



## LIMIT STATES FOR PERFORMANCE-BASED DESIGN

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### SUMMARY

A true performance-based design procedure must be based on the reliability-based design principles that have evolved over the past few decades. One issue that must be addressed is the number of limit states that should be explicitly checked. This paper examines this issue in the context of a two-dimensional “design space” suggested by other researchers. The vertical axis of the space represents probability of exceedance and the horizontal axis represents a structural response quantity such as interstory drift. The qualitative limit states (such as immediate occupancy) are associated with specific values of structural response along the horizontal axis. In this space, a performance objective curve (PO curve) is defined by the code which specifies limiting exceedance probabilities for each limit state. Each specific structure has its own structural performance curve (SP curve) that reflects the actual probability of exceeding values of structural response. The paper discusses the possible shapes of the PO and SP curves, and shows how their shapes must be considered by code-writing organizations when formulating the number of limit state checks to be performed by designers. Finally, a simulation study is performed to explore how design decisions can affect the shape of a structure’s SP curve.

### INTRODUCTION

Today, it is widely recognized that seismic design codes need to incorporate performance-based design criteria. As noted in the Vision 2000 report prepared by the Structural Engineers Association of California [SEAOC, 1995], “Performance-based engineering methodology encompasses the full range of engineering activities necessary to create structures with predictable seismic performance within established levels of risk.” Conceptual frameworks for the new codes have been proposed and reviewed by the engineering profession in various countries. These conceptual frameworks for performance-based design are very convenient to illustrate the general philosophy of how design levels and structure performance levels can be combined to form structure performance objectives. Now, the difficult task of converting these conceptual frameworks into practical, quantifiable, design guidelines must be done.

A true performance-based design procedure must be based on the reliability-based design principles that have evolved over the past few decades. The premise behind performance-based design is that the design process should put more emphasis on predicting structural performance. However, given the many sources of uncertainty that are inherent in earthquake-resistant design, this can only be done in a probabilistic sense. In many of the proposed frameworks for performance-based design, probabilities have been used to define the design ground motion. However, as noted by Collins *et al.* [1995, 1996] and Wen and Foutch [1997], associating probabilities with the input earthquake excitation does not provide direct information about the probability of successful structural performance. Probabilities must be applied directly to the measures of structural performance considered important by the engineering profession.

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## PERFORMANCE CURVES AND LIMIT STATES

### Performance Objective (PO) Curves

As noted above, a true reliability-based, performance-based design procedure should associate target probabilities directly with suitable measures of structural performance. Therefore, one way to specify structural performance objectives might be as shown in Figure 1. This format for specifying performance objectives has been used in the past by Wen and Foutch [1997]. On the vertical axis, the probability of exceedance is shown on a logarithmic scale. This probability is an unconditional probability that accounts for both the likelihood of earthquake occurrences as well as the severity of earthquakes when they do occur. On the horizontal axis, a measure of structural performance is shown. This measure of performance should be, conceptually, any observable structural response quantity that can be directly measured in real structures and analytically evaluated with acceptable accuracy using customary computer models. The qualitative limit states (such as immediate occupancy) are associated with specific values of structural response along the horizontal axis. Within this “performance space” or “design space”, the code would prescribe a “performance objective” curve, or PO curve, which relates probabilities of exceedance to structural response. Theoretically, the PO curve could be specified as a continuous curve. However, for practical code application, several discrete combinations of exceedance probability and response would be prescribed. In Figure 1, it is assumed that five specific combinations are specified by the code, and these combinations are assumed to be interconnected by straight lines. The “safe” or “satisfactory design” region in this space would be the region below the PO curve.

