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ASSESSMENT OF SEISMIC VULNERABILITY TO OUT-OF-PLANE COLLAPSE OF MASONRY WALLS

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SUMMARY

Simplified collapse mechanisms are proposed for the study of seismic vulnerability in T-connections between the façade and transversal walls of historical masonry buildings. Masonry work is modelled by a regular assembly of rigid blocks and elastic plastic joints with friction and without cohesion. A comprehensive analytical formulation is developed for the structure subject to a ground peak velocity. Finally, the analytical predictions are compared with numerical results obtained by means of a discrete element method.

INTRODUCTION

When investigating the response of historical masonry buildings to earthquake, the role of transversal walls as bracing elements for façade walls - which are most vulnerable to seismic forces - is highly important. The literature is rich in studies of the contribution of these walls in buildings with rigid diaphragms or other connections that spread horizontal seismic forces among resistant shear walls [Tomazevic et al., 1991; AA.VV., 1995]. Various experimental tests and models are also available to study the behaviour of individual walls or panels subject to in-plane loads [Page, 1981; Anthoine, 1991; Magenes and Calvi, 1997; Gambarotta and Lagomarsino, 1997]. However, traditional masonry buildings often lack these connections; nor are their floors able effectively to redistribute the seismic loads among resistant walls. Resistance to earthquakes is left in these cases to the continuity of the masonry fabric between the façade and lateral walls. The better the connections, the greater the restraining forces with respect to the out-of-plane motion of the façade.

In order to estimate seismic resistance, simple failure mechanisms are proposed, following an approach introduced by Giuffrè [Giuffrè, 1993] in the mechanics of historical masonry buildings. A similar method was applied in an earlier work [de Felice, 1999] to check the resistance of masonry buildings to static horizontal forces. Here the analysis is extended to study the response of the fabric to an impulse, in the belief that an instantaneous force gives a better approximation of seismic action, while also providing a parameter for the structure's resistance that can be directly compared with the expected peak of the ground seismic velocity.

Two classes of mechanism are examined: (1) the detachment of the façade wall from the inside wall due to a vertical fracture at the abutment and (2) collapse due to the opening of a diagonal crack on the transversal wall. A careful examination of traditional stone-masonry houses damaged by earthquakes often reveals the presence of cracks close to the connection with the façade, increasing from bottom to top, which are related to the mechanisms of failure proposed in the present study.

The walls are represented as a regular assembly of rigid and infinitely resistant blocks and elastic plastic joints with friction but no cohesion. This is an extremely simple model of masonry, which is nonetheless frequently used for analysing dry-stone or weak-mortar masonry, since it possesses the main features of historical fabrics [Baggio and Trovalusci, 1993]. Once the class of mechanism is specified, the failure load is obtained by keeping its expression to a minimum in relation to the free parameters in the mechanism concerned. This makes it possible to express the earthquake resistance as a function of the geometric and mechanical features of the

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structure. The analytical results are finally compared with the numerical analysis performed using a discrete element method.

ANALITICAL PREDICTION OF COLLAPS LOAD

Most research aimed at evaluating seismic vulnerability of masonry structures, is devoted to obtaining the collapse load of the structure with respect to static horizontal forces. However, the value of static resistance is difficult to correlate with a seismic intensity factor. Comparison with the peak ground acceleration leads to an overestimation of vulnerability and is not suitable because, as is well known, the peak acceleration is not representative of the damage potential of ground motion. Among scalar representations, a more convenient description of ground motion intensity is given by the peak velocity. Let us consider, then, a masonry structure consisting of the exterior wall of a building, joined to the transversal walls and subjected to out-of-plane seismic action represented by an impulse with peak velocity $v_g = a_g\Delta t$, where a_g is the value of the acceleration which is applied for a short time Δt ($\Delta t \rightarrow 0$). The overturning motion of the exterior wall is restrained by its own weight and by the connections with orthogonal walls. The motion takes place only when the friction due to block texture is overcome and a crack forms in the transversal walls.

