

Determination of the dynamic behaviour of soils using surface waves: Spanish experiences

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ABSTRACT: In this paper a review is made of the different surface wave surveying methods used in Spain for Civil Engineering purposes over the last fifteen years. The principles lying behind each procedure are discussed and examples are given of how those methods can be used to obtain information concerning the elastic properties of soil deposits. A description is made of the techniques used to interpret the dispersion curves obtained in the field, and the results obtained in some cases are compared with those provided by more conventional methods i.e.: plate loading and cross-hole tests. Finally, the usefulness of the surface wave methods to locate anomalous zones in earthworks such as tunnels, embankments and urban waste deposits is emphasized, illustrating, at the same time, their capacity to evaluate the adequacy of some ground treatments.

1. INTRODUCTION

The use of seismic geophysical methods for geotechnical purposes has always been favored by civil engineers, who for many years have found them to be the most suitable procedures for obtaining "in situ" information concerning the true linear elastic behaviour of soils under static and dynamic loading conditions. In many cases, the shear elastic modulus of the soil G_0 becomes a parameter of paramount importance for the design of the supporting structure foundation and its accurate determination requires the use of "in situ" methods, free of all the sample disturbance effects inherent to laboratory testing. For this purpose seismic methods aimed to obtain the velocity V_s with which shear waves propagate within the soil mass are the most reliable methods, since the interpretation of all the other mechanical procedures often used i.e.: SPT and CPT tests, are based on empirical correlations between the shear wave velocity and other geotechnical indices. Once the V_s value has been obtained, the G_0 parameter can be worked out directly from the following

expression borrowed from the theory of Elasticity:

$$G_0 = \rho V_s^2 \quad (1)$$

where ρ represents the soil mass density including the pore water mass.

At present, out of the different existing seismic methods, cross-hole and down-hole tests are more commonly used in the field of Foundation Engineering to obtain V_s values. However, these methods do require the drilling of boreholes and, in many cases, special treatment of the space between the casing and the borehole walls and for these reasons they tend to become expensive and time-consuming methods. With a view to overcoming such drawbacks, seismic refraction methods have sometimes been used. Nevertheless, the calculation of the ground shear wave velocity profile using seismic refraction is not simple, given that the arrival of the shear waves is usually hidden by that of faster compression waves and the method becomes blind when competent layers overlie softer ones. Therefore, it is hardly surprising that surface wave methods, not requiring the

drilling of boreholes and being free of the abovementioned disadvantages, are now becoming widely accepted among engineers. A review will be made in the following paragraph, of the different methods of that type which have been used in Spain for Civil Engineering purposes over the last 15 years.

2. STEADY STATE METHOD

The use of surface waves for engineering purposes was introduced in Spain by Santamaria et al. (1973, 1975). They were applied to the analysis of multilayer road systems using the same surveying technique as that established by Bernhard (1939) and Long et al. (1945) in U.S.A., Jones (1958, 1962) in U.K. and Blain (1968), and Guillemín et al. (1971) in France.

The method consisted of applying vertically stationary senoidal vibrations on the ground surface, within a wide range of frequencies, and finding for each frequency ω the points on that surface which were in phase with the vibrator (see Figure 1). In this way, it was possible to calculate the wavelength associated with each value ω , imposed on the vibrator. Knowing these two values, velocity c at which frequency ω propagates along the ground surface, i.e.: the phase velocity, can be obtained using a very simple expression:

$$c = \frac{\omega}{2\pi} \lambda \quad (2)$$

substituting $f = \omega/(2\pi)$ the following equation results:

$$c = f \cdot \lambda \quad (3)$$

By plotting c against f or λ , it is possible to obtain the so called "dispersion curve", which reflects the geometrical and elastic properties of the different ground layers at the observation point.

