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STATISTICAL CHARACTERISTICS OF LONG PERIOD RESPONSE SPECTRA OF EARTHQUAKE GROUND MOTION

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SUMMARY

The theory of response spectra is the fundamental of earthquake response analysis for structures. The design method based on response spectra has been adopted in seismic design codes in most countries. With the increase of long period structures such as ultra-tall buildings, large span bridges and so on, the reasonable indication of long period response spectra has been deeply concerned by structural engineers.

In this paper, 1735 horizontal components of strong ground motion accelerograms recorded in the US are used to analyze the characteristics of long period response spectra. These components are grouped into four types in terms of the site categorization corresponding to the current design spectra in China. Then the average response spectra of each type are regressed with least square method, and the natural periods together with the periods of maximum response are found. Based on the results of average response spectra fitted by least square method, the corner periods and declining principles of long period response spectra at velocity control stage and displacement control stage are presented. That the corner periods vary with the site types is also revealed in the paper. On the other hand, the method of finding the corner frequency of velocity spectra proposed by M. D. Trifanac is discussed and the corner periods computed by Trifanac method are compared with the results proposed by this paper.

The process method presented above can reveal the principles that the corner periods vary with the site types and the corner periods can be determined. The method proposed by Trifanac suggested that the corner periods vary with the magnitude. And the long period response spectra suggested by the paper can be used as references for the revision of the code for seismic design of buildings in China.

INTRODUCTION

With the rapid economic development of China, ultra-tall buildings, large span bridges and more complicated structures sprung up with the trend of taller, bigger and more slender. In this case, horizontal loads are commonly controlled by seismic actions. That the common characteristics of these structures have is long natural period. At present, the natural period of many ultra-tall buildings constructed exceeded the maximum period range of design spectrum stipulated by Code for Seismic Design of Buildings in China (GBJ11-89) [1989], which was 3.0 sec.

For the structures with period exceeding 3.0 sec, special researches are suggested by GBJ11-89 for the lack of deep investigations then. Because the theory of response spectrum is the principal theory of seismic design of structures, it is very important to determine long period spectrum for design of super and complicated structures. Actually, the indication of design response spectrum with long period is becoming popular and some research results are achieved by international studies [Xiang, 1995; Trifunac, 1993 and 1995]. Some stipulations of design response spectrum with long period are made in some seismic codes, for example Eurocode 8 [1994]. But

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research results that reflect the characteristics of long period components are fairly fewer. This paper presents the statistical characteristics of response spectrum with long period through statistical analysis of large actual strong ground motions' records based on previous researches.

APPLICATIONS OF RESPONSE SPECTRA IN SOME SEISMIC CODES

The concept of response spectrum is accepted by engineering scope and commonly used in many countries' seismic codes since 1940s, when Biot and Housner used response spectrum to illustrate the spectral characteristics of strong ground motions. From then on, the theory of response spectrum is becoming a fundamental method for analyzing earthquake actions.

The current seismic code of China GBJ11-89 stipulates that the period range of response spectrum is from 0 to 3 sec. The special researches are required for structures with periods exceeding 3 sec.

Eurocode 8 [1994] presents design spectrum with period exceeding 3 sec, which attenuation exponent is –5/3 and a minimum value of $0.2\alpha(\alpha=a_g/g)$ is required. At the same time, the value d_g of the peak ground displacement is estimated by means of the following expression: $d_g = 0.05a_gST_cT_d$. Where a_g is design ground acceleration for the reference return period, S is soil parameter, T_c is value defining the ending of the constant spectral acceleration branch, T_d is value of defining the beginning of the constant displacement range of the spectrum.

UBC-94 [1994] of US presents normalized acceleration response spectrum curve, namely β spectrum, given β_{max}=2.50. Soil is classified three types. Soil type 1 stands for rock and stiff soils, soil type 2 for deep cohesionless or stiff clay soils, and soil type 3 for soft to medium clays and sands. The maximum period is also 3 sec.