Let b and H be respectively the thickness and the height of the exterior wall, while s and l are the thickness and the relative distance of transversal inner walls respectively (Figure 1). Masonry work is represented as a regular assembly of blocks with height a, length $2c$ and depth equal to that of the wall. Assume that the blocks are rigid and infinitely resistant, while the contacts among them are cohesionless joints with Coulomb friction. Therefore, the properties of the masonry are defined only by a mechanical parameter f, which is the friction coefficient in the joints, and by the shape and dimension of the blocks.

In order to define some analytical expressions of seismic resistance, two simple collapse mechanisms are investigated:

1. the out-of-plane collapse of the façade, which breaks away from the transversal wall with a vertical crack at the abutment.
2. the rigid body out of plane overturning of the façade with part of the transversal wall, breaking away with a diagonal crack at an angle ψ with respect to the vertical.

Both mechanisms are sketched in Figure 1 where h is the height of the part of the façade subject to the tilting motion.

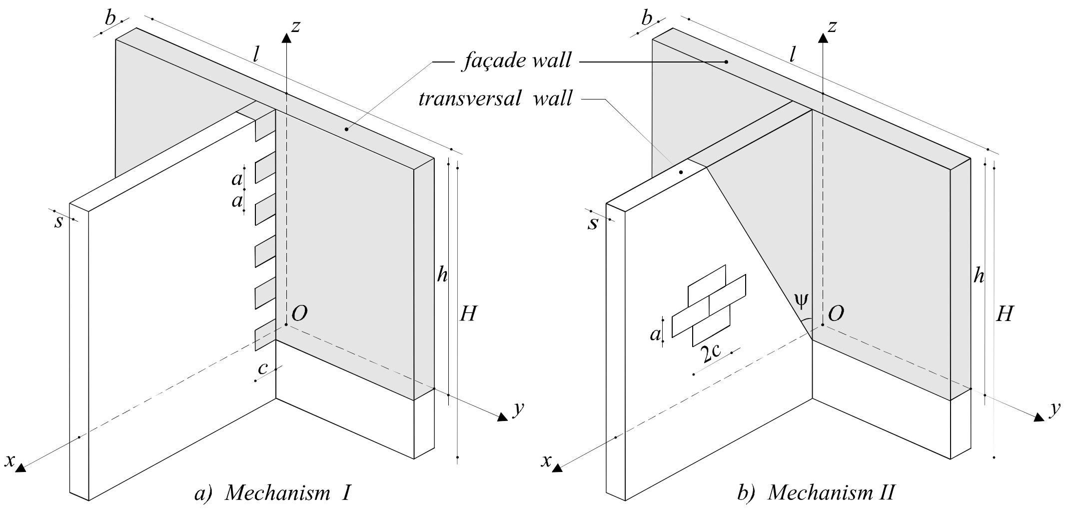

Figure 1: Failure mechanisms: a) detachment of the façade from the lateral wall with a vertical crack at the abutment; b) overturning of the façade and part of the transversal wall with the formation of a diagonal crack.

Since for both mechanisms a part of the structure fails by rotating around axis y while the rest of the fabric follows the ground motion rigidly, the following balance equation can be written:

$$I\ddot{\theta}+M_R(\theta)+M_V(\dot{\theta})=mz_G a_g \quad (1)$$

where m and I are respectively the mass of the failing part of the structure and the inertia moment with respect to axis y, M_R and M_V are the moment of the restraining and viscous forces respectively, θ is the rotation angle, z_G is the height of the centre of gravity in the reference *Oxyz*, and a_g is the ground motion acceleration.
By integrating (1) between 0 and Δt, with initial conditions $\theta(0)=\dot{\theta}(0)=0$, for $\Delta t \to 0$, we obtain:

$$I\dot{\theta}(\Delta t)=mz_G v_g \quad (2)$$

where

$$v_g = \lim_{\Delta t \to 0}\int_0^{\Delta t} a_g(t)dt \quad (3)$$

is the peak velocity representing the out-of-plane impulse applied to the front wall. Equation (2) gives the value of the rotation velocity $\dot{\theta}_0$ of the failing part of the fabric at the end of the impulse:

$$\dot{\theta}_0 := \dot{\theta}(\Delta t)=\frac{z_g v_g}{i} \quad (4)$$

with $i = I/m$. If no external forces, other then gravity, remains after the impulse, according to equation (1) and neglecting the viscous term, the variation in kinetic energy must be equal to the work of restoring forces $W(\theta)$. This equality allows us to express the angular velocity $\dot{\theta}(t)$ as a function of $\dot{\theta}_0$ and $W(t)$:

$$\dot{\theta}(t)=\sqrt{\dot{\theta}_0^2-2\frac{W[\theta(t)]}{I}} \quad (5)$$

Let us now suppose that the function $W(\theta)$ increases and peaks at a certain value $\theta = \theta_m \geq 0$. This conjecture is explained by the fact that gravity, which is a large part of the restraining force, reaches a maximum when the centre of gravity passes through the vertical line on the rotation centre *O*. Therefore if $\dot{\theta} \leq 0$ for $\theta = \theta_m$, the motion is confined, otherwise it becomes boundless and overturning ensues. The minimum value of the rotation velocity that produces the collapse of the wall is obtained from (5) by putting $\dot{\theta}=0$:

$$\dot{\theta}_0^2 = 2\frac{W(\theta_m)}{I} \quad (6)$$

Consequently, the minimum $\bar{v}_g$ ground velocity that leads to the failure of the structure is reached by substituting the expression of $\dot{\theta}_0$ in equation (4):

$$\bar{v}_g = \frac{i}{z_G}\sqrt{2\frac{W(\theta_m)}{I}} \quad (7)$$

Failure mechanism I: detachment of the façade

Two kinds of restraining forces are developed in this mechanism: (a) the weight of the façade; (b) the friction on the horizontal joints among blocks at the abutment between the front and transversal walls. Since the friction is proportional to the weight of the masonry above, the density of friction force distributed along the abutment, is:

$$f_a(z)=\begin{cases} g\gamma s f\,\dfrac{(h-z)(c-u(z))}{a} & \text{if } u(z)<c \\ 0 & \text{if } u(z)\geq c \end{cases} \quad (8)$$

where γ is the mass density of masonry, z is the height with respect to the rotation axis y and $u(z)$ is the horizontal component of the relative displacement between the front and transversal walls.
Assuming θ<<1 and describing the motion with a linearised kinematics, so that $u(z) = \theta z$, the power of friction forces for the rotation velocity $\dot{\theta}$ results:

$$\dot{W}_a = \int_0^h f_a(z)\dot{\theta}z\,dz = \begin{cases} g\gamma sfh^3 \dfrac{2c-h\theta}{12a}\dot{\theta} & \text{if} \quad \theta < \dfrac{c}{h} \\ g\gamma sfc^3 \dfrac{2h\theta-c}{12a\theta^3}\dot{\theta} & \text{if} \quad \theta \geq \dfrac{c}{h} \end{cases} \tag{9}$$

The work expended by friction forces in the rotating motion of the front is given by integrating the last expression with respect to $dt = d\theta/\dot{\theta}$. Adding to this term the work done by gravity, the complete expression of the restraining work W is obtained:

$$W(\theta) = \frac{1}{2} g\gamma hlb^2 \left(1 - \frac{h}{2b}\theta\right)\theta + g\gamma sfc^2 \frac{6h^2\theta^2 + c^2 - 4ch\theta}{24a\theta^2} \tag{10}$$

and, by introducing the dimensionless parameters

$$\lambda = \frac{h}{b} \qquad \rho = \frac{l}{s} \qquad \alpha = \sqrt{\frac{cf}{a}} \qquad \beta = \frac{c}{b} \tag{11}$$

becomes:

$$W(\theta) = 24g\gamma sb^3 \left(-6\rho\lambda^2\theta^2 + 12\rho\lambda\theta + 6\alpha^2\lambda^2\beta - \frac{4}{\theta}\alpha^2\beta^2\lambda + \frac{1}{\theta^2}\alpha^2\beta^2\right) \tag{12}$$

