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ENVELOPES FOR SEISMIC RESPONSE VECTORS IN NONLINEAR STRUCTURES

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SUMMARY

In earthquake engineering, the effects of responses acting simultaneously in a structure must often be considered. In this paper, a parametric investigation is conducted using time-history analyses to simulate the envelopes that bound selected vectors of seismic responses in a nonlinear steel moment frame building. New and practical insight into the behavior of nonlinear structures and the effectiveness of conventional seismic design methodologies is gained by studying these response envelopes. Particularly noteworthy are (1) the adverse effects of near-field ground motions on the nonlinear response of the structure, (2) the relationship between the yield levels of the moment frame connections and the variability in the displacement demands, (3) the effectiveness of plastic hinges as fuses to limit the shear forces and bending moments transmitted to the moment frame columns and (4) the use of simulated response envelopes to identify the spatial distribution of plastic hinges in the structure and the expected sway mechanism under severe seismic loading.

INTRODUCTION

In the seismic design or analysis of a structure, the simultaneous action of multiple responses must be considered. For example, a column in a moment frame must resist an axial force and bending moment that act concurrently and vary in time. It is natural to describe such response combinations as a vector $\mathbf{x}(t)$, in which each component corresponds to one of the time-varying responses. The adequacy of a structural element is evaluated by comparing the applicable response vector to the limit state surface (interaction diagram) that defines the boundary in the response space between acceptable and unacceptable performance. If $\mathbf{x}(t)$ out-crosses the limit state surface, then the element is said to have "failed"; otherwise, the element is said to be "safe".

Due to the probabilistic nature of seismic excitation and response, there is almost always a finite probability that an element will fail. Recognizing this fact, a design is considered adequate if the probability of failure does not exceed a prescribed threshold. The determination of the probability of the failure event described above is the well-known vector out-crossing problem in the field of structural reliability. Unfortunately, the problem does not have an exact solution. However, an upper bound on the probability p_f that $\mathbf{x}(t)$ out-crosses the limit state surface ∂S in time interval $0 \le t \le T$ is given by [Shinozuka, 1964]

$$p_f \le p_f(0) + \int_0^T \upsilon(\partial S, t)\, dt \,, \tag{1}$$

where $p_f(0)$ is the probability that $\mathbf{x}(0)$ lies in the failure domain and $\upsilon(\partial S, t)$ is the mean rate at which $\mathbf{x}(t)$ out-crosses ∂S per unit time. In general, the evaluation of $\upsilon(\partial S, t)$ is difficult, as it involves the joint distribution of the response vector and its time derivative and an integration over the limit state surface [Belyaev, 1968]. Consequently, there are only a few closed-form solutions available for $\upsilon(\partial S, t)$ for restricted classes of processes

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and limit state surfaces [e.g., Veneziano et al., 1977]. Because this approach couples the computation of the response vector statistics and the limit state surface, it is not convenient for practical engineering applications.

An alternative means of evaluating the adequacy of a structural element is to predict an envelope that bounds $\mathbf{x}(t)$ and compare this envelope, rather than the response vector itself, to the relevant limit state surface. If the envelope is encompassed by the limit state surface, then the element is deemed adequate. The advantage of this approach, over that based on the out-crossing formula (1), is that the demand and capacity evaluations are uncoupled. Naturally, due to the stochastic nature of seismic responses, any envelope on $\mathbf{x}(t)$ has a finite probability of being exceeded; i.e., $\mathbf{x}(t)$ can only be bounded in a statistical sense. However, this uncertainty in the envelope is fundamentally the same as that associated with the peak value of any scalar seismic response quantity, which is usually addressed by building codes through prescribed load and performance factors.

For linear structures, the envelope on $\mathbf{x}(t)$ is an ellipsoid whose coordinates can be computed using values available in conventional response spectrum analyses, i.e., the response spectra of the principal components of ground motion and the modal properties of the structure [Menun, 1999]. The ellipsoid is inscribed within the rectangular envelope defined by the peak values of the individual response components. It has been demonstrated by means of a comprehensive series of time-history analyses that the elliptical envelope has a level of accuracy that is commensurate with its response spectrum bases. Furthermore, it has been shown that improved economy in structural members can be achieved by using the elliptical envelope, rather than the conventional rectangular envelope, in the design calculations [Menun and Der Kiureghian, 1998; Menun, 1999]. Linear analyses, however, are only appropriate for serviceability limit states associated with moderate-intensity ground motions. When a structure is subjected to high-intensity ground motions, its behavior is nonlinear and, hence, the response spectrum method is not valid. Consequently, time-history analyses must be used to simulate the response envelopes for such structures. As we will see, these envelopes usually are not elliptical.

