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Response spectrum of ground shear strain

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ABSTRACT: This paper introduces two types of ground shear-strain response spectra based on a simplified model of vertically propogating shear waves in an uniform half-space. With a free-field earthquake motion specified at the ground surface, both spectra express the relationship between the normalized maximum ground shear-strain induced and the characteristic wave travelling time of the ground. They can be used to estimate the maximum ground shear-strain at any depth of the ground once the wave travelling time needed from that depth to the ground surface can be accurately estimated.

1 INTRODUCTION

When a structure or a piece of equipment is subjected to earthquake motions, its maximum response can be conveniently characterized by the response spectrum of a single-degree-of-freedom structural system. For many years, the response spectrum has been recognized as one of the most fundamental basis for earthquake resistant design of structures, especially for building structures. However, the usefulness of response spectrum can not be extended to the design of underground structures because their seismic responses are very different from those of super- structures. The structure buried in the ground will generally move along with the surrounding soils (Kuesel 1969; Okamoto and Tamura 1973). When the soil is stiff compared to the structure, the response of the buried structure will conform with the free-field earthquake deformation of soil. For very soft soils, interaction between the soil and the structure may have to be considered. In any case, the free-field deformation of soil during the earthquake excitation becomes the basic information for earthquake resistant analysis and design of underground structures.

For underground lifelines such as transportation tunnels in metropolitan areas, their cross-section lining is usually rather flexible as compared to surrounding soils. During earthquakes, the free-field ground motion environment will cause the lining cross-section to undergo racking (shear-type) deformation as shown in Fig. 1, where γ represents the angle of shear distorsion of a rectangular tunnel. Therefore, the lining has to be designed to have sufficient ductility to absorb the shear deformation imposed by soils. In engineering practice, it is thought that the most severe condition is that produced by the vertical travelling shear waves in soil layers. The design values of shear deformation are usually estimated empirically by

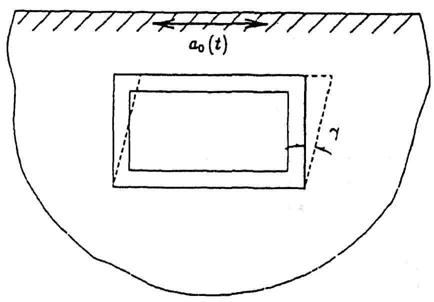


Fig. 1 Racking deformation of a ractangular tunnel.

$$\gamma = \frac{v_{max}}{c} \tag{1}$$

where v_{max} is the horizontal peak ground velocity of design earthquake and c is the shear wave velocity of in-situ soils (DORTS 1988). This practice is quite simple but is thought to be very conservative, especially for the shallower depth of ground (Penzien et al. 1992). To provide a rational estimation for maximum ground shear strain induced by specified earthquake, this paper introduce two types of innovative spectra which can be used as a quick reference for calculating the free-field responses of maximum shear strain at various depths of ground.

2 GROUND RESPONSE

The propagation of seismic waves is rather complex depending on source mechanism of the earthquake, travelling path and local site conditions. However, for simplicity, a simple model assuming vertically propagating shear waves within a homogeneous soil layer is usually used in engineering applications. The purpose of this paper is to estimate the ground

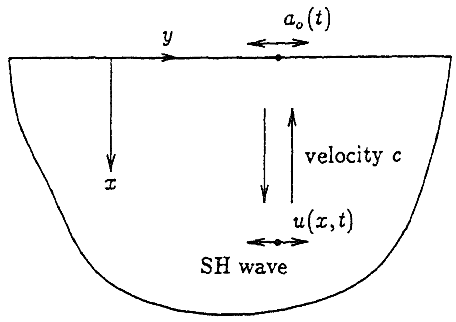


Fig. 2 Vertical propagating shear wave in an uniform half-space.

shear-strain for engineering design, the same model is therefore adopted herein.