The rotation angle that corresponds to the maximum of the work of restraining forces can be obtained by solving the equation $dW/d\theta = 0$, which explicitly gives:

$$\theta^4 - \frac{1}{\lambda}\theta^3 - \frac{\alpha^2\beta^2}{3\rho\lambda}\theta - \frac{\alpha^2\beta^3}{6\rho\lambda^2} = 0 \tag{13}$$

The exact solution of equation (13) is long and difficult to handle, but an approximate solution is easily obtained by developing equation (12) in a Taylor series around the point $\overline{\theta} = 1/\lambda$ and neglecting terms of degree greater than two:

$$\theta_m \approx \frac{1}{\lambda}\left[\frac{5}{6} + \frac{(\alpha\beta\lambda)^2}{18\rho} + \frac{1}{18\rho}\sqrt{9\rho^2 + (\alpha\beta\lambda)^2\rho(30+18\beta) + (\alpha\beta\lambda)^4}\right] \tag{14}$$

The expression of θ_m allows us to compute, through equation (12), the kinetic energy and then, through equation (7), the velocity peak required to overturn the façade. The normalised value of the velocity $\overline{v}_g\,(gb)^{-1/2}$, whose expression is omitted for the sake of brevity, is a function of the dimensionless parameters λ, β and ρ/α^2.

The height h of the part of the wall subject to tilting, which is not defined *a priori*, can be obtained by seeking the value of the slenderness λ that corresponds to the minimum velocity peak leading to the failure of the structure. This minimum, which is difficult to evaluate analytically, is obtained by means of a numerical minimization procedure. The results are sketched in Figure 2 as functions of the ratio $\rho = l/s$, for different values of the parameters α and β, together with the results of mechanism II and those obtained by the numerical analysis described below.

Failure mechanism II: diagonal crack in the transversal wall

A correct estimate of the stability condition of the wall with respect to mechanism II requires a careful choice of the slope of the crack in the transversal wall. This can be done by resorting to a homogenisation method, since, by assuming that the dimensions of the blocks are much smaller than those of the wall (i.e. $a \ll h$), it can be shown [de Buhan and de Felice, 1997] that masonry is equivalent to a homogeneous anisotropic continuum medium whose strength domain is defined by the following inequalities:

$$\begin{aligned} \pm\tau_{xz} + f\,\sigma_z &\leq 0 \\ \frac{a}{c}\sigma_x \pm \left(\frac{fa}{c} + 1\right)\tau_{xz} + f\,\sigma_z &\leq 0 \end{aligned} \tag{15}$$

in the space of plane stress ($\sigma_x\ \sigma_z\ \tau_{xz}$). According to the anisotropy of this fictitious medium, the contact forces that resist the opening of a crack depend on the slope of the crack; in particular it can easily be shown that, for the rotating motion considered in mechanism II, no friction arises if the inclination of the crack fulfils the condition:

$$\tan\psi \geq \sqrt{cf/a} \tag{16}$$

Therefore, the limit value $(cf/a)^{1/2}$ defines a preferential attitude of discontinuity in the wall, for which the restraining force in the failure mechanism is given exclusively by the own weight of masonry the block involved in the motion, whose centre of gravity in the reference *Oxyz* has coordinates:

$$x_G = \frac{b}{3}\frac{3\rho + \lambda^2\alpha^2 + 3\alpha\lambda}{2\rho + \lambda\alpha} \qquad y_G = 0 \qquad z_G = \frac{b}{3}\frac{\lambda(3\rho + 2\lambda\alpha)}{2\rho + \lambda\alpha} \tag{17}$$

where $\alpha = \tan\psi = (cf/a)^{1/2}$. The work of the restraining force, which is simply equal to the variation in potential energy, peaks when the centre of gravity is aligned vertically through the rotation centre *O*:

$$W(\theta_m) = mg(r_G - z_G) \tag{18}$$

where $r_G = \sqrt{z_G^2 + x_G^2}$ and m is the mass of the part of the structure undergoing the rotating motion.
The velocity peak producing the failure of the structure is given by substituting eq. (18) in (7):

$$\bar{v}_g = \sqrt{\frac{2gi}{z_G}\left(\frac{r_G}{z_G} - 1\right)} \tag{19}$$

By expressing $i = I/m$ as a function of the non-dimensional parameters ρ, α, λ previously introduced:

$$i = \frac{b^2}{6}\frac{(3\alpha + \alpha^3)\lambda^3 + 4(\rho + \alpha^2)\lambda^2 + 6\lambda\alpha + 4\rho}{2\rho + \lambda\alpha} \tag{20}$$

and considering eq. (17), the minimum value of velocity that causes the overturning according to mechanism II is obtained as a function of the (normalised) height λ.
The critical value of the height corresponds to the minimum velocity $\bar{v}_g$. As in the previous case, the search for this minimum is performed by a numerical procedure whose results are illustrated in Figure 2.

Table 1: Results of the numerical analysis

Test n°	Wall dimensions				Block dimensions		Joint friction				Results			
	H	*L*	*s*	*b*	*a*	*c*	*f*	ρ	β	α^2	h_v	λ_v	v_g	$v_g\,(gb)^{-1/2}$
B1	9	6	0,50	0,50	0,30	0,20	0,6	12	0,4	0,4	6,9	13,8	0,83	0,377
B2	9	6	0,50	0,50	0,20	0,20	0,6	12	0,4	0,6	6,8	13,6	0,98	0,443
B3	9	6	0,50	0,50	0,15	0,20	0,6	12	0,4	0,8	5,9	11,7	1,13	0,509
B4	9	6	0,50	0,25	0,30	0,20	0,6	12	0,8	0,4	3,9	15,6	0,88	0,564
B5	9	6	0,50	0,25	0,20	0,20	0,6	12	0,8	0,6	2,8	11,2	1,13	0,720
B6	9	6	0,50	0,25	0,15	0,20	0,6	12	0,8	0,8	2,3	9	1,32	0,846
C1	9	6	0,50	0,50	0,30	0,20	0,6	5	0,4	0,4	6,9	13,8	1,32	0,598
C2	9	6	0,50	0,50	0,20	0,20	0,6	5	0,4	0,6	4,4	8,8	1,72	0,775
C3	9	6	0,50	0,50	0,15	0,20	0,6	5	0,4	0,8	3,8	7,5	1,91	0,864
C4	9	6	0,50	0,25	0,30	0,20	0,6	5	0,8	0,4	2,7	10,8	1,37	0,877
C5	9	6	0,50	0,25	0,20	0,20	0,6	5	0,8	0,6	2,0	8	1,72	1,096
C6	9	6	0,50	0,25	0,15	0,20	0,6	5	0,8	0,8	1,4	5,4	1,96	1,253