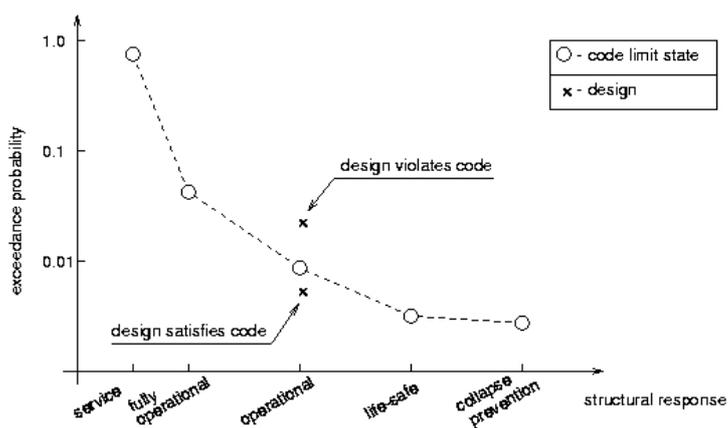
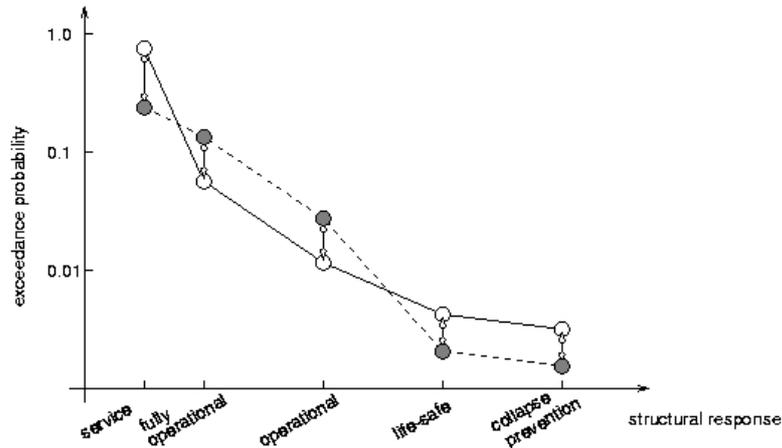


Figure 1. Reliability-Based Definition of Performance Objectives.

From a reliability-based design standpoint, the design space defined by Figure 1 is consistent with current LRFD design procedures (*e.g.*, AISC [1994]). LRFD codes are calibrated so that structural elements have the same reliability (or  $\beta$  value) regardless of the magnitude of the applied loads. In the seismic design context, the PO curve specifies the minimum reliability that a structure (or a particular class of structural systems) must have regardless of the seismic loading environment. For example, the probability of exceeding 1% drift in a given time period (for example, 50 years) for a particular structural system should be the same everywhere in the United States even though the seismic hazard varies greatly from location to location. Compare, for example, a 6-story moment frame designed for New York and Los Angeles in the United States. Clearly, from a seismic design standpoint, the two designs will be different due to the differences in the seismic hazard. However, it is reasonable to require that the unconditional probability of exceeding 1% drift in 50 years should be approximately the same for both designs.

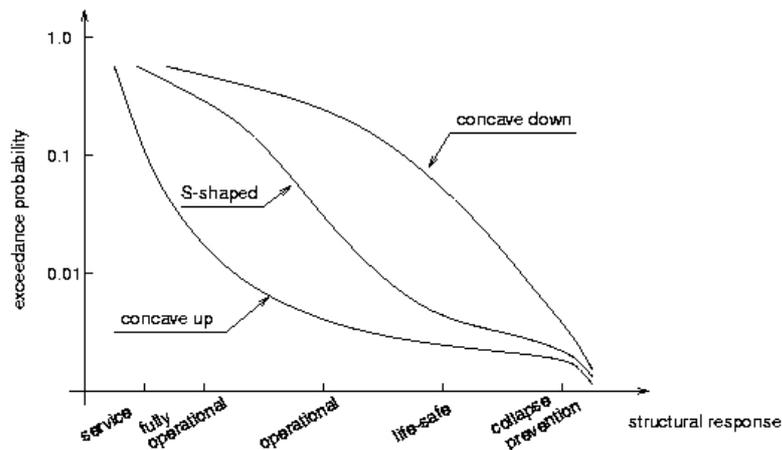
The specific points on the PO curve are chosen by code committees. Thus, the shape of the PO curve depends on the combinations of probabilities and response measures specified by the code. Figure 2 shows two possible PO curves. As will be discussed next, the shape of the PO curve is significant when determining the number of limit states to check in design.