Assume a vertically propagating shear wave producing horizontal motions in the x-y plane of a homogeneous elastic half-space, as shown in Fig. 2. The resulting ground motions can be characterized by the one-dimensional wave equation as follows

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2} \tag{2}$$

where $u(x,t)$ is the horizontal displacement of soil and c is the shear wave velocity of soil. For harmonic vibrations with time dependency $e^{i\omega t}$, the governing wave equation can be transformed into the frequency domain to get

$$\frac{\partial^2 U_x(\omega)}{\partial x^2} = -\frac{\omega^2}{c^2}U_x(\omega) \tag{3}$$

where $U_x(\omega)$ is the amplitude of displacement response at depth x measured from the ground surface. Find the general solution of Eq. (3) and impose the zero shear-stress condition at the free-surface, one can get

$$U_x(\omega) = 2E\cos(\frac{\omega x}{c}) \tag{4}$$

where the coefficient E can be determined by prescribed earthquake motion in the half-space. Suppose the prescribed motion is a free-field accelerogram at the ground surface as usual, the given acceleration time-history can be expressed in the frequency domain by using the Fourier transform as follows

$$a_0(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} A_0(\omega)e^{i\omega t}d\omega \tag{5}$$

$$A_0(\omega) = \int_{-\infty}^{+\infty} a_0(t)e^{-i\omega t}dt \tag{6}$$

where $a_0(t)$ is the prescribed acceleration time history at the ground surface and $A_0(\omega)$ is the corresponding Fourier amplitude of acceleration at excitation frequency ω. For each harmonic vibration of frequency ω, the amplitude of ground surface displacement $U_0(\omega)$ is related to the amplitude of prescribed acceleration by

$$U_0(\omega) = -\frac{A_0(\omega)}{\omega^2} \tag{7}$$

Substitude Eq. (7) into Eq. (4) and differenciate Eq. (4) with respect to x to get

$$\Gamma_x(\omega) = \frac{1}{\omega c}\sin(\frac{\omega x}{c})A_0(\omega) \tag{8}$$

where $\Gamma_x(\omega)$ represents the amplitude of shear strain at depth x from the ground surface. Equation (8) gives the response of shear-strain, in the *frequency domain, corresponding to the surface* acceleration specified. Its time-history response can be obtained through the application of Fourier transform as defined in Eq. (5).

3 $x\gamma$ RESPONSE SPECTRUM

Define the characteristic wave travelling time τ as the time needed for a shear wave travelling from depth x to the ground surface, *i.e.*,