Cuéllar and Escario (1977), used that method to obtain the soil parameters needed for the design of the foundation blocks of a pumping station forming part of the water supply system of the Northern part of the city of Madrid. The site consisted of slightly cemented

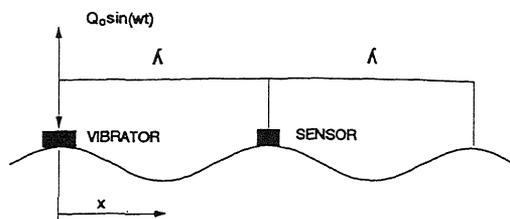


Figure 1. Determination of wavelength-frequency values by the steady state method

dense granular material, known locally as "Arena de Miga" that was excavated until the foundation level was reached. At that level, steady state surface wave tests were conducted using a Goodman vibrator. The interpretation of the dispersion curve at that time, was consistent with the observations made by Heukelom and Foster (1960), who found that when dividing by a factor of two the wavelengths obtained in a road system a new plot was obtained where layers with different elastic properties could be easily identified. In addition, a plate loading test was performed on a 30 cm square plate located at the same level. A Young modulus of 95 MPa was calculated using the data obtained from the repeated loading and unloading cycles carried out as part of the plate loading test (see Figure 2). It can be shown that this value is associated with a 0.60 m. thick soil layer below the plate, for which a mean value of 10^{-3} was estimated as being representative of the strain field created by the maximum load imposed on the plate. According to Heukelom and Foster that thickness correspond to wavelength values ranging from 0 to 1.2 m., which define a phase velocity of 200 m/s in Figure 3. Assuming values of 0.35 and 21 KN/m^3 for the Poisson ratio and unit weight of the Arena de Miga respectively, very simple elastic calculations give a Young modulus of 270 MPa associated to that velocity and to an estimated strain value of 10^{-6} . Plotting those results in the Seed and Idriss (1970) graph that relates elastic modulus ratio with strain values for granular materials (see Figure 4), a point was obtained consistent with the fact that, the denser the material the closer it plots to the

upper curve of the range suggested by Seed and Idriss for different void ratios.

Those preliminary results encouraged the use of this method to obtain the dynamic elastic properties of soils at that time. However, improvements made in digital electronics over the last two decades, have fostered the appearance of new surveying techniques replacing the more time consuming steady state method.

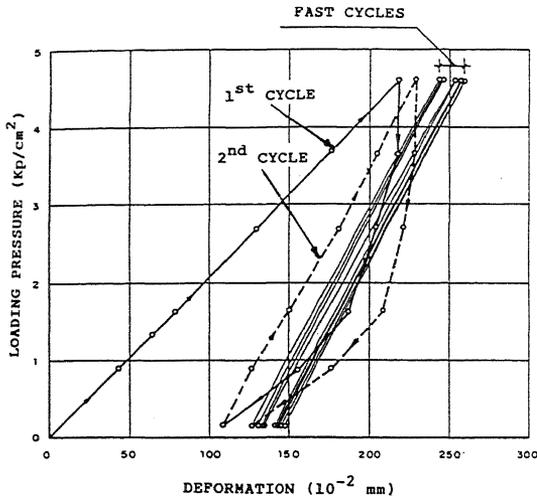


Figure 2. Plate loading test on Arena de Miga (Cuéllar and Escario, 1980)

3. SPECTRAL ANALYSIS OF SURFACE WAVES

In the last decade, Stokoe and his coworkers at Texas University, in Austin, introduced the spectral analysis of surface waves (SASW) into the field of Geotechnical Engineering, see Heisey et al. (1982), Nazarian (1984), Stokoe and Nazarian (1984). The test is conducted by placing two receivers on the ground surface at a preselected spacing. A vertical impulse is then applied to the surface and this generates a transient signal $F(t)$ containing a certain range of frequencies. The group of waves is monitored by the receivers and captured in the time domain as $X_1(t)$ and $X_2(t)$ signals, with a recording device (see Figure 5). They are transformed into the frequency domain by utilising the Fourier transform:

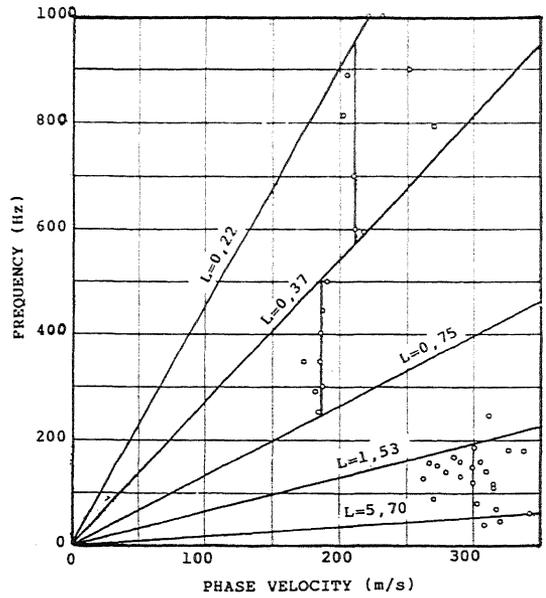


Figure 3. Phase velocity for different ranges of wavelength in Arena de Miga (Cuéllar and Escario, 1980)

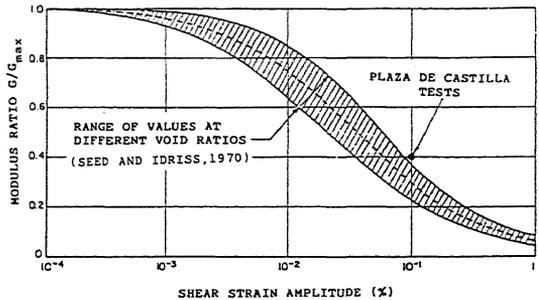


Figure 4. Modulus ratio of Arena de Miga from plate loading and surface wave tests.

$$X_1(f) = \int_{-\infty}^{\infty} X_1(t) e^{-i2\pi ft} dt \quad (4)$$

$$X_2(f) = \int_{-\infty}^{\infty} X_2(t) e^{-i2\pi ft} dt$$

The transfer functions $H_1(f)$ and $H_2(f)$, characterizing the dynamic systems made up by the soil layers between the source and each one of the receivers, are obtained through dividing those functions by

the linear spectrum $F(f)$ of the input signal:

$$H_1(f) = \frac{X_1(f)}{F(f)} = \frac{X_1(f)}{F(f)} \cdot \frac{F^*(f)}{F^*(f)} = \frac{G_{X_1F}}{G_{FF}} \quad (5)$$

$$H_2(f) = \frac{X_2(f)}{F(f)} = \frac{X_2(f)}{F(f)} \cdot \frac{F^*(f)}{F^*(f)} = \frac{G_{X_2F}}{G_{FF}}$$

where $F^*(f)$ represents the complex conjugate of the input signal linear spectrum and G_{X_1F} and G_{X_2F} are the cross-spectra between the input signal and the signals captured by the receivers.

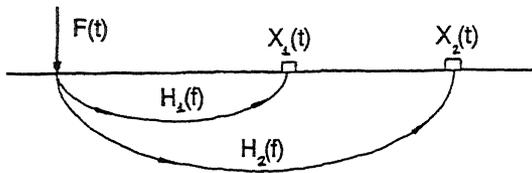


Figure 5. Input and receiver signals in SAW tests.

In Eqs. 5 the cross-spectra G_{X_1F} and G_{X_2F} appear normalised by the function G_{FF} , consequently the transfer functions $H_1(f)$ and $H_2(f)$ only depend on the transmission characteristics of the systems, and not on the input signal. On the other hand, as G_{FF} is a real valued function, and bearing in mind that:

$$X_1(f) = H_1(f) \cdot F(f) \quad (6)$$

$$X_2^*(f) = H_2^*(f) \cdot F^*(f) \quad (7)$$

it can also be shown that the phase of the cross-spectrum $G_{X_1X_2}$ does not depend on the input signal:

$$G_{X_1X_2} = X_1(f) \cdot X_2^*(f) = H_1(f) \cdot H_2^*(f) \cdot G_{FF} \quad (8)$$

Those two features are basic to the application of the method for obtaining the dispersion curve in the field, since they permit the use of any wave source as long as it generates the range of frequencies needed for each case.