Prof. Yayong Wang [Xiang 1995] presents design response spectrum with long period based on normalized design response spectrum obtained from analysis of actual strong ground motions, which is suitable for petro-chemistrical structures. Within the period less 3 sec, amplification factor is identical to GBJ11-89, that is, the response spectrum conforms to the principle of $(T_1/T)^{0.9}$. While in the displacement control range, the principle of $(3.5T_1/T^2)^{0.9}$ is used. Where T_1 is the first corner period (The period corresponding to the intersection of constant spectral acceleration and declining part). The normalized response spectrum for design is average β spectrum obtained from statistical analysis. The period range of velocity control stage is from 0.35s to 3.5s and the period of displacement control stage from 3.5s to 15s. This means the second corner period is suggested 3.5s. The maximum period extends to 15 sec.

GBJ11-89 presents seismic effect coefficient (α) with 5% damping ratio. When $T_g \le T \le 3.0s, \alpha = (T_g/T)^{0.9}\alpha_{max} \ge 0.2\alpha_{max}$, T_g is characteristic period (corner period) taken from Table 1. GBJ11-89 supposed that the maximum of dynamical coefficient β_{max} has less relationship with seismic intensity, soil, and near/far-earthquake based on statistical results from earthquake ground motions, then taken β_{max} =2.25. If the seismic effect coefficient curve of GBJ11-89 is converted into β spectrum, then $\beta_{min}=0.2\beta_{max}$.

It should be noted that the response spectrum of many soil types at medium period is actually a constant line, namely, $\beta(T)=0.2\beta_{max}$. Table 1 presents the characteristic period (T_g) of design response spectrum of GBJ11-89 and the starting point period of minimum spectral value (Termed as T_s). Table 1 also states the β value of spectrum at the maximum period 3.0 sec.

From the regulations of response spectrum in typical codes above, we found that most codes did not include response spectrum with long period. Eurocode 8 stipulates a minimum value for long period spectrum, but the value is a little larger than that of statistical analysis and does not reflect the fact that response spectrum with long period declines more quickly than that with medium period.

Table 1 Characteristic Period of Design Response Spectrum T_g(s), Starting Point Period T_s(s) of Minimum Spectral Value, and Minimum β Value

Site category	Near/far earthquake	Characteristic period T_g(s)	Starting point period T_s(s)	Minimum value of β
1	Near-earthquake	0.20	1.20	0.45
	Far-earthquake	0.25	1.50	
2	Near-earthquake	0.30	1.79	
	Far-earthquake	0.40	2.39	
3	Near-earthquake	0.40	2.39	
	Far-earthquake	0.55	3.29	0.49
4	Near-earthquake	0.65	3.89	0.57
	Far-earthquake	0.85	5.08	0.72

RECORDS OF STRONG GROUND MOTIONS AND GROUPING OF RESPONSE SPECTRUM

Source of Records

Records of earthquake ground motions used for this research work are mostly from the US. Through sorting out, the records of ground motions which magnitude is less than 4.5 and peak acceleration of ground motion is less than 25gal are excluded, then 1735 records of horizontal components are selected.

Grouping Method of Response Spectra

Response spectra are sorting out in terms of characteristic period based on the following reasons.

(1) Most records of ground motions lack in detailed profile information about the site of investigation, so it is difficult to determine the type of given soil, and they are not available within the short period of time.

(2) The revised code should be of continuity when determining the index of grouping.

(3) It is well known that statistical analysis requires enough samples of response spectrum in each group, and the amount of samples in each group cannot be varied greatly. It is difficult to achieve this purpose by grouping in terms of intensity, near/far-earthquake and soil types.

(4) Much more records of ground motions can be used when grouped in terms of characteristic period.

The boundary line of characteristic period is determined in accordance with T_g of GBJ11-89 with adjustments (See Table 2). The characteristic period of sample response spectrum is determined according to commonly used method, taking the corresponding period of the intersection point of the horizontal line $\beta=0.707\beta_{max}$ and the right end of β spectrum curve. Considering the available records of ground motions are mostly analogue ones, the maximum period of response spectrum is taken as 7 Sec. The interval of controlled points is 0.05 sec, then the amount of points is 141.

It is noted that the index of multi parameters is simplified into that of single parameter by grouping in terms of characteristic period, the statistical results show it has little dispersion and makes analysis simple.

Grouping Results

The 1735 samples of response spectrum are divided into four groups, and the amount of each group is no less than 350 (See Table 2). The period of peak acceleration and characteristic period of average β spectrum with 5% damping ratio in each group is also given in Table 2. Figure 1 illustrates the average β spectrum of each group with 5% damping ratio.