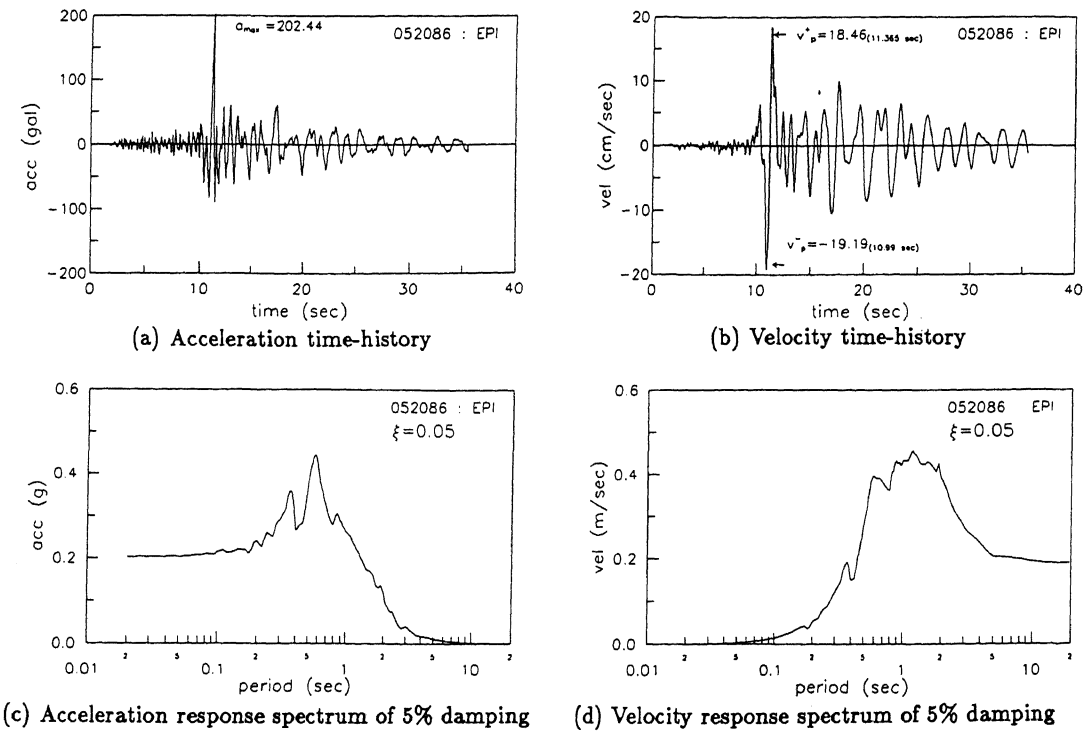
$$\tau = \frac{x}{c} \tag{9}$$

Multiply both sides of Eq. (8) by x and substitute Eq. (9) into Eq. (8), one gets

$$x\Gamma_x(\omega) = [\frac{\tau}{\omega}\sin(\omega\tau)]A_0(\omega) \tag{10}$$

Equation (10) shows that the transfer function between $x\Gamma_x(\omega)$ and $A_0(\omega)$ is function of ω and τ only. That is to say, the product of depth and shear-strain amplitude can be fully characterized by wave travelling time τ, irrelevant to any *specific depth x or shear wave velocity c of the ground.* Therefore, giving a vlaue of τ, one can get the time-history response of $x\gamma(t)$ from the Fourier transform of Eq. (10). Locating the maximum value of $x\gamma(t)$ of the earthquake duration for a series values of τ constitutes a spectrum which will be called the $x\gamma$ spectrum thereafter. This spectrum relates the normalized maximum shear-strain ($x\gamma_{max}$) with respect to the characteristic wave travelling time for the prescribed surface motion.

For illustration, a free-field accelerogram recorded at the LSST (Large Scale Seismic Test) site of Lotung, Taiwan on the May 20, 1986 earthquake is chosen as an example (Tang 1987). Lotung is located on the Lanyang Plain where the well known strong motion array SMART-1 is installed. The Lanyang Plain is a deltaic fan which is almost completely covered by thick alluvial deposits of thickness ranged from two hundred meters to more than a thousand meters. At the LSST site, the ground can be characterized as a soft site with interbedded layers of sandy silts and silty sands. The shear



(a) Acceleration time-history
(b) Velocity time-history
(c) Acceleration response spectrum of 5% damping
(d) Velocity response spectrum of 5% damping

Fig. 3 Responses at Lotung, Taiwan on the May 20, 1986 earthquake.

wave velocities, obtained from cross-hole geophysical surveys, ranged from 100 to 300 m/sec for the soils near the ground surface. The horizontal acceleration and velocity time-histories along the epicentral direction of this earthquake are shown in Fig. 3. For reference, their structural response spectra of 5% critical damping are shown in Fig. 3.

Based on Eq. (10), the normalized maximum shear strain induced by the given earthquake can be calculated for each τ value assigned. The $x\gamma$ *spectrum generated is shown in Fig. 4. It shows* that the spectral value of $x\gamma$ increases with the increasing of characteristic time τ. Since the τ values are usually very small in engineering application, the spectrum for τ ranged from 0 to 0.4 seconds is also shown in Fig. 4 in arithmetic coordinate. Since the spectrum generated is function of τ only, it can be used to calculate the maximum shear strain produced by the free-field surface motion chosen, for all values of shear wave velocity and depth of the site. In engineering applications, if the wave velocity for a given site can be estimated, the maximum shear-strain at any depth x can then be estimated by first calculating the characteristic time τ and then finding the corresponding maximum value of $x\gamma$ from the spectrum generated.

