN (soil site)

We conclude from the above discussion that the path and local site effects on Fourier phase spectra have a close relationship to these effects on amplitude spectra: i.e., the characteristics observed in the amplitude spectrum also are observed in the phase spectrum. Further study is needed to clarify the mechanism of the path and local site effects on the Fourier phase spectra.

## 5. CONCLUSION

1. A new method for representing the Fourier phase spectrum, which is based on the average and variance spectra of the group delay time, is introduced.
2. The observed average and variance spectra of the group delay time are described by the summation of the spectra of the source, path and local site effects.
3. A new method by which to identify the characteristics of the path and local site effects from Fourier phase spectra is proposed.
4. Effects of the path and local site on the phase spectrum are identified from observed ground motion records of small events.
5. The identified variance spectrum of the path effects takes a large value in the frequency range where  $1/Q$  is large.
6. The identified variance spectrum of the local site effects takes a large value in the frequency range where the amplification factor is large.

Our proposed method can be extended to identify the phase characteristics of source spectrum. The average spectrum of the group delay time,  $\mu_{T_{gr}}(\omega)$ , can be identified by the same method.

## REFERENCE

- Cohen, L. (1995), *Time-Frequency Analysis*, Prentice Hall, Englewood Cliffs, New Jersey.
- Iwata, T. and Irikura, K. (1988), "Source parameters of the 1983 Japan sea earthquake sequence", *Journal of Physics of Earth*, 36, 155-184.
- Izumi, M. and Katsukura, H. (1983), "A fundamental study on extraction of phase-information in earthquake motions", *J. Struct. Constr. Eng., AIJ*, 327, 20-28 (in Japanese).
- Katsukura, H., Watabe, T. and Izumi, M. (1984), "A study on the Fourier analysis of non-stationary seismic wave", *Proceedings of 8th World Conference of Earthquake Engineering*, II, 525-532.
- Lawson, C.L. and Hansen, R.J. (1974), *Solving Least Squares Problems*, Prentice Hall, Englewood Cliffs, New Jersey.
- Ohsaki, Y. (1979), "On the significance of phase content in earthquake ground motions", *Earthquake Engineering and Structural Dynamics*, 7, 427-439.
- Soda, S. (1986), "Basic study on application of probability characteristics of phase inclination to nonstationary random vibration analysis", *J. Struct. Constr. Eng., AIJ*, 365, 48-57 (in Japanese).
- Papoulis, A. (1986), *The Fourier Integral and Its Application*, McGraw Hill, Maidenhead.
- Toki, K., Irikura, K. and Kagawa, T. (1995), "Strong motion records in the source area of the Hyogoken-nanbu earthquake, January 17, 1995, Japan", *Journal of Natural Disaster Science*, 16, 2, 23-30.