0730

A PROCEDURE TO MODIFY THE FREQUENCY AND ENVELOPE CHARACTERISTICS OF EMPIRICAL GREEN'S FUNCTION

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SUMMARY

Semi-empirical method, which divides the fault plane of large earthquake into elements and uses small ground motion records as empirical Green's function, is often used to synthesize the ground motions of large earthquakes. The method adjusts the amplitude of the ground motion records to satisfy the geometrical attenuation for different focal distances between the records and the elements. The different focal distance, however, changes not only the amplitude but also the frequency contents and envelope shape of ground motion. A procedure that modifies the frequency contents and envelope shape of the ground motion records according to the relation of the focal distances of the records and the element is suggested. The *Q* factor and the response spectrum obtained by attenuation relationship were used to modify the frequency contents. The ground motions of a scenario earthquake were synthesized as an example. The modification by the *Q* factor and that by the response spectrum gave similar results from the viewpoint of response spectrum of the synthesized ground motion. Based on the concept that the envelope can be represented by Fourier phase spectra given by the integration of group delay time, the standard deviation of the group delay time was employed to modify the envelope shape for different focal distances of the records and elements. With the modification of the frequency contents and envelope shape, the frequency contents and envelope shape are not the same for each element and a more realistic ground motion can be synthesized

INTRODUCTION

Ground motion time history is necessary for the seismic design of critical structures when the dynamic response of them is performed. The time history can be strong ground motion records if it fulfil the design earthquake load or artificially synthesized ground motion if the required ground motion is not available. For synthesizing of ground motion, stochastic process method and semi-empirical method are often used. The stochastic process method, which is the summation of a series of sinusoidal waves, is simple but difficult to give an earthquake- and site-specific solution. The semi-empirical method, which divides the fault plane of a large earthquake into small elements and uses the ground motion records as empirical Green' function to consider the fault rupture process and the path effects, are possible to incorporate the source, propagation and local site effects.

The semi-empirical method originated from Hartzell [1978], where the ground displacement of a mainshock was synthesized by the records of an aftershock, is often used to synthesize the ground motions of large earthquakes for engineering purposes. There are several versions of semi-empirical method. In this paper, the procedure proposed by Irikura [1986] was employed. According to Irikura, the ground motion is synthesized by

$$a_{syn}(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{R_{rec}}{R_{ij}}\left[a_{rec}\left(t-t_{ij}\right)+\frac{1}{n}\sum_{k=1}^{(N-1)n}a_{rec}\left(t-t_{ijk}\right)\right] \quad (1)$$

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where $a_{syn}(t)$ is the ground motion to be synthesized, $a_{rec}(t)$ is small ground motion records, N is scaling factor, R_{rec} and R_{ij} are the focal distances of the ground motion records and element ij, respectively. Appling Fourier transform to Equation (1), it becomes

$$F_{syn}(\omega) = \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{R_{rec}}{R_{ij}}A_{rec}(\omega)e^{-i\phi_{rec}(\omega)}\left[e^{-i\omega t_{ij}} + \frac{1}{n}\sum_{k=1}^{(N-1)n}e^{-i\omega t_{ijk}}\right] \tag{2}$$

where $A_{rec}(\omega)$ and $\phi_{rec}(\omega)$ are the Fourier amplitude and phase angle, respectively. In Equation (2), the Fourier amplitude is modified by the ratio of the focal distance of the records and that of the element ij to satisfy the geometrical attenuation of seismic wave and the frequency contents and envelope shape are identical for all of the elements. On the other hand, the difference of focal distance changes not only the amplitude but also the frequency contents and envelope shape due to the physical absorption of seismic waves, scattering and dispersion effects during the propagation of seismic wave. For a large earthquake, whose fault dimension could be more than a hundred kilometers, the differences of the frequency characteristics and envelope shape caused by the different focal distance among elements might not be negligible.

The physical absorption, which is caused by the imperfect elasticity of the medium, can be measured by the Q factor. The scattering caused by inhomogeneities is difficult to consider with mathematical expressions. Izumi and Katukura [1983] have pointed out that there was a close relationship between the envelope shape and the standard deviation of the group delay time of ground motion. In order to take into account the physical absorption and scattering effects, a procedure, which modifies the frequency contents and envelope shape for each element with the Q factor, the response spectrum and the standard deviation of group delay time, is suggested in this paper.