4 $c\gamma$ RESPONSE SPECTRUM

From Eq. (8), multiply both sides of this equation by c and substitute τ for x/c, one gets

$$c\Gamma_x(\omega) = [\frac{1}{\omega}\sin(\omega\tau)]A_0(\omega) \qquad (11)$$

It shows that the product of shear wave velocity and shear-strain amplitude can also be fully characterized by wave travelling time τ. Thus, using the technique of Fourier transform, one can establish the relationship between the maximum value of $c\gamma(t)$ and the characteristic time τ to construct a spectrum which will be called $c\gamma$ spectrum thereafter.

Alternatively, Eq. (11) can be rewritten as

$$c\Gamma_x(\omega) = [i\sin(\omega\tau)]V_0(\omega) \qquad (12)$$

where $V_0(\omega)$ is the Fourier amplitude of velocity of the prescribed surface motion. Equation (12) has closed-form Fourier transform given by

$$c\gamma(t) = \frac{1}{2}[v_0(t+\tau) - v_0(t-\tau)] \qquad (13)$$

where $v_0(t+\tau)$ and $v_0(t-\tau)$ represent the ground surface velocities at time $(t+\tau)$ and $(t-\tau)$, respectively. Therefore, for each specific value of τ, the response of $c\gamma(t)$ can be obtained directly in the time domain by superimposing two time histories of ground surface velocity with time offsetting of $\pm\tau$, respectively. From the time-history response obtained, one can locate the maximum value of $c\gamma(t)$ to construct the $c\gamma$ spectrum desired. Therefore, if the ground velocity is known, the $c\gamma$ spec-

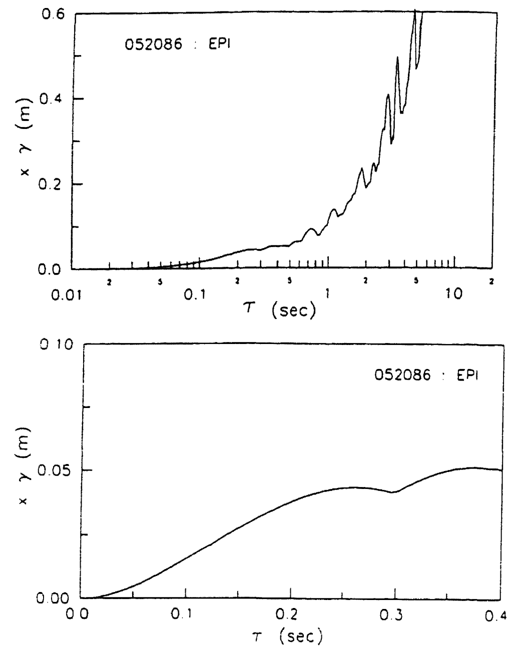


Fig. 4 Elastic $x\gamma$ spectra.

trum can be constructed by using Eq. (13) very efficiently. Furthermore, it can be shown that the $c\gamma$ spectrum obtained will have its peak and asymptotic values closely related to the peak velocity at ground surface as follows:

(i) For a ground velocity time history of duration T, it can be shown that

$$\max_t |c\gamma(t)| = \frac{1}{2}|v_p| \quad \text{when } \tau > \frac{1}{2}(T - t_p) \qquad (14)$$

where v_p is the peak ground velocity of prescribed earthquake and t_p is the occurring time corresponding to peak ground velocity. Equation (14) indicates that the $c\gamma$ spectrum will approach a constant value, the half of the peak ground velocity, for large values of τ.