After defining the range of frequencies for which both signals $X_1(t)$ and $X_2(t)$ correlate well, the

phase of the cross spectrum $G_{X_1X_2}$ is used to find out the phase shift ϕ between receivers for each frequency f selected (see Figure 6). By knowing the distance d between receivers, the phase velocity c is calculated by means of the following equation:

$$c = \frac{360 \cdot d \cdot f}{\phi} \quad (9)$$

where ϕ is expressed in degrees and f in Hz.

The corresponding wavelength is calculated by substituting c and f into Eq. (3). The dispersion curve can then be obtained using that procedure for different values of f .

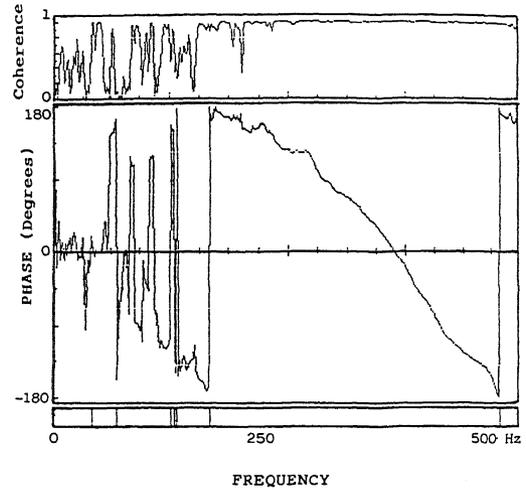


Figure 6. Range of frequencies with coherence close to one in typical SASW test.

A program called FORWARD, developed by Sanchez Salinero (1987) has been used for the interpretation of the dispersion curves. It is based on the first mode of propagation of in plane S and P waves through a layered half-space with constant elastic properties within each layer. Using the stiffness matrix formulation suggested by Kausel and Roesset (1981) for a complex domain, forces per unit area at the boundaries of each layer are expressed in terms of displacements. By assembling the stiffness matrix of all the layers

a global stiffness matrix for the complete system is then obtained. Setting the determinant of that matrix to zero, it is possible to obtain the dispersion curves that correspond to the different modes of propagation of the in-plane motion; that equation represents an eigenvalue problem in which for each frequency ω there are several values of phase velocity or eigenvalues that satisfy the equation. For any one of those values, there is a dispersion curve that characterizes the corresponding mode of propagation. The way in which different modes combine in a real test depends on several factors. It is usually assumed that the first mode is the one which makes the greatest contribution to the shape of the dispersion curve, but this is not necessarily always the case. That assumption holds for a system in which stiffness increases with depth. In such a case, the FORWARD program can be relied upon to interpret the dispersion curves obtained in the field. With a few slight limitations, it can also be used to interpret dispersion curves for the case of only one layer overlying a less stiffness half-space, but for the case of a multilayered system in which the stiffness decreases with depth, the FORWARD program only gives an acceptable fit of the low frequency branch of the real dispersion curve i.e.: the part of that curve which is controlled by the first mode of propagation. In view of the fact that the highest frequency branch of the "in situ" curve can be also reproduced fairly well by FORWARD assuming only one layer over a less stiffness half-space, the following trial and error procedure was used to invert the dispersion curves obtained in the field (see Figure 7).

Firstly, different stages are identified along the dispersion curve, and the coordinates (phase velocity and wave-length) of the extreme points defining each stage are taken directly from the "in situ" data.

Secondly, using the coordinates c_1, λ_1 and c_2, λ_2 of the two points that define the first stage, a system consisting of only one layer over a half-space is determined assuming a layer thickness of $\lambda_1/2$ and trying different shear wave

velocities to match the coordinate values.