In this paper, a parametric investigation is carried out using time-history analyses to simulate the envelopes that bound vectors of seismic responses in a nonlinear structure. Attention is focused on how the size, shape and variability of the elliptical envelope bounding a response vector in a linear structure change when hysteretic elements are introduced into the structure. New and practical insight into the behavior of nonlinear structures and the effectiveness of current seismic design methodologies is gained by examining these response envelopes. To the authors' knowledge, all research conducted to date on the nonlinear seismic response of structures has been directed at scalar response quantities. Thus, the approach taken in this paper, namely to examine the envelopes bounding vectors of nonlinear responses, is unprecedented.

EXAMPLE STRUCTURE AND GROUND MOTIONS

The three-story steel building shown in Figure 1, which is representative of a large class of structures that are commonly encountered in practice, is considered in this study. The lateral force resisting system consists of four moment frames on the perimeter of the building. For the dynamic analyses, each floor plate is modelled as a rigid diaphragm that has a seismic mass of approximately 6.4×10^5 kg. The center of mass is offset from the center of the floor plate by 5% of the building dimension along both horizontal structure axes. The building, which is analyzed using DRAIN-3DX [Prakash et al., 1994], has a fundamental period of 0.68 s and modal damping ratios ranging from 2% to 5% for the nine modes included in the linear analyses. Gravity loads, which are based on a uniform dead load of 5.75 kN/m^2 on each level of the building and are consistent with the framing plan shown in Figure 1, are applied to the structure prior to each dynamic analysis.

The moment connections at the ends of the beams and at the column bases of the moment frames are modelled using zero-length elements. These connection elements are the only locations within the structure where yielding can occur. Each connection is modelled as an elastic-perfectly-plastic, non-degrading hysteretic element that has a plastic moment capacity

$$M_p = \eta F_y Z \tag{2}$$

where F_y is the yield stress of the steel listed in Figure 1, Z is the plastic section modulus of the member that the connection supports and η is a prescribed strength ratio. The positive and negative moment capacities of a connection are equal. In the current practice, such connections are commonly used with $\eta \geq 0.7$ recognized as a practical lower limit when seismic loads govern the design. In this study, the nonlinear time-history analyses are performed using $\eta = 0.4$, 0.7 and 1.0. Except when noted, all connections in the structure are assigned the same value of η. For the linear analyses, η is set to a large value that prevents any yielding from occurring in the structure.

The structure is subjected to two ensembles of horizontal ground motions. The first ensemble consists of 25 pairs of simulated accelerograms that are representative of a far-field $M_w = 7.5$ earthquake, during which the structure is expected to be severely damaged [Menun, 1999]. Each pair of ground motions represent the principal components of an earthquake, with the intensity of the minor component is approximately 85% of that of the major component. The mean pseudo-acceleration response spectra of these ground motions are shown in Figure 2a. The second set of accelerograms considered consists of five pairs of synthetic near-field ground motions that are representative of a $M_w = 7.1$ event on the Elysian Park fault near Los Angeles, CA [Somerville et al., 1997]. The components of each near-field event are oriented in the fault-normal and fault-parallel directions. The mean pseudo-acceleration response spectra of this ensemble are shown in Figure 2b.

For the time-history analyses conducted using the far-field earthquakes, the major component of ground motion is directed along the z_1 axis of the building shown in Figure 1. For the near-field ground motions, the fault-normal component is directed along the z_1 axis.

NONLINEAR RESPONSE ENVELOPES

Three response vectors are considered. First, the horizontal roof displacements are examined. Displacements are often used to characterize the performance of structures subjected to intense ground motions; thus, the envelope that bounds such response quantities is of practical interest. Second, we study the envelopes that bound the axial force and bending moments acting in one of the ground floor columns. These envelopes are considered in an effort to assess the effectiveness of the conventional design methodology for structures of this type, which is to limit the forces acting in critical structural elements by forcing nonlinear behavior to occur at only selected locations. Finally, we consider the envelope that bounds the horizontal roof displacement and the plastic hinge rotation at the base of a column. We will see that a simple geometric interpretation of this envelope provides useful information about the performance of the building that might otherwise be difficult to infer from time-history results alone.