Table 2 Grouping Results of Response Spectrum Samples In terms of Characteristic Period

Site category	GBJ11-89 T_g(s)	Statistical range(s)		Amount of samples	Average β spectrum with 5% damping ratio	
		Lower	Upper		Peak period(s)	Characteristic period(s)*
1	0.20, 0.25	0.000	0.265	384	0.150	0.231
2	0.30, 0.40	0.265	0.420	486	0.250	0.372
3	0.40, 0.55	0.370	0.570	514	0.350	0.516
4	0.65, 0.85	0.570	2.000	488	0.550	0.958

* The characteristic period is determined by $0.707\beta_{max}$.

STATISTICAL ANALYSIS OF RESPONSE SPECTRUM

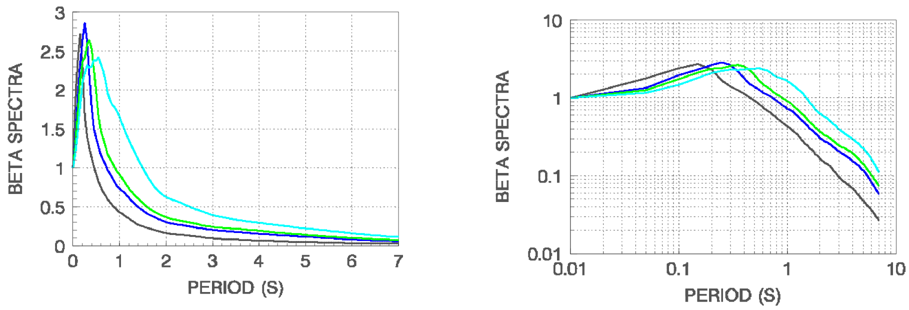
Corner Period of Response Spectrum

The determining method of corner periods

There are two types of principal methods for determining the corner period of response spectrum. One is the regression of spectrum curves, the other is decided by the relations of peak value of ground motions. The first one is more rational. The corner period of average β spectrum of each group is determined by using least square curve regression method. The clue line of the method is: given formula of response spectrum β(T)=f(T)at a frequency range, regressed forward from a point until the spectrum formula does not agree with the form or the error escalated steeply, then the period value can be considered as the corner period of declining part of response spectrum. The following spectrum equation is used for analyzing,

$$\log\beta = a\cdot\log T + b \qquad (1)$$

where *a* and *b* are under-determined factors. *a* is an attenuation exponent. In the regression, if absolute value |*a*| is obviously greater than the given *a* value (Termed as [*a*]), then the corresponding period is the corner period. Theoretically, acceleration response spectrum changes with 1/T at the velocity control stage, then *a*=-1. Figure 1 illustrates the average β spectrum of each group with 5% damping ratio at the logarithmic coordinate. In the actual application, |*a*| is often taken as |*a*|<1, for the consideration of safety factors. For example, GBJ11-89 takes *a*=-0.9, Eurocode 8 takes *a*=-2/3. Rosenblueth [1980] also suggested *a*=-2/3 should be used, which makes structures with long period safer. Therefore, there are four patterns used to determine corner period by regressing average β spectrum, that is, [*a*]=-1,-0.9,-0.8,-2/3.



(a) Rectangular coordinate **(b) Logarithmic coordinate**

Figure 1 Average β Spectrum of Each Group with 5% Damping Ratio

There are three corner periods to be determined at the curve of response spectrum. That is, the beginning point period of acceleration controlled stage, the starting point period of velocity control stage and the beginning point period of displacement control stage. Generally, many research results show that the beginning point period of acceleration control stage varies between 0.1 and 0.5 sec, therefore, it is not set as a regression parameter. The starting point period (T_1) of velocity control stage is also the ending point period of constant acceleration of response spectrum or the starting point period of attenuation stage of response spectrum. Two types of value are used to determine the period. One is the peak acceleration period of average β spectrum, the other is characteristic period of average β spectrum (Decided according to Section 3).

The boundary point period of different attenuation exponent at the declining part of response spectrum is taken as approximately the starting point period of the displacement controlled stage, which is termed as T_2 and obtained from regression of least square method.

Statistical results of corner periods

The computation and analysis shows regression results of T_2 when taking peak period and characteristic period as T_1 are close to that taking characteristic period as T_1. Therefore, the regression results of later case are given in this paper (See Table 3).