MODIFICATION OF THE FREQUENCY CONTENTS

Two methods were considered to modify the frequency contents of ground motion records. One uses the Q factor and another makes use of the response spectrum obtained by attenuation relationship. While the modification by the Q factor is mathematically explicit and simple, the modification by the response spectrum is based on the observation data.

Modification by the *Q* factor

The semi-empirical method requires a appropriate ground motion records at the site to serve as the empirical Green's function. In order to estimate the difference of the frequency contents between the records and the Green's function of an element, a statistical model that gives the estimation of the Fourier amplitude of acceleration for a site with focal distance R

$$A(\omega) = CA_S(\omega)A_P(\omega)A_L(\omega) \tag{3}$$

was used. In Equation (3), C reflects the effects of radiation pattern and is a constant for a site, $A_S(\omega)$, $A_P(\omega)$ and $A_L(\omega)$ denote the source spectrum, attenuation effects and local site amplification, respectively, and the attenuation effects is represented by

$$A_P(\omega) = \frac{1}{R}\exp\left(-\frac{\omega R}{2QC_s}\right) \tag{4}$$

where C_S is the velocity of shear wave.

Although we know its Fourier spectrum for the ground motion records, it is difficult to separate it into source spectrum, attenuation effects and local amplification clearly. Suppose the Fourier amplitude of the ground motion records can be represented by Equation (3), it is expressed as

$$A^{rec}(\omega) = C^{rec}A_S^{rec}(\omega)A_P^{rec}(\omega)A_L^{rec}(\omega) \tag{5}$$

In the same way, the Fourier amplitude for an element is given by

$$A^{ele}(\omega) = C^{ele}A_S^{ele}(\omega)A_P^{ele}(\omega)A_L^{ele}(\omega) \tag{6}$$

Since the C, $A_S(\omega)$ and $A_L(\omega)$ can be considered approximately the same for the records and the Green's function of element, the ratio of Equation (6) to Equation (5) becomes

$$\frac{A^{ele}(\omega)}{A^{rec}(\omega)} = \frac{A_P^{ele}(\omega)}{A_P^{rec}(\omega)} \tag{7}$$

Substituting Equation (4) into Equation (7) results in

$$\frac{A_P^{ele}(\omega)}{A_P^{rec}(\omega)} = \frac{R^{rec}}{R^{ele}} \exp\left[-\frac{\omega(R^{ele}-R^{rec})}{2QC_S}\right] = \frac{R^{rec}}{R^{ele}}\alpha(\omega) \tag{8}$$

The ratio of the focal distances in Equation (8) has been included in Equation (1) and $\alpha(\omega)$ is neglected there. As we know, the Q factor and C_S are strongly area dependent. In practical application, the Q factor and C_S of that area should be used. A value of 3.0 *km*/*sec* for the shear wave velocity and the results of the Q factor from Kiyono [1992]

$$\log Q = 0.7\log(\frac{\omega}{2\pi}) + 2.2 \tag{9}$$

were used here as an example. Equation (9) is an average relationship for Japan. Figure 1 shows the changes of $\alpha(\omega)$ with respect to ω for the cases of R^{ele} - R^{rec} being equals to -50, 0 and 50 *km*. From Figure 1, considerable differences among $\alpha(\omega)$'s are observed in high frequency range.