Envelopes Bounding Roof Displacements

Consider the roof displacements, $\Delta_1(t)$ and $\Delta_2(t)$ in the z_1 and z_2 directions, respectively, at corner G7 shown in Figure 1. Plotted in Figure 3, for each ensemble of ground motions, are the mean and mean-plus-one-standard-deviation simulated envelopes that bound $\mathbf{x}(t) = [\Delta_1(t),\ \Delta_2(t)]^T$ for $\eta = 0.4$, 0.7 and 1.0. Also shown are the corresponding response-spectrum-based elliptical envelopes for the linear structure ($\eta = \infty$).

The envelopes plotted in Figure 3 indicate that the nonlinear response of a structure is sensitive to the input ground motions. For the ensemble of far-field records, the envelope bounding $[\Delta_1(t),\ \Delta_2(t)]^T$ contracts in a uniform manner and remains roughly elliptical when nonlinear elements are introduced. In contrast, for the near-field motions the peak roof displacement in the fault-normal direction (along the z_1 axis) is amplified several times more than the displacement in the fault-parallel direction when nonlinear elements are introduced into the building. This pronounced difference in the nonlinear displacement response in the fault-normal and fault-parallel directions, which cannot be predicted from the elliptical envelope for the linear structure, suggests that the temporal characteristics of the ground motion (e.g., the velocity pulse present in the fault-normal component) are as important to the nonlinear response of a structure as the frequency content. Therefore, it is unlikely that analytical procedures or empirical rules (like the well-known "equal displacements" rule) that relate the nonlinear envelope to the response-spectrum-based envelope of the linear structure can be developed. Simulation appears to be the only reliable way of predicting the envelope that bounds a response vector in a nonlinear structure.

The far-field results plotted in Figure 3 indicate that for $\eta = 1.0$ the variability in the envelope is comparable to that predicted for the linear structure. As η is decreased, the variability in the envelope increases. A similar trend is evident for the near-field motions, although the variability in the fault-normal direction (Δ_1) is considerably larger than that of the linear structure for all values of η. This observation has an interesting implication on the design of the structure. An underlying objective of conventional seismic design is to reduce the uncertainty in the structural response, usually by allowing damage to occur at selected locations within the structure that, in turn, limits the stresses in critical structural elements. We discuss the effectiveness of this approach in the next section for the moment frame columns. Here we note that there is an economical advantage to be gained by reducing the yield strength of the plastic hinges and, hence, the stresses in the columns. Obviously, by reducing the stresses, it may be possible to use smaller sections. However, the results plotted in Figure 3 indicate that this advantage is offset by increased variability in the displacement demands imposed on the structure. This trade-off must be addressed when selecting the yield strength of the nonlinear elements in the structure.

Envelopes Bounding Column Responses

For seismic loading, a widely used design methodology is to restrict any nonlinear behavior to predefined locations that are evenly distributed throughout the structure. During an intense earthquake, these locations of yielding act as fuses that limit the shear forces and bending moments transmitted to the members in the structure that are intended to remain undamaged. The design of conventional buildings usually forces plastic hinges to form in the beams and prevents inelastic action in the columns, except at their bases. Given the location and moment capacity of the plastic hinges, reasonable estimates of the maximum axial force and bending moment that can be experienced by a column can be made and used to size the element. It is interesting to examine the envelopes that bound the column responses in a nonlinear structure to better understand the benefits of this design methodology and it potential shortcomings.

Consider the ground floor column at node B7 in Figure 1. We denote the axial force in this column $P(t)$ and the bending moments at its top and bottom $M_{\text{top}}(t)$ and $M_{\text{bot}}(t)$, respectively. Plotted in Figure 4a are the simulated mean and mean-plus-one-standard-deviation envelopes that bound $\mathbf{x}(t) = [M_{\text{top}}(t),P(t)]^T$ and $\mathbf{x}(t) = [M_{\text{bot}}(t),P(t)]^T$ for $\eta = 0.4$, 0.7, 1.0 and ∞ when the building is subjected to the ensemble of far-field ground motions. The corresponding envelopes for the near-field motions are plotted in Figure 4b.