Table 3 Regressed Corner Periods and Their Corresponding Parameters
Taken [*a*]=-1, -0.9, -0.8 and -2/3

Patterns	Site category	T_1 (s)	T_2 (s)	a	b	R
[a]=-1	1	0.23	3.00	-0.999	-0.435	-0.96
	2	0.37	-----	-0.973	-0.239	-0.96
	3	0.52	-----	-0.929	-0.172	-0.96
	4	0.96	-----	-0.808	-0.018	-0.96
[a]=-0.9	1	0.23	2.10	-0.897	-0.398	-0.94
	2	0.37	5.10	-0.900	-0.236	-0.96
	3	0.52	5.95	-0.899	-0.172	-0.95
	4	0.96	-----	-0.808	-0.018	0.90
[a]=-0.8	1	0.23	1.60	-0.794	-0.350	-0.92
	2	0.37	3.00	-0.800	-0.216	-0.92
	3	0.52	3.80	-0.798	-0.161	-0.92
	4	0.96	6.05	-0.797	-0.018	-0.90
[a]=-2/3	1	0.23	1.15	-0.666	-0.277	-0.89
	2	0.37	1.95	-0.657	-0.162	-0.87
	3	0.52	2.35	-0.660	-0.119	-0.85
	4	0.96	4.15	-0.669	-0.006	-0.85

Note: 1. R is correlation coefficient; 2. T_1 is characteristic period [$\beta(T_1)=0.707\beta max$];
3. "----" indicates the parameters are regressed from T_1 to 7.0s when T_2 is greater than 7.0s.

The characteristics of corner periods

Table 3 shows that there are common characteristics of corner periods among different patterns.

(1) The corner periods of response spectrum are varied with soil category. They increase with the soil types.

(2) The second corner period regressed from peak period is close to that from the period corresponding to $0.707\beta_{max}$. Because the period corresponding to $0.707\beta_{max}$ agree with the current code GBJ11-89 except for the sites of category 4, the later regression will be started from the period corresponding to $0.707\beta_{max}$.

(3) The larger [*a*] is, the more slowly the response spectrum attenuates and the smaller the corner period is.

The above characteristics agree with the knowledge of spectral performance of ground motions. It is seen that the corner period is influenced with site category. It should be noted that the above results do not reflect the influence of magnitude. Section 4.1.4 will cite Trifunac's typical attenuation principles of ground motions to illustrate the influence of magnitude on corner periods.

Computational results of corner periods by M. D. Trifunac

M. D. Trifunac [1993] states the empirical equations for scaling Fourier amplitude spectra can be extrapolated to describe the long period strong motion amplitudes. The theoretical expressions for the corner frequencies (f_2) of velocity controlled stage and the corner frequencies (f_1) of displacement controlled stage are given as follows,

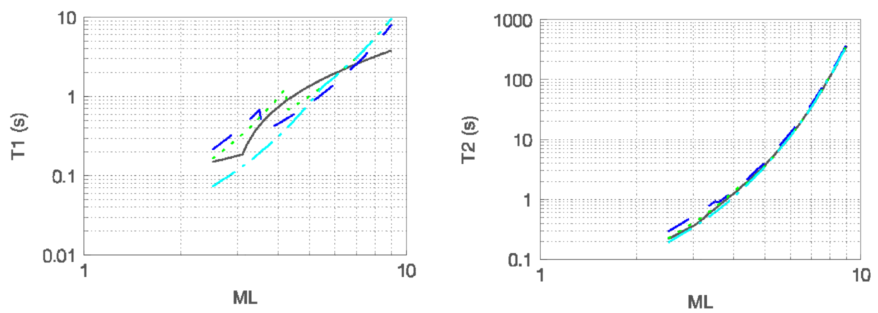
$$f_1 = \left(\frac{L}{2.2} + \frac{W}{6}\right)^{-1}, \quad f_2 = \frac{2.2}{W} \tag{2}$$

Where L is fault length (km), $L=2L_{min}$,$L_{min} = a\times10^{bM}$, L_{min} is minimum fault length (km), W is fault width (km), $W=c\times10^{dM}$, M is magnitude. a, b, c, d are empirical scaling coefficients.