In a third step, a two layer system is determined conserving the properties of the first layer, assuming a thickness of $(\lambda_2 - \lambda_1)/2$ for the second layer and using the coordinates of the two points that define the second stage in the dispersion curve to obtain the shear wave velocities of the two last layers.

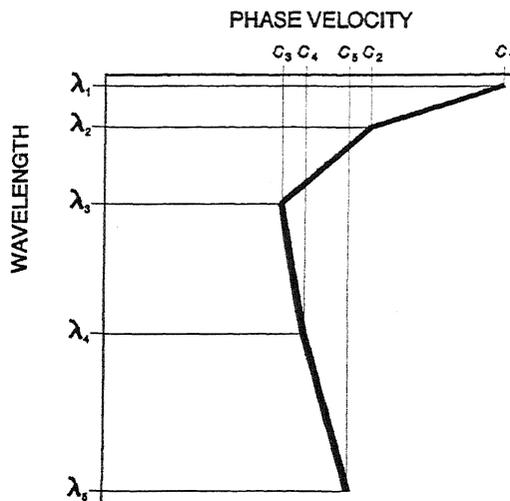


Figure 7. Stages used to invert dispersion curves.

Proceeding in that way, the thickness and shear wave velocity of the remaining layers are obtained. Once the stages where stiffness increasing with depth are reached, the whole process speeds up, since several stage point coordinates can be adjusted at the same time.

The abovementioned procedure was used to interpret the dispersion curves at Los Barrios, a site on the Spanish Coast of the Straits of Gibraltar, which was filled with fly ash, a by-product from a nearby power station. Figure 8 shows the layer thicknesses and shear wave velocities obtained at a point where there was a hard desiccated crust on the surface. Cross-hole data obtained at the same point, are also included in that figure.

The same technique was used to obtain the Young modulus of

decompressed marly rock around the concrete lining of a hydraulic tunnel situated in the Alcubierre Mountains in the North-East of Spain. In order to identify weak zones behind the lining, S.A.S.W. tests were performed on the top and lateral walls of the tunnel (see Fig. 9) and the dispersion curves obtained at different cross-sections were compared.

Figure 10 shows the dispersion curve obtained from the wall of a railway tunnel linking Spain and France under the Pyrenees in an area of considerable seismic hazard. In that particular case, natural rock seems to be more competent than the lining of the tunnel made up of masonry over a rough stone layer.

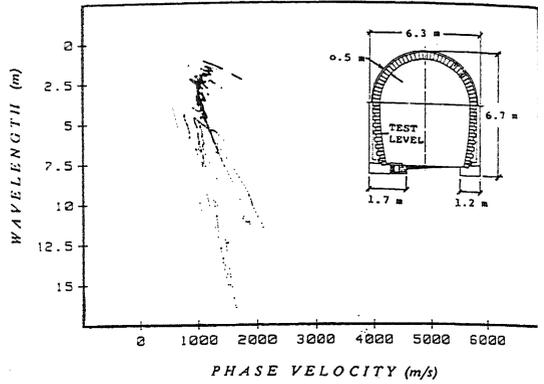


Figure 10. Dispersion curve from the wall of Somport railway tunnel (Cuéllar, 1990)

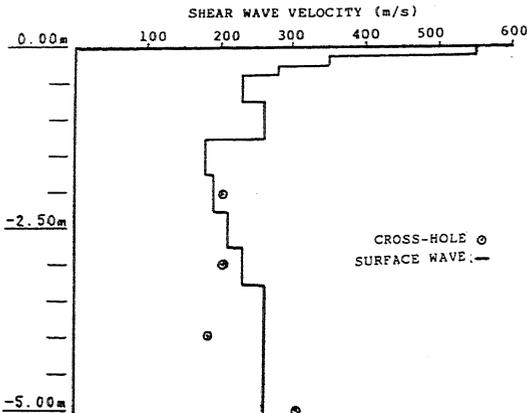


Figure 8. Shear wave velocity profile at Los Barrios (Cuéllar, 1988a).