A comparison of Figures 4a and 4b reveals that, for a given value of $\eta \leq 1.0$, the envelopes that bound the responses at the base of the column in the nonlinear structure are similar for the two ensembles of ground motions, even though the elliptical envelopes for the linear structure ($\eta = \infty$) differ notably in size. Furthermore, there is little, if any, uncertainty in the peak values of $P(t)$ and $M_{\text{bot}}(t)$. This, of course, is the chief benefit of the design methodology described above, since the designer can remove almost all of the uncertainty in the column response and select the required column size with confidence. In this case, the bound on $M_{\text{bot}}(t)$ is the plastic capacity of the column base while the bounds on $P(t)$ are

$$P_0 - \frac{2}{L}\sum_{i=1}^{3} M_{pi} \;\leq\; P(t) \;\leq\; P_0 + \frac{2}{L}\sum_{i=1}^{3} M_{pi} \,, \tag{3}$$

where P_0 is the gravity load acting in the column, M_{pi} is the plastic capacity of the beam connection at level i and L is the length of the beam.

The primary shortcoming of this design methodology can be seen in the envelopes that bound the responses at the top of the column. While $P(t)$ is again bounded by the values given in (3), there are no strict bounds on $M_{\text{top}}(t)$ because the columns, which are assumed to remain elastic in the superstructure, can attract additional bending moments during intervals of severe yielding in the beams. Consequently, not all of the uncertainty in the column moments can be eliminated. This effect is apparent in Figure 4b for the near-field records and is undoubtedly related to the severe response of the structure to the fault-normal component of the ground motion. If the uncertainty in $M_{\text{top}}(t)$ is not accounted for, a plastic hinge may form in the column and a "soft-story" sway mechanism, in which damage is concentrated at one level of the building, may develop. Such sway mechanisms are undesirable because they are inefficient for dissipating the seismic energy imparted to the building and, more importantly, can cause the structure to become unstable. Current building codes address this problem in an approximate manner by requiring that the sum of the moment capacities of the columns that frame into a joint be greater than the sum of the moment capacities of the beams. However, it is well known that this design criterion does not guarantee that yielding will not occur in the columns. Reliable procedures for predicting the envelopes that bound column responses in nonlinear structures need to be developed.

Envelopes Bounding Roof Displacements and Column Hinges

The response vectors considered above consist of "interacting" components that can combine in an adverse way that must be guarded against. However, the components of a response vector do not have to be related by some design criterion. In this section, the response vector $\mathbf{x}(t) = [\Delta_1(t), \theta(t)]^T$ is considered, in which $\theta(t)$ is the plastic hinge rotation at the base of column B7 in Figure 1. For design purposes, these deformation demands do not have to be considered together since their acceptable limits are independent of each other. However, we will see that the envelope bounding this response vector provides information about the performance of the building under severe seismic loading that might otherwise be difficult to infer from scalar time-history results.

In the following analyses, η is varied over the height of the building as shown in Figures 5a, 5b and 5c. Recall that as η is increased, the yield level of a connection is increased. Hence, for the configuration shown in Figure 5b, we would expect the yielding elements to be concentrated near the top of the moment frame as indicated. The reverse is true for the configuration shown in Figure 5c.

When designing a moment frame, inefficient sway mechanisms like those sketched in Figures 5b and 5c are usually avoided in favor of a sway mechanism that closely matches the one shown in Figure 5a, in which all the plastic hinges participate in the dissipation of the seismic energy imparted to the building. The only reliable way of predicting the sway mechanism that will form in a moment frame is by performing nonlinear time-history analyses using an appropriate ensemble of ground motions. This time-history approach is not problem-free though. It requires that the spatial distribution of plastic hinges be recorded in time (e.g., by plotting the yield events on an elevation of the moment frame) to confirm that a desirable sway mechanism is attained. Moreover, since an ensemble of ground motions should be considered, the analyst is also faced with the problem of incorporating the results of several time-history analyses into the evaluation. The amount of data that must be recorded and stored can become unmanageable unless an efficient procedure for interpreting the time-history results is available. As we will see, the envelope bounding $\mathbf{x}(t) = [\Delta_1(t), \theta(t)]^T$ is well suited to this problem.

Consider the ideal sway mechanism shown in Figure 5a. It can be seen from the geometry of this figure that

$$\frac{\theta_{\max}}{\Delta_{1\max}} = \frac{1}{H}, \tag{4}$$

where $\theta_{\max} = \max[\theta(t)]$, $\Delta_{1\max} = \max[\Delta_1(t)]$ and H is the total height of the building. Note that $\Delta_{1\max}$ and $\theta_{\max}$ occur simultaneously in this case; hence, $[\Delta_{1\max}, \theta_{\max}]^T$ lies on the envelope that bounds $\mathbf{x}(t) = [\Delta_1(t), \theta(t)]^T$. It is clear from Figures 5b and 5c that the equality in (4) does not hold for the sub-optimal sway mechanisms depicted in these figures. Furthermore, the sense of the inequality is indicative of where in the moment frame the yielding elements are located; e.g., if the plastic hinges are concentrated near the roof, as in Figure 5b, then $\Delta_{1\max} > H\theta_{\max}$. Thus, we argue that the envelope bounding $\mathbf{x}(t) = [\Delta_1(t), \theta(t)]^T$, which provides information about the relationship between $\Delta_{1\max}$ and $\theta_{\max}$, can be used to identify the spatial distribution of plastic hinges in the moment frame.