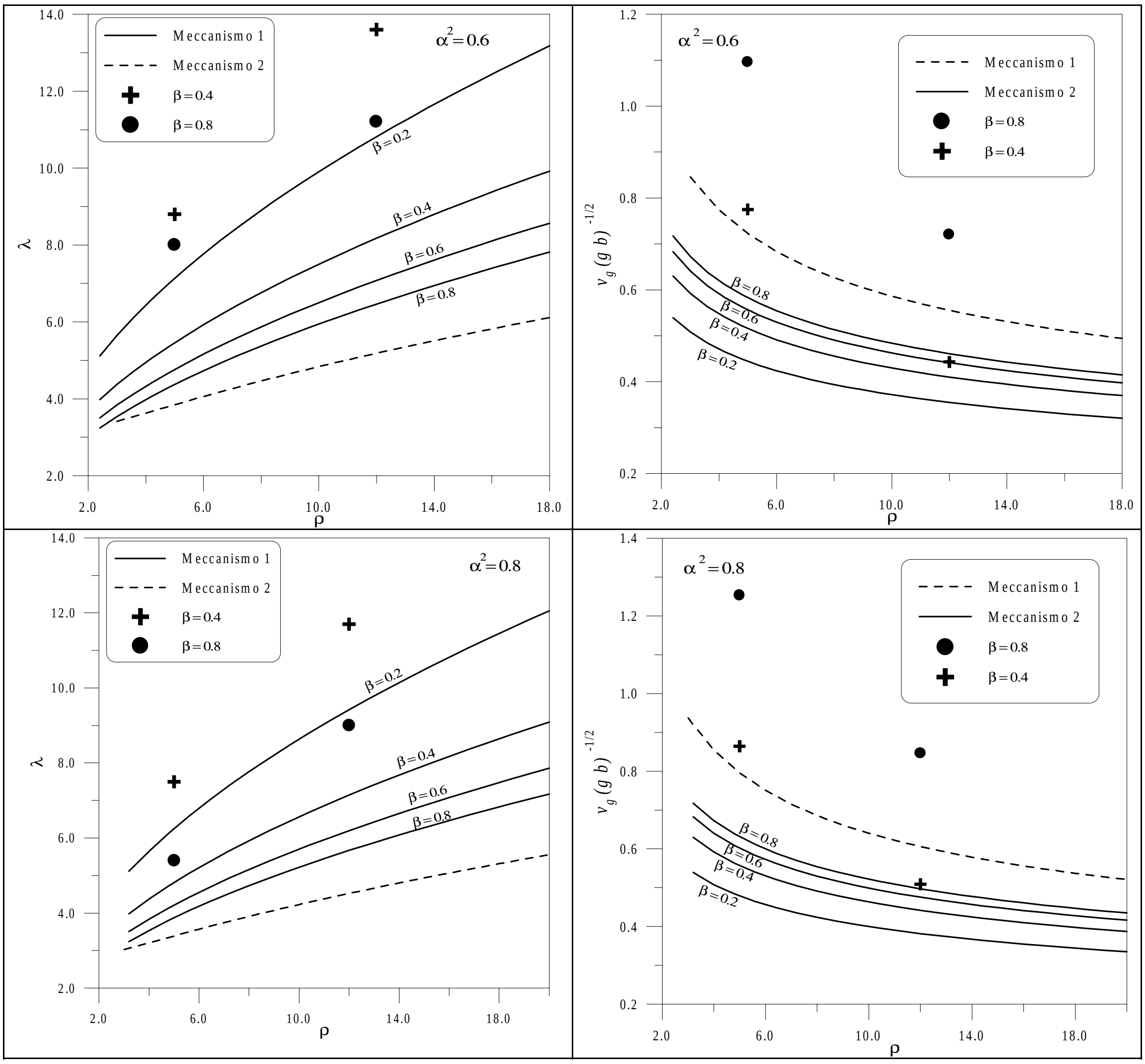

Figure 2: Slenderness and normalised collapse acceleration of the façade subject to an impulsive out of plane force: comparison between the analytical expressions corresponding to the two failure mechanisms proposed and the values of the numerical analysis performed using a discrete element method.

NUMERICAL PREDICTION OF COLLAPSE LOAD

The results of the limit analysis for the chosen failure mechanisms were compared with those of numerical analysis performed using a discrete element model (Udec). The method originally developed in the framework of the mechanics of fractured rocks and granular media, consists in solving the equations of motion of the system using an integration algorithm explicit in the time domain.

In our analysis each block is represented by an indeformable discrete element, while the joints are described using non linear punctiform contact elements: respectively a no-tension elastic normal spring and an elastic-plastic tangential spring with a Coulomb friction coefficient f= 0.6.