(ii) The $c\gamma$ spectrum generated will have a peak value given by

$$\max_\tau \Big\{ \max_t |c\gamma(t)| \Big\} = \frac{1}{2}\Big|v_p{}^+ - v_p{}^-\Big| \qquad (15)$$

where $v_p{}^+$ and v_p^- are the positive and negative ground velocities of specified earthquake, respectively, and this peak value occurs at τ value given by

$$\tau = \frac{1}{2}|t_p^+ - t_p^-| \qquad (16)$$

where t_p^+ and t_p^- are the occurring times of v_p^+ and v_p^-, respectively. Equations (15) and (16) show that the the peak value of $c\gamma$ spectrum is equal to the half of the absolute difference of the positive and negative peak surface velocities, and the corresponding characteristic time τ is equal to the half of the absolute difference of the occurring time of these two peak surface velocities.

For the same earthquake described above, the ground surface velocity along the epicentral direction is shown in Fig. 3(b) which has $v_p{}^+ = 18.46$ *cm/sec* occurred at $t_p{}^+ = 11.365$ *sec* and $v_p{}^- = -19.19$ *cm/sec* occurred at $t_p{}^- = 10.99$ *sec*. Based on the methods described above, the $c\gamma$ spectra generated by using the frequency domain method and the time domain method are exactly the same as shown in Fig. 5. By using Eqs. (15) and (16), it can be found that the $c\gamma$ spectrum generated will have a peak value of 18.82 *cm/sec* corresponding to the characteristic wave travelling time τ of 0.190 *sec*. Besides, for large values of τ, the spectral value approaches a constant value of 9.60 *cm/sec* which is equal to the half of the peak ground velocity.

The application of $c\gamma$ spectrum is exactly the same as the $x\gamma$ spectrum constructed. However, from the view point of the efficiency and the close relationship with the peak ground velocities, the $c\gamma$ spectrum should be more useful for engineering applications.

5 DAMPED SPECTRUM

To consider the effect of material damping on the responses of ground shear strain, the complex-valued shear modulus can be as represented by

$$G^* = G(1 + 2i\beta) \qquad (17)$$

where G^* and G are the complex-valued and the elastic shear moduli, respectively, β is the damping ratio of hysteretic type and $i = \sqrt{-1}$. The associated complex-valued shear wave velocity can then be written as

$$c^* = c\sqrt{1 + 2i\beta} \qquad (18)$$

Substitute Eq. (18) into Eqs. (10) and (11), the transfer functions in frequency domain can be rewritten as

$$x\Gamma_z(\omega) = [\frac{\tau}{\omega\sqrt{1+2i\beta}}\sin(\frac{\omega\tau}{\sqrt{1+2i\beta}})]A_0(\omega) \qquad (19)$$

$$c\Gamma_z(\omega) = [\frac{1}{\omega\sqrt{1+2i\beta}}\sin(\frac{\omega\tau}{\sqrt{1+2i\beta}})]A_0(\omega) \qquad (20)$$

Based on the above two equations, the damped $x\gamma$ and $c\gamma$ spectra can be constructed through the frequency domain analysis as before. For the earthquake recording chosen (Fig. 3), the damped spectra of various damping ratios are shown in Fig. 6. Since the sine term in both the transfer equations, $\sin(\omega\tau/\sqrt{1+2i\beta})$, is an exponentially increasing function with respect to its argument, the damped spectra shown were calculated by choosing the cut-off frequency equal to 10 Hz and characteristic

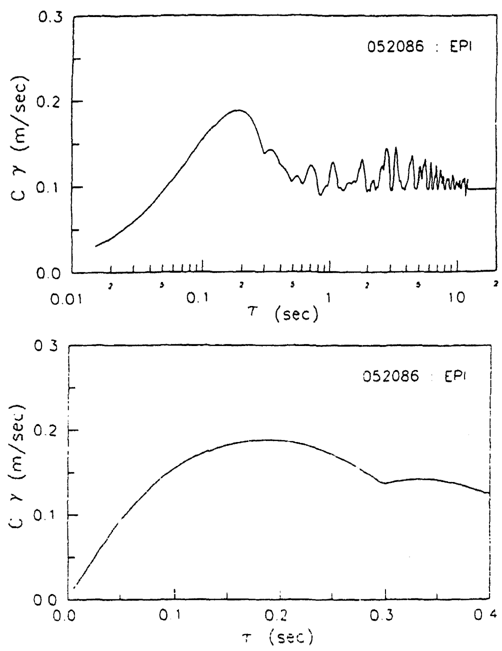


Fig. 5 Elastic $c\gamma$ spectra.