To support this claim, the simulated mean envelopes that bound $\mathbf{x}(t) = [\Delta_1(t), \theta(t)]^T$ when the building is subjected to the far-field ground motions are plotted in Figure 6 for the moment frames shown in Figure 5. The qualitative nature of the results obtained for the near-field ensemble is similar to that seen in Figure 6, so they are not presented. Also plotted in Figure 6 is a line that has a slope of $1/H$. As noted above, if the structure attains the ideal state depicted in Figure 5a, then the point $[\Delta_{1\max}, \theta_{\max}]^T$ should lie on this diagonal line as well as on the envelope. As seen in Figure 6a, this situation nearly occurs for the case in which $\eta = 0.4$ for all connections. The results plotted in Figures 6b and 6c, for the cases in which η varies over the height of the building, further confirm that the envelope bounding $\mathbf{x}(t) = [\Delta_1(t), \theta(t)]^T$ can be used in the way described above to determine where in the moment frame the yielding elements are located. The chief advantage of using this response envelope to identify the sway mechanism lies in the fact that the yield events need not be recorded and plotted as they occur in time. Furthermore, the results from any number of time-history analyses can be rationally included in the assessment by computing statistical envelopes such as those corresponding to the mean and mean-plus-or-minus-one-standard-deviation of the ensemble.

SUMMARY AND CONCLUSIONS

In earthquake engineering, the simultaneous action of multiple responses must often be considered. For most structural engineering applications, the envelope that bounds the responses as they evolve in time provides all the information necessary to carry out the design calculations. In this paper, the envelopes that bound selected vectors of seismic responses in a nonlinear three-story steel moment frame building were simulated and examined. The results of this investigation provide insight into the nonlinear behavior of structures and the effectiveness of current seismic design methodologies. In particular, the following observations were made:

1. The size, shape and variability of nonlinear response envelopes are strongly dependent upon the characteristics of the ground motion. For example, for the near-field ground motions, the changes in the size and variability of the roof displacement envelope upon the introduction of nonlinear elements into the system were more pronounced in the fault-normal direction than in the fault-parallel direction. This observation suggests that the temporal characteristics of the ground motion are as important to the nonlinear response of the building as the frequency content.

2. The uncertainty in the displacement demands is dependent upon the level of nonlinear behavior experienced by the structure. In particular, it was noted that decreasing the yield strengths of the moment frame connections caused increased variability in the roof displacement envelope.

3. The use of plastic hinges as fuses to limit the stresses in moment frame columns is only partially effective. It was observed that under severe seismic loading, the uncertainty in the peak column bending moments in the superstructure could not be eliminated. Thus, there is a finite probability that the stress in a column will exceed the yield stress of the steel.

4. Nonlinear response envelopes can provide information about the seismic performance of a structure that might otherwise be difficult to infer from scalar time-history results. In particular, we demonstrated how a simple geometric interpretation of a particular nonlinear response envelope can be used to identify the spatial distribution of plastic hinges in the moment frame and, hence, the expected sway mechanism.

