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Hybrid non-linear response of soft soils

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ABSTRACT: Two techniques are used for the computation of the one-dimensional seismic response of stratified media with non-linear behavior. One of them makes use of a finite-difference technique and step by step time integration. This allows to follow stress-strain paths prescribed by backbone curves and load-reload criteria. Under the assumpions imposed by these behavior models. its solution is regorous. The other technique is the linear equivalent method, which is based on the adjustment of viscoelastic dynamic properties to make them compatible with the strain level. Comparisons between both methods show the advantages of the linear equivalence to describe the most important features of the response in layered media with non-linear properties. Hybrid modeling is also considered to evaluate the non linear response of a soft soil layer overlying a 2D viscoelastic valley. Results show that lateral irregularities can modify the non-linear response in both, amplitude and frecuency content.

1 INTRODUCTION

In soil mechanics, earthquake engineering and seismology it is of interest to know the seismic response of soil deposits when the materials behave non-linearly. For most soils, the stress-strain relationships can undergo strong non-linear behavior under moderate shaking. Many authors have pointed out that dynamic behavior of soils, outside the elastic range, is extremely complicated. However, it has been observed that this phenomenon is, in practice, controlled by the variations of internal damping and stiffness modulus in terms of strain. This explains the success of the linear equivalent method (Seed & Idriss, 1969). It is very simple and efficient as it captures the essential physics of the problem. It requires the adjustment of these two relevant mechanical parameters (stiffness modulus and damping) to make them consistent with the strain level.

In this work, we implement the Seed and Idriss linear equivalent method in the framework of the Thompson-Haskell propagators for layered media (Aki & Richards, 1980). To calibrate our exercise we established a reference that we will regard as "exact". We used a finite-difference scheme in time and space in which the backbone stress-strain curves and the load-reload criteria are specified. We studied the one-dimensional response of some soil layer configurations and the results, obtained with both, the linear equivalent method and the rigorous one are presented. In most cases, a very good agreement is achieved both in acceleration time series and frequency spectra.

Finally, a hybrid approach is proposed to evaluate the non-linear response of a soft soil horizontal layers overlying a 2D viscoelastic valley. The approximation is better when the impedance contrast is large. This hybrid scheme is applied to study in a simplified way the combined effects of relatively large-scale lateral irregularities and the non-linear response of the uppermost, softer layers.

2 NON-LINEARITY OF SOILS

Shear stiffness modulus and internal damping have great variations due to non-linearity. The complexity of such variations are usually simplified in order to take them into account for numerical modeling. Typically, prescribed stress-strain paths are used to guide the non - linear behavior in terms of backbone curves and load-reload criteria, which mimic the real behavior of soils under large strain. Several backbone curve models have been proposed. All of them reflect the behavior during initial load, from elastic range up to the failure. The most important are the experimental model of Seed & Idriss (1970) and the analytical ones of Hardin-Drnevich (1972) and Ramberg-Osgood (Richart, 1975). The loading-reloading criteria account for plasticity effects during dynamic load. Stress-strain paths, which must be follow after loading inversion, are described with them. The most diffused criteria are due to Masing, Pyke and Iwan. According to Chen (1981), the last one is better to represent the non-linear behavior of the shear modulus and internal damping, since these criteria includes yielding elements which simulate the material's plasticity after certain applied stress level. This allows to reproduce significant stiffness reductions which limit maximum stress. Figure 1 shows the stress-strain paths obtained with these criteria.

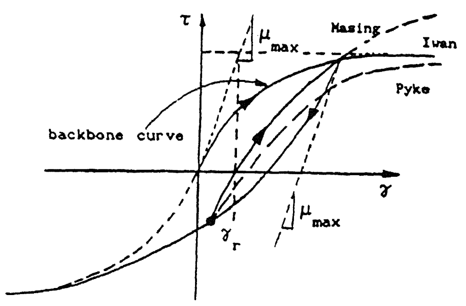

Figure 1. Reload paths by using Masing, Pyke and Iwan criterion. τ = stress, γ = strain, γ_r = reference strain, μ = stiffness modulus.

It is clear that to know the values of shear stiffness at any time, or at any strain level, is necessary to follow the stress-strain paths. To this end, we used a step-by-step time integration scheme. Note that internal damping due to plasticity is taken into account, automatically, in the equations of motion.

An alternative way is to express equivalent information in terms of effective stiffness and total damping, that is to say, the damping due to viscosity of soil and the one due to its plastic behavior. In this representation it is assumed that the values are the average for a harmonic input motion. They are obtained from the backbone curve and load-reload criteria. In these plots, the slope of the secant to the origin at any strain represents the effective stiffness as well as the area into the hysteretic cycles is proportional to the total damping. Figure 2 shows

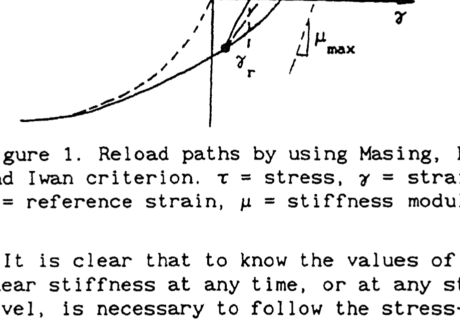

Figure 2. Stiffness-strain and damping-strain relations for Hardin-Drnevich backbone curve and Masing load-reload criterion.

shear stiffness modulus and effective damping versus strain for the Hardin-Drnevich backbone curve and Masing load-reload criterion. Note that for this case, linear range cover up to .01 in a reference strain scale (γ/γ_r). In the linear equivalent method this information is used to adjust dynamic properties to the reached strain level in an iterative procedure on viscoelastic linear models.