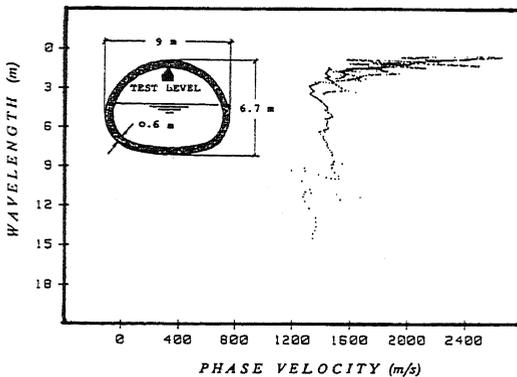


Figure 9. Dispersion curve from the top of Alcubierre hydraulic tunnel (Cuéllar, 1988b).

4. LINEAR FREQUENCY SWEEP METHOD

Recently, Valerio (1990) devised a data-collecting technique similar to the one suggested by White (1972) and White and Pinnington (1982), as an alternative to the impact-type methods for the dynamic analysis of structures. In this method, a vibrator is fed with an input signal $F(t)$ of constant amplitude, but with frequencies ranging linearly between an initial value ω_1 , and a final one ω_2 , over a fixed time length T . To achieve that, the following expression for $F(t)$ holds:

$$F(t) = F_0 \sin \psi(t) \text{ for } 0 \leq t \leq T \quad (10)$$

where F_0 = constant amplitude

$$\omega(t) = \frac{d\psi(t)}{dt} = \text{instant frequency} \quad (11)$$

In the case of a linear sweep of frequencies:

$$\omega(t) = 2at + b \quad (12)$$

$$\text{in which } b = \omega_1 \text{ and } a = \frac{\omega_2 - \omega_1}{2T}$$

Eqs. (11) and (12) lead to the following expression for function $\psi(t)$:

$$\psi(t) = at^2 + bt \quad (13)$$

Figure 11, shows function $F(t)$ generated for the case of unit amplitude $F_0 = 1$, a time length of 1 second and values ω_1 and ω_2 , which correspond to frequencies of 1 Hz and 10 Hz, respectively. The linear spectrum of that signal has been drawn at the lower part of the figure. It is rectangular in shape, with a constant mean value given by:

$$|F(f)| = \frac{F_0}{2} \sqrt{\frac{2\pi T}{\omega_2 - \omega_1}} \quad (14)$$

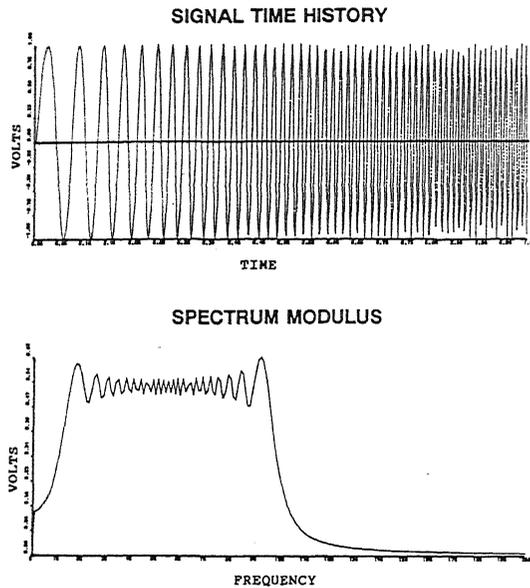


Figure 11. Spectrum for linear range of frequencies.

Expression (14) shows that with this procedure, it is possible to control not only the frequency band of the excitation, but also its mean spectral level.