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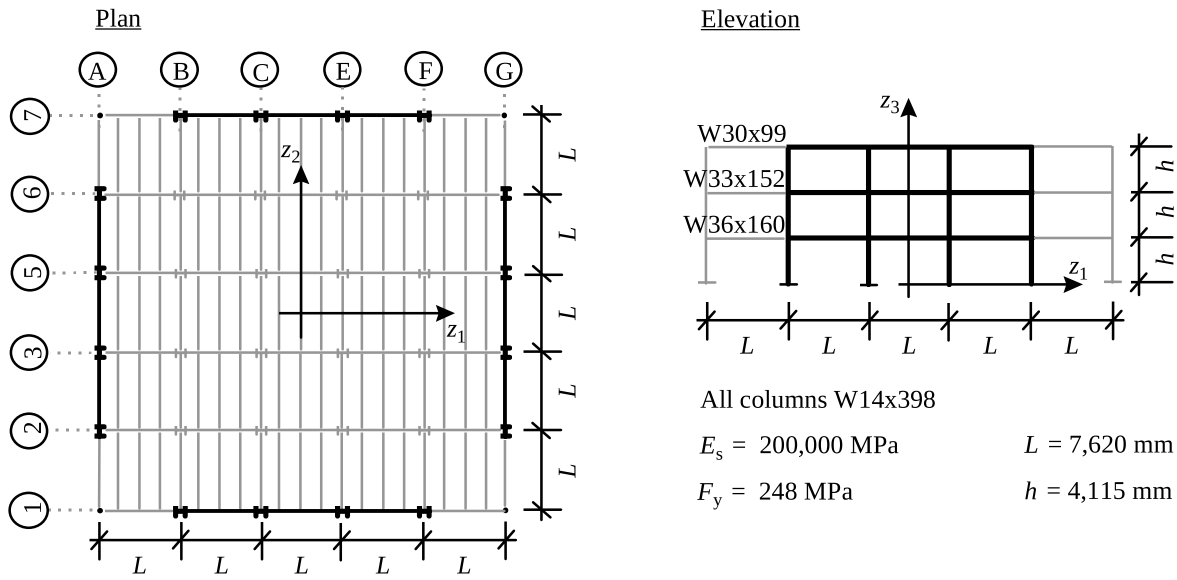

Figure 1. Example three-story steel moment frame building.

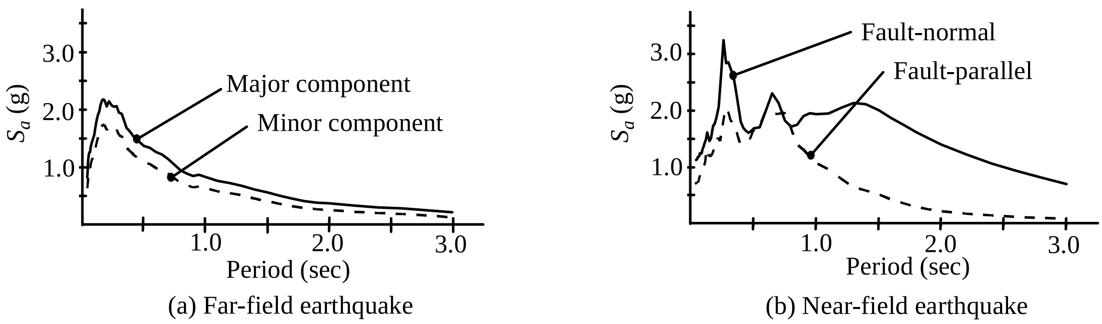

Figure 2. Mean pseudo-acceleration response spectra of the ground motions (ζ = 5%).

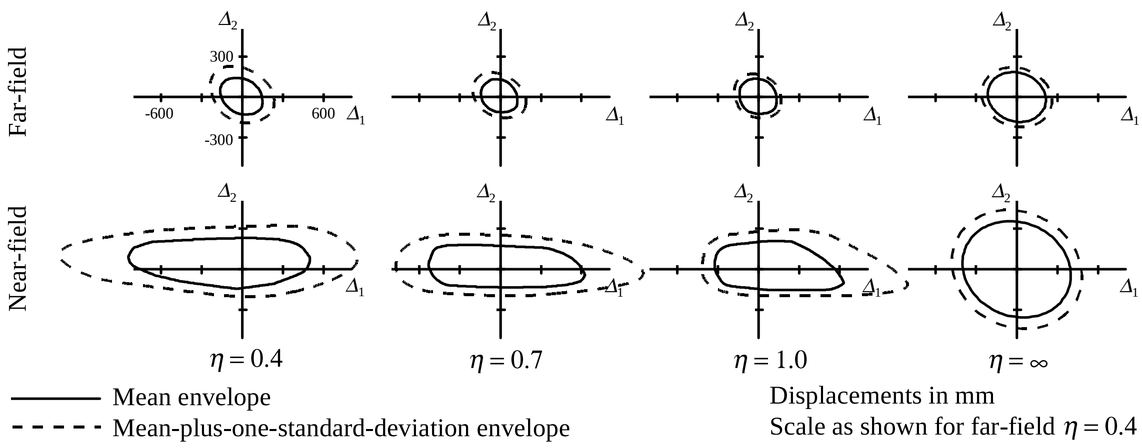

Figure 3. Roof displacement envelopes.