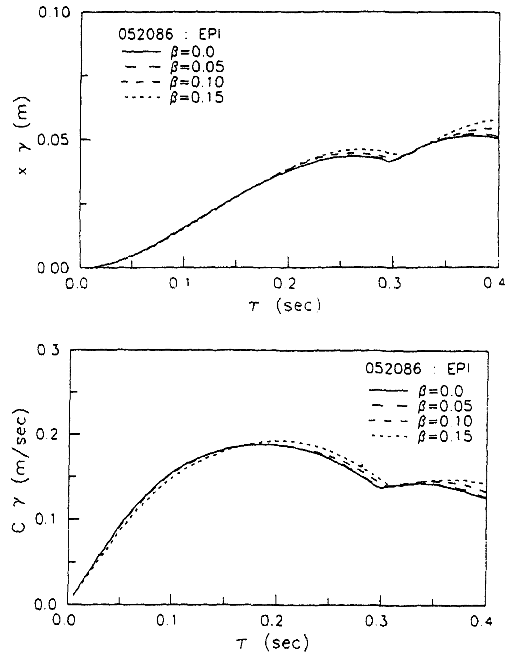


Fig. 6 Damped $x\gamma$ and $c\gamma$ spectra.

wave travelling time τ up to 0.4 seconds only. For larger values of the τ, the spectral values will become extremely large which are out of the range of engineering applications.

From the examples shown above, it can be found that both the damped $x\gamma$ and $c\gamma$ spectra for τ values less than 0.4 seconds are very similar to the elastic (zero damping) spectra generated, indicating that the soil damping is insensitive to the maximum responses of ground shear strain. For engineering applications, the characteristic wave travelling time τ is usually smaller than 0.3 seconds even for a very soft site; thus, it is satisfactory to use the elastic spectra directly instead of the damped spectra.

6 DISCUSSIONS

The $x\gamma$ and $c\gamma$ spectra proposed are the graphical representations of the normalized maximum shear strains of ground produced by earthquake motions specified at the ground surface. The spectra generated are very neat and complete because the parameter τ, representing the wave travelling time needed from depth x to the ground surface, is adopted as the abscissa. Theoretically, the parameter τ governs the vibrational characteristics of the soil layer over the depth considered. For example, the fundamental period T_1 of a soil layer underlain by rigid bedrock is related to the shear wave travelling time τ_1 of that layer by

$$T_1 = 4\,\tau_1 \tag{21}$$

Therefore, the parameter τ can be regarded, in some way, as a parameter related to the vibrational period of the layer between the depth x and the ground surface. Under this consideration, the physical meaning of the proposed $x\gamma$ and $c\gamma$ spectra is very similar to the structural response spectrum conventionally used which relates the maximum response of a single-degree-of-freedom system to its structural period of vibration as shown in Fig. 3. This explains why the $x\gamma$ and $c\gamma$ spectra generated are very general and apply for any case of which the parameter τ can be estimated, i.e., regardless of specific values of depth x and shear wave velocity c of the ground.

The model used for generating $x\gamma$ and $c\gamma$ spectra is an uniform elastic half-space. Although it is simple, it is not necessary that the resulting spectra can only be applied for half-space cases. At least, they can be applied for the following three cases without doubt:

(1) Assuming vertical propagating shear wave model, the seismic response of ground at any depth x is irrevelant to the soils below that depth once the surface motion is prescribed, i.e. pre-determined. Therefore, the $x\gamma$ and $c\gamma$ spectra can represent the maximum shear-strain at depth x no matter what the soil conditions below that depth are.