Several tests were conducted at different locations in the South of Spain, using this type of vibration source. Figure 12 shows the dispersion curves obtained from the backfill of an old retaining wall located in the Sultana Patio at La Alhambra, in Granada. The 5 m. high wall showed evidence of large displacements, probably reflecting the effect of large magnitude earthquakes that took place in the past. Owing to the historical value of the wall, and in order not to

interfere with visitors to the palace, it was decided to stabilize the wall by strengthening the backfill material. Accordingly it was grouted using sleeve pipes, the pressure and flow of the grouting mixture being controlled very strictly. With a view to checking the efficiency of the treatment, surface wave tests were carried out before and after the soil was grouted. The interpretation of the dispersion curves obtained at different points led to post-grouting shear moduli ranging from 5 to 10 times the values obtained before the injection took place.

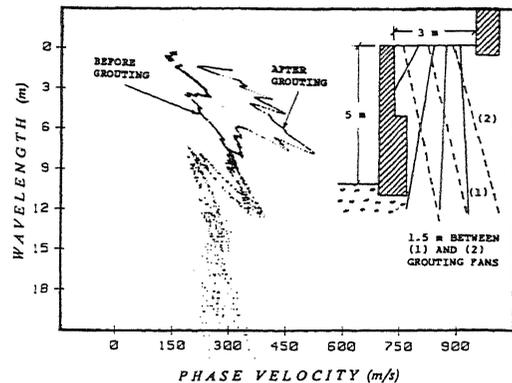


Figure 12. Dispersion curves from grouted backfill in Sultana Patio at La Alhambra (Cuéllar, 1992).

Different procedures have been used to improve the foundation conditions of the Juan Carlos I highway connection at EXPO 92, in Seville, where there was a very poor soil, consisting of a sandy clay mixed with urban waste deposits. In some zones, the foundation soil was compacted by letting a heavy weight fall from the top of a crane, and then using gravel to fill the sink-holes created by the impact. To obtain an even ground surface, a last layer was spread and compacted with pneumatic rollers. After this, a 2.5 m. high embankment was constructed on the top. The dispersion curves in Figure 13, show that above a depth of 3 m., there was a notably improvement in the soil after compaction, and that the construction of the embankment mainly affected the uppermost meter. Other zones were improved by inserting gravel columns into the

soil mass using a torpedo attached to a crane. The torpedo was helped to advance by jetting with water. After reaching a prescribed depth, the torpedo was taken out and the hole filled with gravel. Afterwards, the torpedo was raised and lowered within the hole to compact the gravel. Finally, an 8 m. high embankment was constructed on top of the columns. The dispersion curves in Figure 14, indicate that the treated zone did not show any substantial improvement until the embankment compressed the gravel columns, thus reinforcing and consolidating the weaker material.

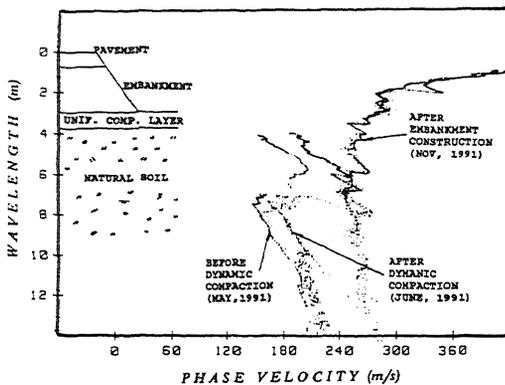


Figure 13. Dispersion curves from dynamically compacted soil at Seville (Oteo, 1992).

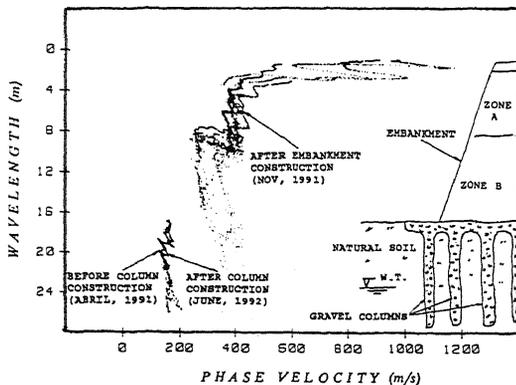


Figure 14. Dispersion curve from soil treated with gravel columns at Seville (Oteo, 1992).