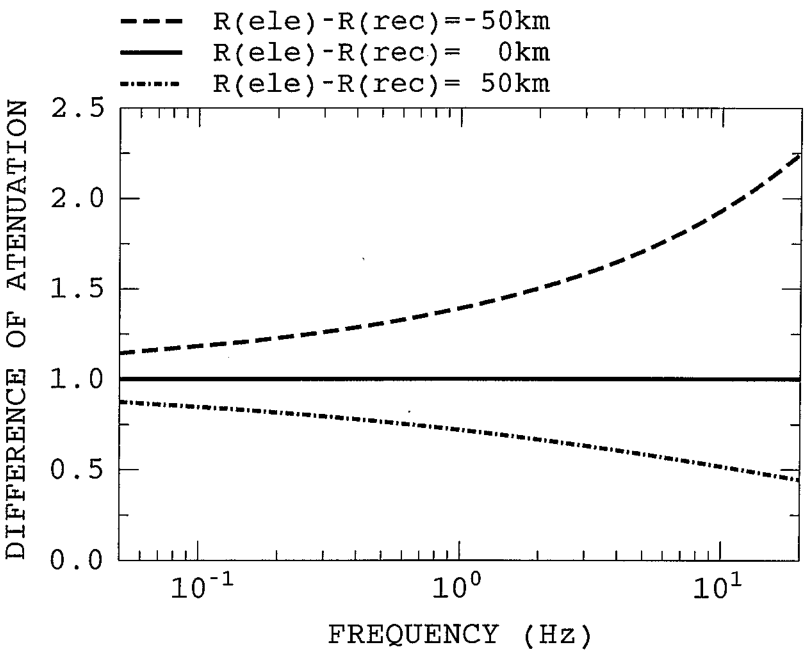

Figure 1: Difference of the attenuation of Fourier amplitude for different focal distance based on the *Q* factor

This difference can be simply considered in the semi-empirical method by introduce $\alpha(\omega)$ into Equation (2), namely

$$F_{syn}(\omega) = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{R_{rec}}{R_{ij}} \alpha(\omega) A_{rec}(\omega) e^{-i\phi_{rec}(\omega)} \left[e^{-i\omega t_{ij}} + \frac{1}{n}\sum_{k=1}^{(N-1)n} e^{-i\omega t_{ijk}} \right] \tag{10}$$

Modification with the shape of response spectrum

The modification by the Q factor takes into account the physical absorption only. It can not consider the scattering effects due to the inhomogeneities and the local site effects if the records and the synthesizing site are not the same site. In order to consider the scattering and local site effects as well, the response spectrum obtained from attenuation relationship was used. There are a number of attenuation relationships available and a site specific attenuation relationship is preferred. In this paper, the attenuation relationship proposed by Annaka and Nozawa [1988] was used to explain how the frequency contents are modified. Their attenuation relationship is expressed in the form

$$\log S(T) = C_m(T)(M) - C_d(T)\log(R) + C_h(T)H + C_0(T) \tag{11}$$

where T is natural period, M is magnitude, H is focal depth, $R = X + 0.35\exp(0.65M)$ and X is focal distance. When Equation (11) is used for the event whose records are used as the empirical Green's function, we have

$$\log S^{rec}(T) = C_m(T)(M) - C_d(T)\log(R^{rec}) + C_h(T)H^{rec} + C_0(T) \tag{12}$$

and for an element, its response spectrum is

$$\log S^{ele}(T) = C_m(T)(M) - C_d(T)\log(R^{ele}) + C_h(T)H^{ele} + C_0(T) \tag{13}$$

The difference of the response spectrum between the element and the records is given by

$$\frac{S^{ele}(T)}{S^{rec}(T)} = 10^{-C_d(T)\log\frac{R^{ele}}{R^{rec}} + C_h(T)(H^{ele}-H^{rec})} = \beta(\frac{\omega}{2\pi}) \tag{14}$$

The ratio, noting as $\beta(\omega)$, is the function of the focal distance and focal depth. Figure 2 shows the changes of $\beta(\omega)$ in the cases of R^{ele} / R^{rec} being equals to 0.5, 0 and 2.0 with H^{ele}-H^{rec}=0. For R^{ele} / R^{rec} equals to 2.0, $\beta(\omega)$ is comparatively constant for different frequencies. There is a considerable difference between R^{ele} / R^{rec} = 0.5 and 2.0 for the frequency range from 0.25 to 20 Hz. Figure 3 shows the changes of $\beta(\omega)$ in the cases of R^{ele} / R^{rec} being equals to 0 and H^{ele}-H^{rec}=-20, 0 and 20km. Comparing with the changes due to focal distance, the changes due to focal depth is minor. $\beta(\omega)$ does reflect the difference of the frequency contents between the records and element but, strictly speaking, not directly the difference of their Fourier amplitudes. Now we assume Equation (14) can be approximately applied to the Fourier amplitude. In this regard, $\beta(\omega)$ can be used to modify the frequency contents of an element. Thus, Equation (2) becomes

$$F_{syn}(\omega) = \sum_{i=1}^{N}\sum_{j=1}^{N}\beta(\omega)A_{rec}(\omega)e^{-i\phi_{rec}(\omega)}\left[e^{-i\omega t_{ij}} + \frac{1}{n}\sum_{k=1}^{(N-1)n}e^{-i\omega t_{ijk}}\right] \tag{15}$$

In Equation (15), The modification by R_{rec} / R_{ij} is not necessary because the geometrical attenuation is already included in the attenuation relationship.