**Figure 2. Effect of a Change of Code Performance Objectives on the Shape of the PO Curve.**

### Structural Performance (SP) Curves

The response of an actual structure during an earthquake is a random variable not only because the earthquake loading is random in nature but also because there is variability in the materials, geometry, and workmanship. In the context of the performance space shown in Figures 1 and 2, it is possible to construct a “structure performance” curve, or SP curve, which shows the actual exceedance probabilities for a chosen response quantity pertaining to a specific structural design. Figure 3 shows three conceptual shapes of these SP curves. Previous studies (such as those summarized by Wen [1995]) suggest that all three shapes are possible. However, it appears that there is no clear understanding of how certain shapes may or may not relate to different structural systems.



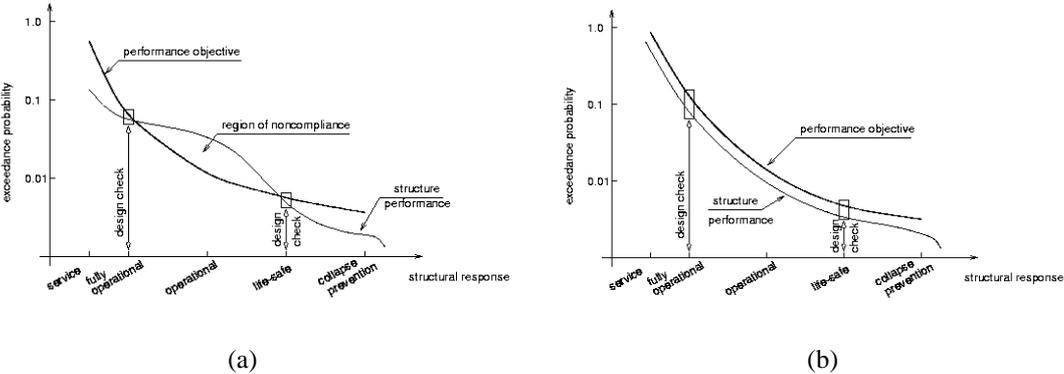
**Figure 3. Possible Shapes of SP Curves.**

The shape of the SP curve depends on the structural system, the geometry of the structure, and the quality of “local” design. The geometry of the structure comprises the distribution of lateral load-resisting systems, the span and height ratios, story setbacks, vertical and horizontal discontinuities, etc. The quality of local design describes the design and detailing of load carrying elements (such as beams, columns, shear walls) and their connections. Good local design insures that structural elements can sustain significant lateral deformations and cyclic deformations without losing their ability to carry gravity loads. Good local design also means that these structural elements are capable of redistributing their loads when they yield.

**Definition of Limit States**

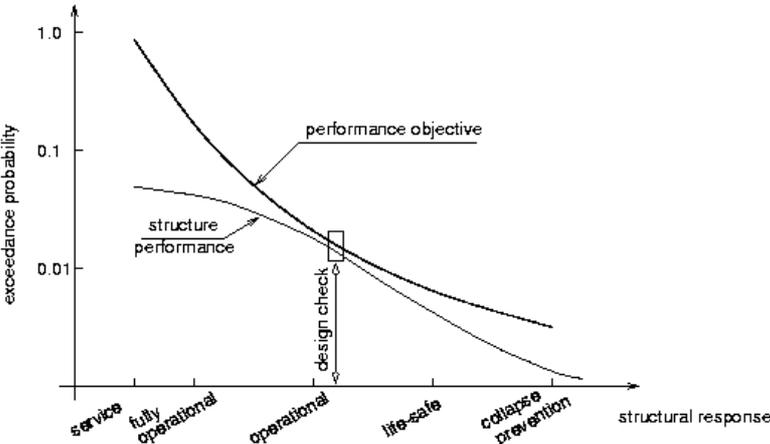
In the most strict interpretation of performance-based design, a satisfactory structural design must have an SP curve that is below the PO curve over the entire range of values of structure response. In other words, the actual probabilities of exceedance must be less than the limiting values prescribed in the performance-based code. However, without doing a detailed simulation study or probabilistic analysis, it is not possible to determine the continuous SP curve. A more feasible approach is to develop some simple design-checks that enable designers to verify that the exceedance probabilities are below the PO curve at a few discrete points. This leads to the question: *How many discrete points must be checked?* The answer depends on the shapes of the PO and SP curves and their relationship to each other.