Figure 2 illustrates the changing principles of corner periods varied with magnitude computed from the above equations. The corner frequencies of four patterns have the same changing trends except for the small magnitude.

From the above results, we find the corner periods (T_1) of Trifunac's equations are larger, which surpass the period range of T_g in GBJ11-89 even at the velocity controlling stage. The corner periods of (T_2) of displacement controlled stage also surpass the period range concerned. It should be noted the corner periods are obtained from Fourier amplitude spectra and adjustments should be made when used in seismic design. That the results applied into the research of pseudo relative velocity spectra shows the corner periods can be used to indicate the attenuation curve of pseudo relative velocity spectra.

From the computational analysis of Trifunac's equations, we can conclude that magnitude influences significantly on the corner periods of response spectra, the corner periods increase with magnitude (See Table 2). T_2 increase quicker than T_1.
GBJ11-89 did not reflect that characteristic period of response spectra varied with magnitude.



(a)Corner period T1 **(b)Corner period T2**

Figure 2 Relationship between Magnitude and Corner Periods Computed from Trifunac's Equations

Statistical Analysis of Elastic Response Spectrum from Records

The declining part of response spectrum is divided into two stages, namely first declining stage [T_1,T_2] and second stage [T_2,7.0], on the basis of the statistical corner period at Section 4.1.2. After regressed by $\log\beta=a\log T+b$, the corresponding parameters are determined with [a]=-1, -0.9,-0.8,-2/3. The parameters of the first stage are shown in Table 3, that of the second stage in Table 4.

From above analysis, we find the attenuation exponent of the second declining stage is less than –1.3 compared with the first declining stage, showing that the declining rate is higher. It should be noted that the period is cut off at 7.0 sec, the regression results deviate theoretical value for short of enough samples when T_2 is close to 6.0 sec.

CONCLUSIONS

The corner periods of the declining part of long period response spectrum are determined and their changing principles are revealed based on statistical analysis from large earthquake ground motions. The attenuation exponents of displacement controlled stage are obtained by least square regression method. From the analysis results of various patterns, we can conclude that the attenuation exponents of displacement controlled stage vary between –1.3 and –1.8, which conforms to the fundamental knowledge of attenuation principles of response spectrum.

The grouping method in terms of characteristic period makes a large quantity of earthquake ground motions available for analyzing. From the grouping results, we know it is an alternate method for analyzing response spectrum.

Table 4 Parameters of Second Stage Regressed with [*a*]=-1, -0.9, -0.8 and -2/3

Patterns	Site category	T_2(s)	a	b	R
[*a*]=-1.0	1	3.00	-1.528	-0.260	-0.997
[*a*]=-0.9	1	2.10	-1.470	-0.302	-0.998
	2	5.10	-2.163	0.593	-1.000
	3	5.95	-2.040	0.593	-0.998
[*a*]=-0.8	1	1.60	-1.435	-0.326	-0.998
	2	3.00	-1.511	0.089	-0.985
	3	3.80	-1.707	0.330	-0.997
	4	6.05	-2.400	1.077	-1.000
[*a*]=-2/3	1	1.15	-1.420	-0.336	-0.999
	2	1.95	-1.292	-0.068	-0.984
	3	2.35	-1.326	0.047	-0.985
	4	4.15	-1.808	0.605	-0.992

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REFERENCES

[1] European Pre-standard (1994), *Eurocode 8*, CEN, ENV 1998-1-1
[2] Hu Yuxian (1988), *Earthquake Engineering*, Seismological Press
[3] National Standard of the People's Republic of China (1994), *Code for Seismic Design of Buildings (GBJ11-89)*, New World Press
[4] Rosenblueth E. (1980), *Design of Earthquake Resistant Structures*, Pentech Press, p34-35
[5] Trifunac M. D. (1995), "Pseudo relative velocity spectra of earthquake ground motion at long periods", *Soil Dynam.. Eearthqu. Engng*, (14) , p331-346
[6] Trifunac M.D. (1993), "Long period Fourier amplitude spectra of strong motion acceleration", *Soil Dynam.. Eearthqu. Engng*, (12), p363-382
[7] UBC-94 (1994), *Uniform Building Code*, Volume 2, Structural Engineering Design Provisions
[8] Xiang Zhongquan and Sun Jiakong (1995), *Resistant Design for Petro-chemistrical Facilities*, Seismological Press