MODIFICATION OF THE ENVELOPE SHAPE OF GROUND MOTION

Envelope shape or duration of ground motion will change as focal distance changes. In stead of directly considering the envelope shape, the concept that the envelope can be represented by Fourier phase spectra given by the integration of group delay time, *tgr*, was used. The group delay time is a differential of Fourier phase spectra in frequency domain.

$$tgr(\omega) = \frac{d\phi(\omega)}{d\omega} \tag{16}$$

The average and the standard deviation of *tgr* is defined as

$$\mu_{tgr} = \frac{1}{E}\int_0^{\omega_N} A^2(\omega)tgr(\omega)d\omega \tag{17}$$

$$\sigma_{tgr}^2 = \frac{1}{E}\int_0^{\omega_N} A^2(\omega)\left(tgr(\omega)-\mu_{tgr}\right)^2 d\omega \tag{18}$$

Izumi and Katukura [1983] pointed out that the average and the standard deviation of *tgr* of strong motions for given frequency window in frequency domain correspond to the centroid and the duration of strong motions filtered with the same window in time domain, respectively. Satoh *et al.* [1997] found that both the average and the standard deviation have a close relationship with the epicentral distance. The regression models they suggested are

$$\mu_{tgr}(\omega) = b_\mu(\omega)D \tag{19}$$

$$\sigma_{tgr}(\omega) = b_\sigma(\omega)D \tag{20}$$

where D is epicentral distance. Because the average of group delay time relates to the arrival of seismic wave and it has been considered in Equation (1), the relationship between the standard deviation and the epicentral distance was used to modify the envelope shape.

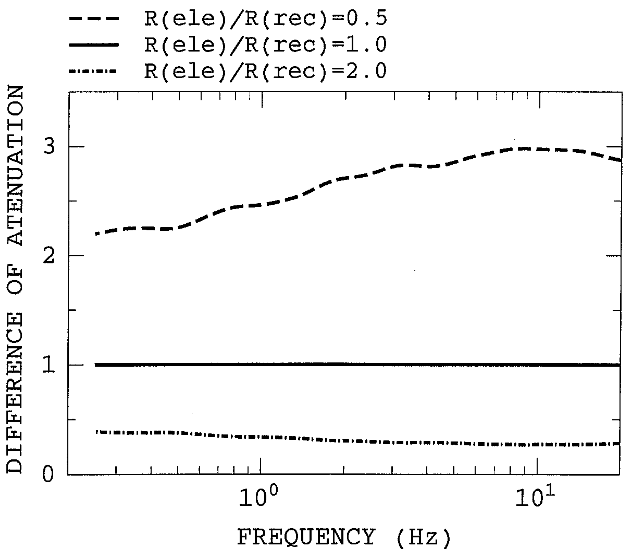

Figure 2: Difference of the attenuation of Fourier amplitude for different focal distances based on

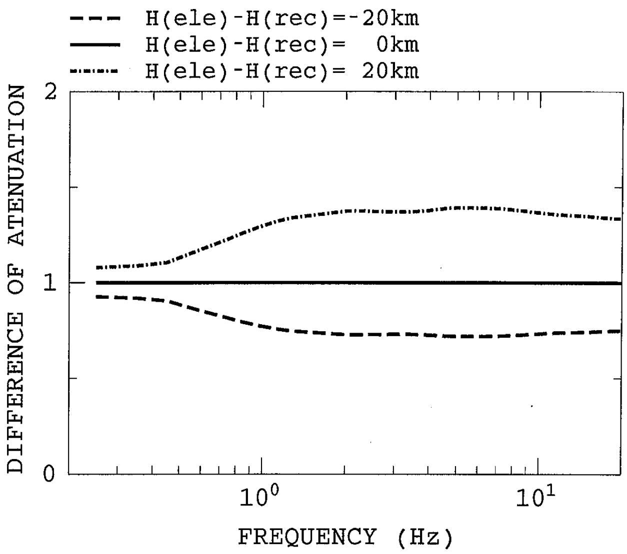

Figure 3: Difference of the attenuation of Fourier amplitude for different focal depth based on

With Equation (20), the standard deviation of group delay time for the records is estimated as

$$\sigma_{tgr}^{rec}(\omega) = b_\sigma(\omega) D^{rec} \qquad (21)$$

At the same time, the standard deviation of group delay time for an element is

$$\sigma_{tgr}^{ele}(\omega) = b_\sigma(\omega) D^{ele} \qquad (22)$$

Taking the ratio of Equation (22) to Equation (21), one obtains

$$\frac{\sigma_{tgr}^{ele}(\omega)}{\sigma_{tgr}^{rec}(\omega)} = \frac{D^{ele}}{D^{rec}} = \gamma \qquad (23)$$