As an example, consider the two situations depicted in Figure 4. In both of these cases, it is assumed that two design checks are done which indicate satisfactory performance. However, in case (a), such a conclusion would be misleading because there is a large intermediate region in which the SP curve is above the PO curve. Thus, it may be concluded that in this region the design does not satisfy the code. Case (b) represents the ideal situation in which the SP curve has the same general shape and concavity as the PO curve. Furthermore, the design is optimized so that the SP curve is very close to the PO curve over the entire range of response values. If this situation is known to exist *a priori*, then one design check would be sufficient to guarantee satisfactory performance.



**Figure 4. Possible Scenarios for a Two-Point Design Check. (a) Incompatible PO and SP Curves. (b) Compatible PO and SP Curves.**

Another possible scenario is shown in Figure 5, where the PO curve and the SP curve exhibit concavity in opposite directions. It can be argued that only one design check is needed in this case, since satisfying one design check automatically means that the SP curve is completely below the PO curve. Obviously, the location of this design check on the structure response axis is critical.



**Figure 5. PO and SP Curves with Opposite Concavity.**

Of course, the occurrence of the situations depicted in Figures 4 and 5 depends on the shapes of the code-specified PO curves and the structure-specific SP curves and their relative locations in the performance space (design space). There are at least three ways that one or more of the undesirable shape combinations and design check locations could be avoided:

1. Add additional design checks at intermediate points. This would increase the amount of work in design and would potentially increase the complexity of the design process.
2. Change the shape of the PO curve by adjusting the performance criteria specified in the code. As shown earlier in Figure 2, by making minor changes in the target exceedance probabilities, the shape of the PO curve can be transformed in a variety of ways.
3. Change the shape of the SP curve by adjusting the design. Conceptually, it seems possible to make some design modifications (such as, for example, the ratio of beam and column strengths at the connection) that can change the overall shape of the SP curve.

The first option above is acceptable if the relationship between the PO curve and the SP curve is as shown in Figure 4(a). However, if the curves have roughly the same degree of concavity as shown in Figure 4(b), then additional checks would not provide much additional information.

The second option is conceptually the easiest to implement, but it is not in the best interest of engineering and society to simply manipulate the code so that structures would tend to satisfy some performance criteria. The performance criteria (*i.e.*, the combinations of probabilities and target structure response values) should be based on sound engineering judgment, experience in past earthquakes, and societal expectations of acceptable risk.

The third option appears to be the most promising. To implement it, the designer must be aware of the possible consequences of a wide range of different design decisions. He or she may carefully consider the consequences of choosing a particular structural system and the geometry of the structure. Once the structural system is chosen, the designer should look into the design and detailing at the local level (*i.e.*, member and connection design details) and examine their influence on overall structural performance.

## SIMULATION STUDY

To gain some insight into the issues and options mentioned above, a small simulation study was performed. The objective of the simulation was to generate SP curves for hypothetical configurations of a moment frame structure to see how sensitive the SP curves are to design changes.

Three different “designs” of a twelve-story steel moment frame were considered. The baseline (reference) design was the same structure considered by Collins *et al.* [1995]. Column sizes ranged from W33x201 to W24x94, and beam sizes ranged from W30x124 to W24x76. The fundamental period of the bare frame was 2.70 seconds. The other two designs were generated by making some extreme modifications to the baseline design. One design, herein referred to as the “weak beam” design, consisted of the same column sizes as the baseline design, but beam sizes were reduced by approximately 50% (in terms of moment of inertia). The other design, referred to as the “strong-beam” design, also had the same column sizes as the baseline case, but the beam sizes were roughly doubled. The fundamental period of the weak-beam design was 3.45 seconds, and the period of the strong beam design was 2.22 seconds. The two modified designs were not intended to represent realistic designs; they were created simply to see how the shapes of the SP curves change for different designs.