The greater length l of the façade wall, compared with the thickness s of the transversal wall, was described by increasing the mass density γ of the façade by the coefficient $\rho = l/s$. The contact stiffnesses were similarly modified so as to obtain a uniform lowering of the wall under its own weight. Whereas earlier studies [Pagnoni and Nisticò, 1993] considered static lateral loading, we examined the dynamic response of the structure to an impulse: loading was obtained by introducing a horizontal acceleration a_g acting for a brief time Δt and monitoring subsequent events. Damping was assumed to be nil, after verifying that the use of a coefficient other than zero noticeably affects the response by generating an additional dissipation mechanism in the structure.

A transversal wall measuring 9 m x 6 m x s= 50 cm was considered together with façade walls of varying thicknesses (b=25:50 cm) and length (l = 2.5:6 m), with three types of masonry, obtained using blocks of constant length $2c$ = 40 cm and varying height a from 15 to 30 cm. Table 1 summarises the results of the tests.

Figure 3 shows the collapse configuration of some of the walls studied. Figure 2 compares the results with the analytical predictions.

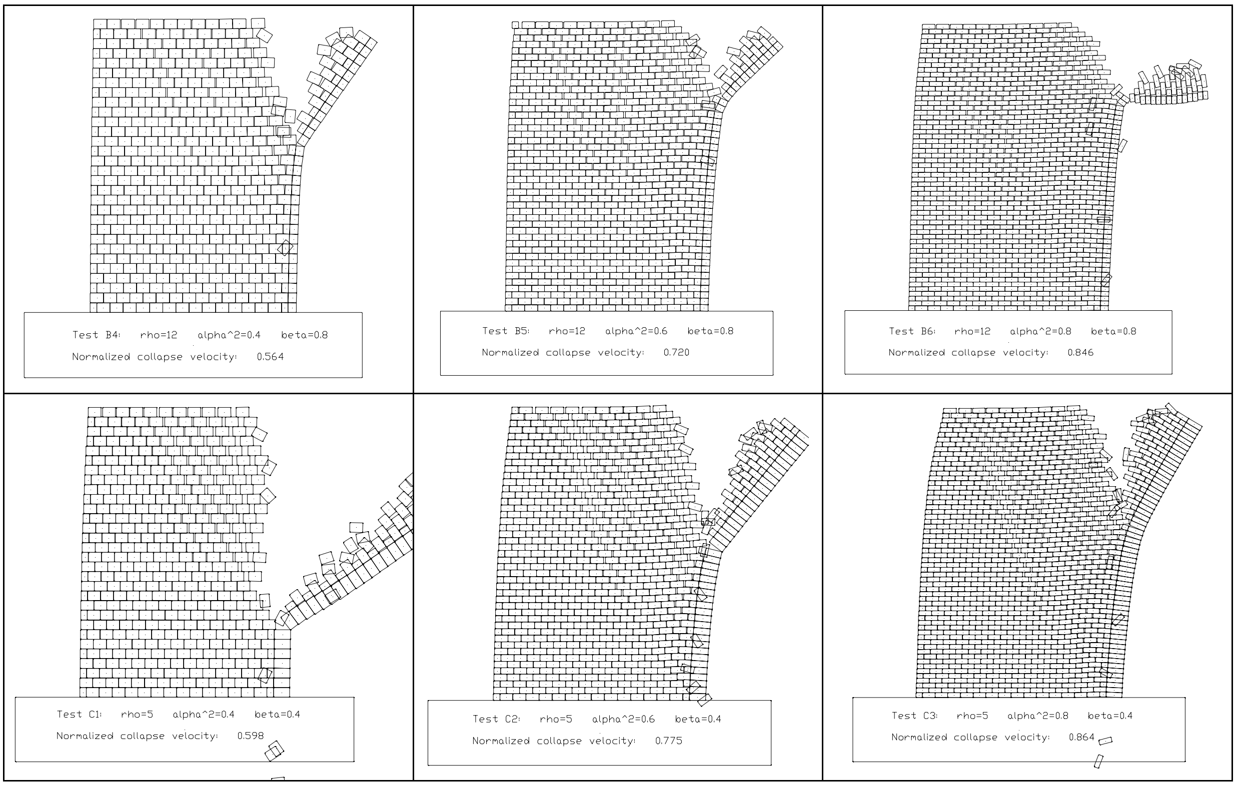

Figure 3. Failure mechanisms and corresponding normalised collapse velocity of six identically-shaped transversal walls (*H*=9 m, *l*=6 m, *s*=50), built of variously sized blocks, when varying the thickness (*b* = 25 : 50 cm) and le length (*l* = 2.5 : 6 m) of the façade.