The ratio depends only on the epicentral distance and is not the function of frequency, indicating that the envelope shape changes simply as the epicentral distance changes. From Equation (23), the standard deviation for an element is calculated by

$$\sigma_{tgr}^{ele}(\omega) = \gamma \sigma_{tgr}^{rec}(\omega) \qquad (24)$$

From statistical theory, Equation (24) is satisfied if the group delay times of the records and element have the relation:

$$tgr^{ele}(\omega) = \gamma tgr^{rec}(\omega) \qquad (25)$$

In practical case, because FFT is used to perform the Fourier transform, the group delay time is calculated by the difference instead of the differential.

$$tgr_i(\omega) = \frac{\phi_{i+1}(\omega) - \phi_i(\omega)}{\Delta\omega} \qquad (26)$$

From Equations (26) and (25), the phase angles of an element can be obtained by

$$\phi_{i+1}^{ele}(\omega) = \phi_i^{ele}(\omega) + \Delta\omega \cdot \gamma \cdot tgr_i^{rec}(\omega) \qquad (27)$$

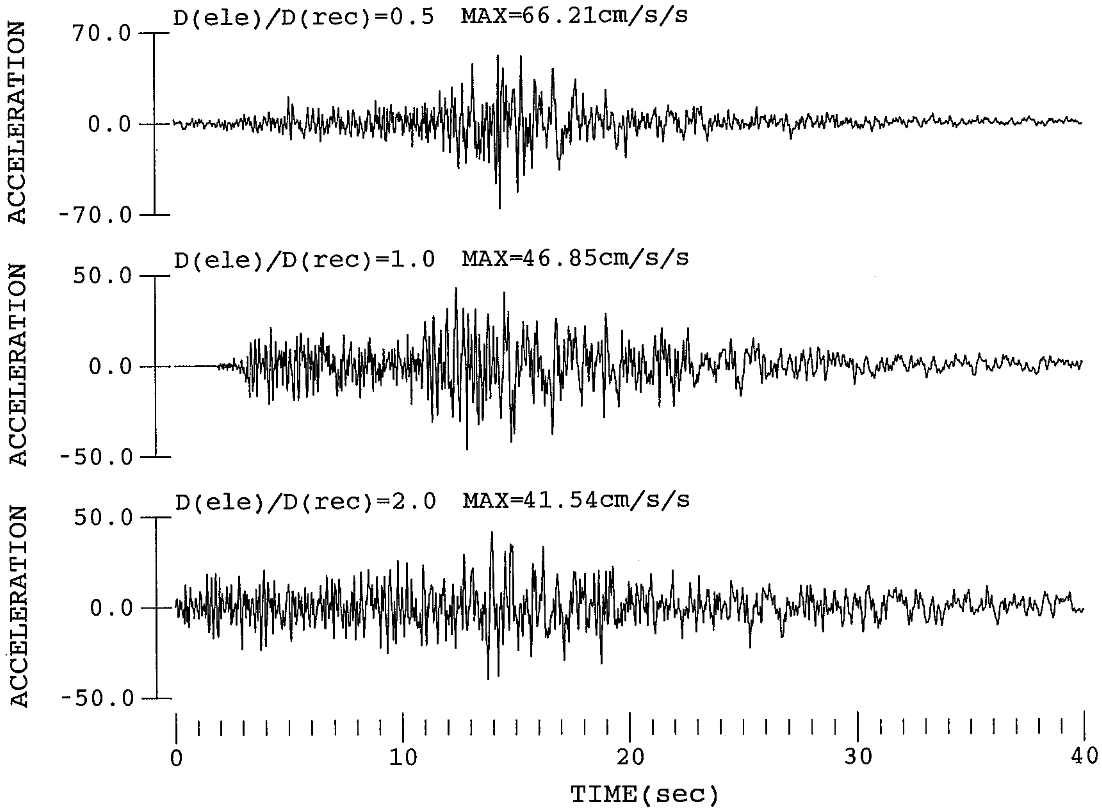

Figure 4: Difference of the envelope shape of time history for different focal distances

The change of the envelope shape, as an example, is shown in Figure 4 for γ = 0.5, 1.0 and 2.0, where γ =1.0 is original ground motion. In the case of γ = 0.5, the duration becomes shorter and the peak value becomes larger comparing with the original ground motion. On the contrary, when γ = 2.0, the duration becomes longer and the peak value becomes smaller. This coincides with the physical phenomenon.