(2) For nonhomogeneous soil layers, the $x\gamma$ and $c\gamma$ spectra generated can still be applied if the equivalent shear wave velocity between the ground surface and the depth considered can be estimated. For example, many empirical methods can be used to approximately estimate the equivalent shear wave velocity for a layered soil system (Donovan 1975; Dobry et al. 1976).

(3) The soils are known to response nonlinearly due to larger strains of vibration during earth-

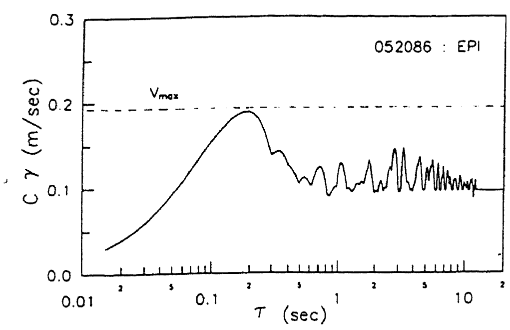


Fig. 7 Comparison of v_{max} and $c\gamma$ spectrum.

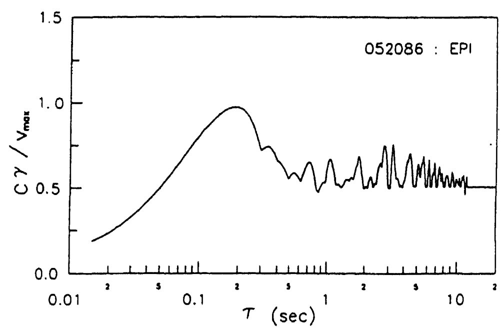


Fig. 8 Normalized $c\gamma$ spectrum.

quakes. The effect of soil nonlinearity can also be included in applying the $x\gamma$ and $c\gamma$ spectra generated when reduced shear wave velocity is used (Seed 1975).

To evaluate the conservativeness of the design value used in common practice, the result of Eq. (1) can be compared with the $c\gamma$ spectrum generated. As shown in Fig. 7, the empirical practice gives a $c\gamma$ value equal to v_{max} which is always larger than the peak value of the $c\gamma$ spectrum generated. The over-estimation by using Eq. (1) is very clear when the excavation depth of an engineering site is very limited, as in usual cases.

From the discussions made above, it can found that the $c\gamma$ spectrum is more convenient than the $x\gamma$ spectrum for engineering applications because the $c\gamma$ value is physically cross-related to the velocity time-history of given earthquakes as shown in Eq. (13). Besides, the $c\gamma$ spectrum can be scaled by v_{max}, the peak ground velocity of given earthquake, to give a normalized spectrum as shown in Fig. 8. For large values of τ, the normalized spectral value equals to 1/2. According to the procedure for generating a site-dependent or site-specific response spectrum of structure, the design response spectrum can be obtained by averaging the spectra of different earthquakes after being normalized by their peak ground accelerations respectively (Newmark and Hall 1982). Following the same philosorphy, a site-dependent or site-specific response spectrum of ground shear-strain can also be developed if sufficient earthquake records and site conditions are available.

7 CONCLUSIONS

Based on the studies presented herein, the following general conclusions can be deduced.

1. By using a simplified model, the maximum ground shear-strain produced by specified ground surface motion can be judiciously represented by either $x\gamma$ or $c\gamma$ spectrum proposed.

2. Both $x\gamma$ and $c\gamma$ spectra are expressed in terms of the characteristic wave travelling time τ, thus they can be applied very conveniently for all values of ground depth and shear wave velocity.

3. The elastic $x\gamma$ and $c\gamma$ spectra are satisfactory for engineering applications because the soil damping is shown to be insensitive to the maximum response of ground shear-strain for small values of characteristic wave travelling time τ.

4. Since the elastic $c\gamma$ spectrum can be directly calculated from the velocity time-history at the ground surface and has peak and asymptotic values closely related to the peak ground velocities, it is thought that $c\gamma$ spectrum has greater potential than $x\gamma$ spectrum to be widely used in engineering applications.

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