DISCUSSION OF RESULTS AND CONCLUSIONS

Simplified failure mechanisms were used to obtain analytic predictions of the resistance of the façade under out-of-plane impulsive forces. The formulation accounts for the connections with lateral walls and assumes that collapse takes place with a rigid body motion of a part of the wall that brakes off from the rest of the structure. Figure 2 represents the tilting slenderness λ and the normalised velocity peak of collapse $v_g(gb)^{1/2}$ in the two mechanisms studied, expressed as a function of the ratio $\rho = l/s$ between the length of the façade and the thickness of the orthogonal wall for different values of $\alpha = (cf/a)^{1/2}$. In the interval of values examined, the velocity peak of collapse given by mechanism I (detachment of the façade) is always lower than that calculated for mechanism II (fracture of the orthogonal wall), so that it is to be presumed that mechanism I is the critical one. For both mechanisms, however, the speed of collapse decreases as ρ increases and increases in parallel with α. The resistance of the wall also depends on the parameter β, which expresses the ratio between the size of the block and that of the wall: the larger the block the greater the resistance. This ratio is not evident in the expressions derived using the collapse mechanism II, which was defined by a homogenising method valid for $\beta \rightarrow 0$.
The figures show the points corresponding to the results of the numerical analysis. The comparison shows that the theoretic model notably underestimates both the peak value and the height of the wall subject to overturning. The effect in relation to ρ, α and β agrees with the numerical results, which are nonetheless more sensitive to the parameters ρ and β. Whereas when ρ=12 and β=0.4 (major distance between transversal walls and relatively small blocks) the results agree well, when β increases and ρ decreases the resistance calculated numerically using the discrete element method is much greater than the theoretic prediction.
The numerical analysis also shows an opening of some of the vertical joints, particularly in the upper part of the wall and near to the fracture. The formation of these cracks does not affect the form of the collapse mechanism (notice that the main fracture falls between the two mechanisms considered), but does influence the value of the impulse that produces the collapse, since some of the kinetic energy is dissipated by friction in the formation of the cracks. This at least partly explains the difference between the analytical forecasts and the numerical results.

The former ignore the formation of these fractures and underestimate the energy required to generate the failure mechanism. Nonetheless, it is possible that the formation of these fractures is associated with the complete absence of cohesion in the joints and that an albeit modest tensile stress-resistance reduces widespread cracking.
With regard to the slenderness λ of the wall that is overturned, it is interesting to remark that in all the cases we considered, both analytical predictions and numerical results show that only the upper part of the wall fails, in contrast to what happens with free-standing walls. The slenderness increases with ρ and decreases with α. While the numerical values are higher than those calculated analytically, the tendencies and the differences are not excessive.
Despite the schematic nature of the mechanisms considered and the differences vis-à-vis the numerical analysis, the proposed method appears to capture the fundamental aspects of the problem and gives an adequately realistic prediction of the resistance of these walls to changes in their geometry and in the masonry fabric. Specifically, the expressions obtained for the velocity peak give a sufficiently clear picture of:

1. The decreasing level of seismic resistance in parallel with the decreasing size of the blocks (in relation to the thickness of the façade) and the quality of the masonry fabric (represented by the increasing height *a* of the blocks for equal length $2c$)
2. The lower level of seismic resistance in parallel with the increase in the distance *l* between the transversal walls (with respect to their thickness *s*).

We therefore feel that, albeit within the limits indicated, these expressions are effective and mechanically sound instruments for predicting damage and estimating the vulnerability of historical masonry buildings.

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