When calculating the phase angle with Equation (27) for each element rather than using the original phase angles for all the elements in the synthesizing of strong ground motion, the change of envelope shape due to the different focal distances among the elements can be considered.

SYNTHESIZING OF GROUND MOTION WITH MODIFIED GREEN'S FUNCTION

The strong ground motions of a scenario earthquake were synthesized with modifying the frequency contents of empirical Green's function by the *Q* factor and the response spectrum. The synthesized ground motion time histories and their response spectra were compared with those without the modification. The parameters of the earthquake are listed in Table 1. The ground motion records from an earthquake with magnitude of 6.6 and focal distance of 86 *km* appeared in Figure 4 was used as the empirical Green's function. According to the scaling law, the fault plane of the scenario earthquake was divided into 4 by 4 elements. The location of the site and fault plane is shown in Figure 5.

Table 1: Parameters of scenario earthquake

Magnitude	Fault length	Fault width	Fault depth	Slip	Rise time	Shear velocity	Rupture velocity
7.8	100 *km*	50 *km*	20 *km*	3.2 *m*	3.2 *sec*	3.0 *km/sec*	2.1 *km/sec*

The Synthesized ground motion time histories with the modification of the frequency contents by the Q factor and the response spectrum and that without the modification are shown in Figure 6. The peak values of the time history with modification are about 30% lager than that without modification. The response spectra of the synthesized ground are compared in Figure 7. The modifications by the Q factor and the response spectrum have close results. Comparing with the response spectrum without modification, the difference is small for long periods and large for short periods. This is because the focal distance of the ground motion records is longer than that of the almost elements. In this case, when the ground motion records is used as it is, the seismic waves are over-attenuated for the elements who has the shorter focal distance than the records. The over-attenuated effects are corrected when the modification of the frequency contents is performed. It is considered that the modifications of the frequency contents and the envelope shape can result in more realistic response spectrum and duration for synthesized ground motion.

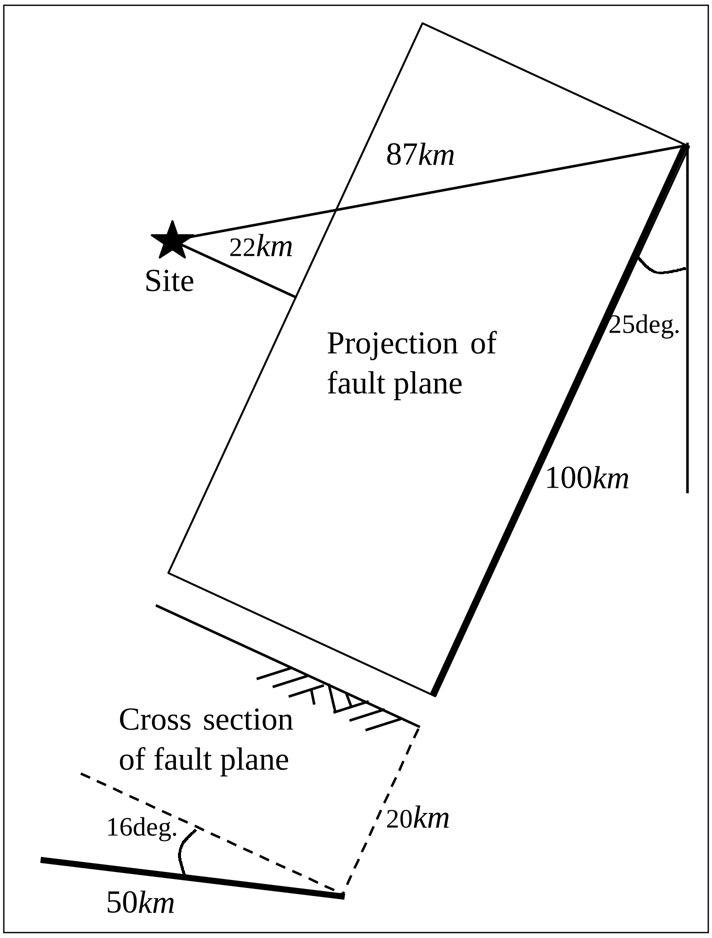

Figure 5: Location of the site and fault plane

CONCLUSIONS

A procedure that modifies the frequency contents and envelope shape of the Green's function by the *Q* factor, the response spectrum obtained by attenuation relationship and the standard deviation of group delay time according to the relation of the focal distance between the records and the element is suggested. Following this procedure, the frequency contents and envelope shape of the Green's function are not the same for each element, which, comparing with the conventional procedure, can result in more realistic ground motion.