The simulation study was conducted using the same set of simulated ground motion records considered by Collins *et al.* [1995, 1996] in their simulation study to generate uniform hazard spectra for a site near Los Angeles, California (USA). A total of 1292 earthquake records were used. For each moment frame configuration and each ground motion record, a nonlinear dynamic analysis was conducted using the DRAIN-2DX [Prakash *et al.*, 1993] computer program. The statistics of the responses were used to calculate 50-year exceedance probabilities of various levels of interstory drift. Interstory drift was chosen as the response measure because it is easy to understand and easy (relatively speaking) to compute.

Figure 6 below shows the SP curve for each of the three moment frame designs. The darkest curve represents the SP curve for the baseline design; the curves for the other two designs are identified in the figure. The step-

like pattern of the curves for drifts above 1.5% is a consequence of the limited sample size used in the simulation. A larger sample size would improve the resolution of the SP curves in the large drift/low probability region.

Figure 6 clearly shows how design decisions can impact the shape and locations of SP curves in the two-dimensional performance space. The curves for both the strong beam design and the reference design are S-shaped whereas the curve for the weak beam design is predominantly concave down. The weak beam design is inherently more flexible than the other two designs; thus, for a fixed value of drift response, the probability of exceedance is higher. A very interesting feature of the SP curve for the strong beam design is that it crosses over the curve for the reference design case. In the large drift/low probability region, the curve for the strong beam design is above the curve for the reference design; the higher probability of exceedance is due to the formation of undesirable plastic hinges in the columns. The formation of these hinges leads to the formation of story “mechanisms” which in turn result in large interstory drifts.

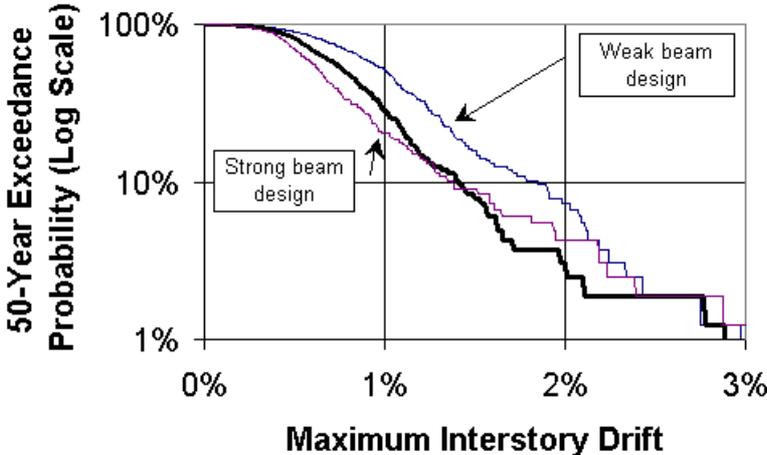


Figure 6. Results of Simulation Study Presented in the Two-Dimensional Performance Space.

CONCLUSIONS

The two-dimensional performance space of exceedance probability versus structural response is an effective way of specifying performance-based code requirements in a reliability-based (probability-based) context. Within this space, the design code specifies a performance objective curve (PO curve) representing combinations of limiting exceedance probabilities and target structural responses. Each structural design has its own structural performance curve (SP curve) within this space. The design is deemed to comply with the performance criteria when its SP curve is at or below the PO curve over the entire range of structural response values.

Since it is not practical for codes to specify continuous PO curves and not reasonable for designers to generate continuous SP curves for each structural design, a finite number of limit state checks must be specified by the code-writing organizations. It is difficult to determine the “optimum” number of design checks because there is a general lack of information available on the shapes of SP curves for typical structural designs. In some cases, only one check may be needed. In other cases, three or more limit state checks may be required. More research is needed to provide more information on the general characteristics of SP curves for a variety of structural systems. Furthermore, research is needed to better understand how design decisions can affect the shapes of SP curves. With this knowledge, code-writing organizations will be able to formulate rational PO curves for classes of structures, and designers will be able to make safer, more economical designs.

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