5. AMPLITUDE MODULATED METHOD

To easily identify the time it takes for each frequency to travel from one receiver to the other, Satoh (1989) and Satoh et al. (1991), suggested to modulate the amplitude of the input signal as sketched in Figure 15. To obtain the dispersion curve they proceed frequency by frequency, keeping constant the distance between receivers and dividing that distance by the value τ that makes

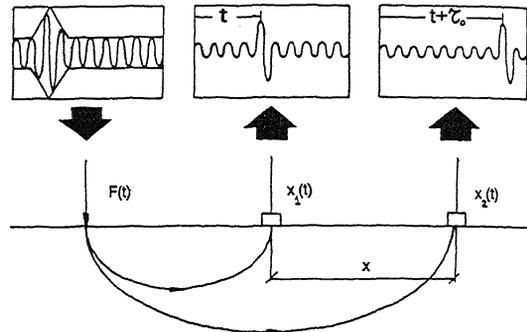


Figure 15. Amplitude modulated signals.

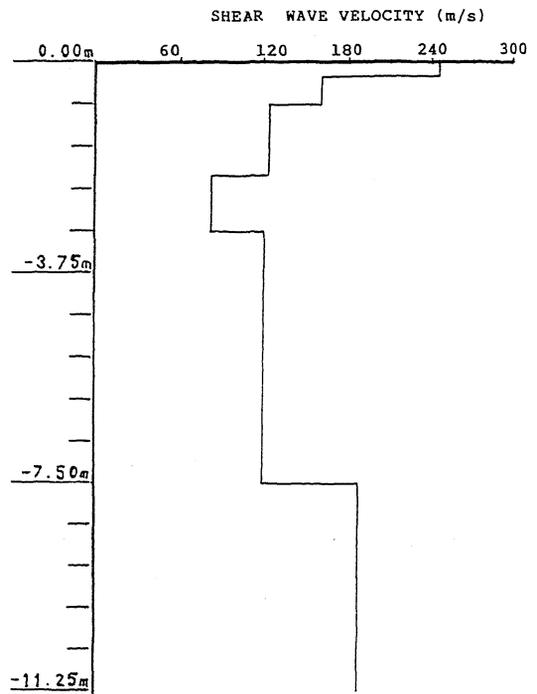


Figure 16. Shear wave velocity profile at Villalba waste deposit.

a maximum of the cross-correlation function $R_{x_1x_2}$ in the time domain:

$$R_{x_1x_2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T X_1(t) X_2(t+\tau) dt \quad (15)$$

Figure 16 shows the distribution of shear wave velocity values with depth obtained using this procedure on the top of an urban waste deposit, situated 80 Km. from Madrid, in the Guadarrama Mountains. Fortunately, the waste deposit lies in a very low seismic zone. Had it been situated in a more hazardous area, it would constitute a real threat to the environment, due to the extremely low shear wave velocity values detected at some levels.

6. CONCLUSIONS

A review has been made of the different surface wave methods used in Spain for civil works over the last fifteen years. All of them have been shown to be reliable procedures to obtain the elastic properties of soil deposits, to detect anomalous zones in earthworks, and to judge the degree of improvement achieved with some ground treatments.

The main differences among the procedures described lie in the type of wave source employed and the kind of domain used to obtain the dispersion curve: The most adequate method being the one capable to generate more efficiently the range of frequencies relevant to each particular case. In certain circumstances highly damped impacts may be used advantageously as an alternative to the very heavy vibrators needed to create low frequency harmonic waves.

In contrast with impact source type methods, the linear sweep frequency procedure allows a greater control of the receiver signals. It is also a more efficient, although less precise, method than the amplitude modulated procedure.

Concerning the interpretation of the dispersion curves suggested in this work, more efficient methods are urgently needed. In this respect, the contributions recently made by Rix and Leipski (1991) Badal et al. (1992) and Roesset (1992) should be mentioned.

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