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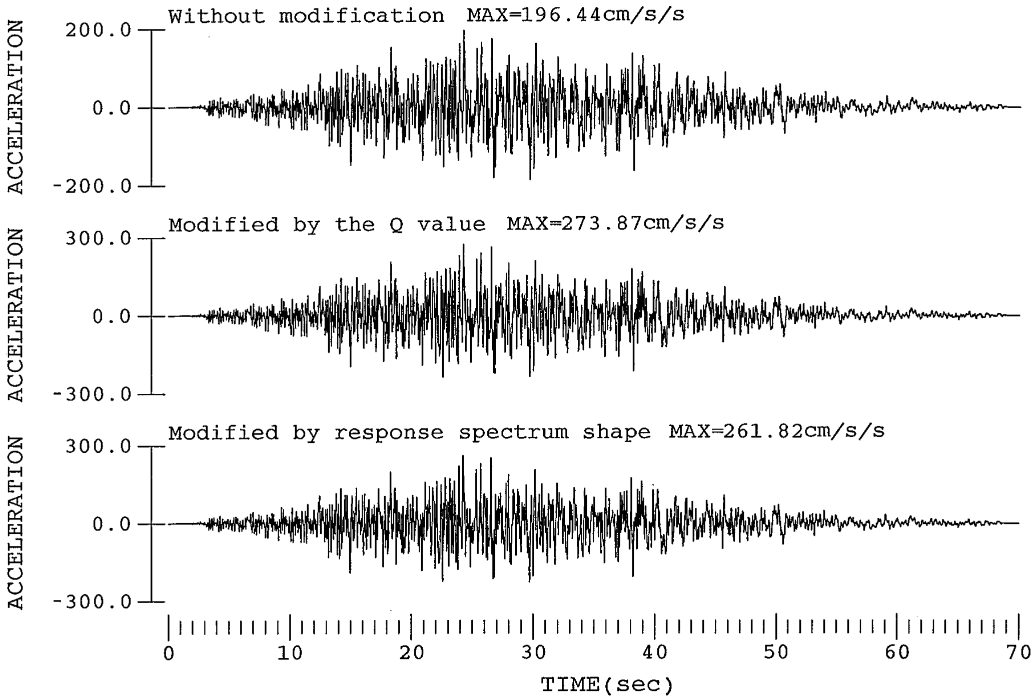

Figure 6: Synthesized ground motions with and without frequency contents modification

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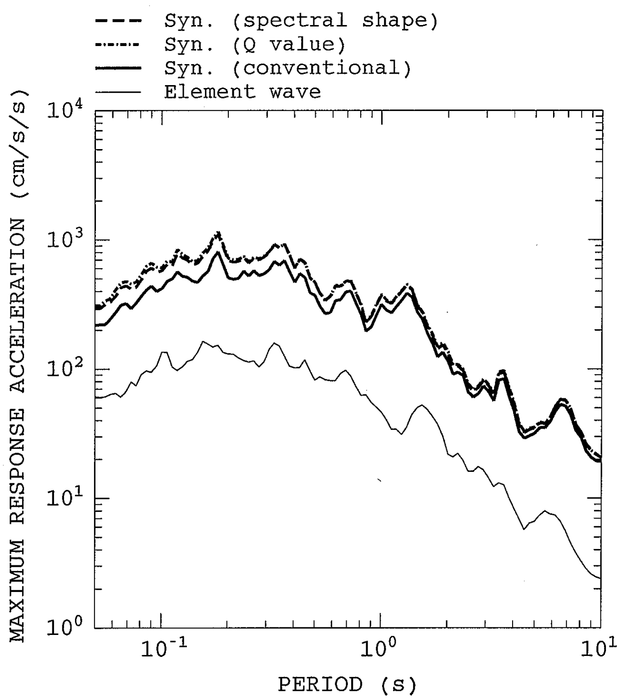

Figure 7: Response spectra of the records and