3 ONE-DIMENSIONAL MODEL

In most cases, the local site conditions can be well described using simple one-dimensional layered media. Because of its simplicity, this assumption allows to include effects related to dynamic soil properties, in particular, non-linearity. We use the one-dimensional shear wave model to study the non - linear effects in the soil response. Let us assume horizontal layered media with infinite lateral extension, overlying an elastic half-space. Each layer is homogeneous and isotropic. On the other hand, the excitation will be given by the vertical incidence of S waves. Two schemes of solution are applied: finite differences with step-by-step time integration and the linear equivalent method.

3.1 Finite-difference method

This method requires vertical discreteness of the medium which can be shown to be equivalent to a lumped-mass system. The size of discreteness depends on the minimum wave length of interest. The equations admit the next matrix expression

$$\mathbb{M}\,\ddot{\mathbb{X}} + \mathbb{C}\,\dot{\mathbb{X}} + \mathbb{K}\,\mathbb{X} = \mathbb{P} \qquad (1)$$

where $\mathbb{M}$, $\mathbb{C}$ and $\mathbb{K}$ are the mass, damping and stiffness matrices of the lumped-mass system, respectively. $\mathbb{X}$ is the displacement vector. $\dot{\mathbb{X}}$ and $\ddot{\mathbb{X}}$ are their first and second derivatives with respect to time. Finally, $\mathbb{P}$ is the vector which contains the input motion. This equation system can be solved by step-by-step time integration. In this work, the Newmark's β-method (Newmark, 1959) is used. It consists in the evaluation of the the velocity and displacement, at the time i+1, in terms of these values at time i and accelerations at the times i and i+1. So, it is possible to express equation 1 at the time i+1

$$\mathbb{M}\ddot{\mathbb{X}}_{i+1} + \mathbb{C}\dot{\mathbb{X}}_{i+1} + \mathbb{K}\mathbb{X}_{i+1} = \mathbb{P}_{i+1} \qquad (2)$$

and it can be written as

$$\mathbb{M}^{*}_{i}\,\mathbb{X}_{i+1} = \mathbb{P}^{*}_{i,i+1} \qquad (3)$$

where $\mathbb{M}^{*}_{i}$ is the effective mass matrix, which contents the constant values of mass and damping and the values of non-linear stiffness at the time i. It is assumed as a constant between the times i and i+1, and it's found by using stress-strain prescribed paths. $\mathbb{P}_{i,i+1}$ is the independent vector which contents all forces related with the motion at the time i and the input motion at the time i+1. Solution yields accelerations at the time i+1 and at each lumped-mass.

3.2 Linear equivalent method

The 1D response of a viscoelastic layered medium is computed by means of Thompson-Haskell method. An iterative procedure according to the linear equivalence is used. The method is based on a matrix formulation which permits to know the displacements and stresses at any depth in terms of a propagation matrix, which contains dynamic properties of the medium, and input motion spectrum, as it is shown in the following expressions

$$v\left(0\right) = B_{11}^{-1}\,\acute{S}(\omega) \tag{4}$$

$$v\left(z_i\right) = R_{11}\,v\left(0\right) \tag{5}$$

$$\tau_{yz}\left(z_i\right) = R_{21}\,v\left(0\right) \tag{6}$$

where $v(0)$ is the superficial displacement, $v(z_i)$ and $\tau_{yz}(z_i)$ are the displacement and stress at the depth z_i, respectively, $\acute{S}(\omega)$ is the Fourier spectrum of the input displacement, R_{11} and R_{21} are components of the effective propagation matrix, from the top to the depth z_i, and B_{11} is a component of the matrix which involves the products of propagator matrices from top to bottom of the stratigraphy and terms related to the radiation towards the half-space. For viscoelastic solids hysteretic damping in the stress-strain relation is assumed:

$$\gamma_{yz} = \frac{1}{\mu\left(1+2iD\right)}\,\tau_{yz} \tag{7}$$

where D represents an internal damping factor and μ is the stiffness modulus. For small values, equation 7 yields elliptic trajectories in the stress-strain plane which remind non-linear typical pat- terns. Figure 3 suggests that it's possible to find stiffness moduli and internal damping factors of an equivalent viscoelastic solid in order to approximately simulate non-linear behavior.

From equations 4-7 it is possible to write

$$\gamma_{yz_i}(\omega) = \frac{1}{\mu_i\left(1+2iD_i\right)}\,R_{21}(z_i,0,\omega)\;B_{11}^{-1}(\omega)\;\acute{S}(\omega) \quad \ldots(8)$$

which allows to compute the strain Fourier spectrum at any depth in terms of the material properties and input spectrum. From the corresponding root-mean-square value of the strain spectra at each control depth, which can be computed by using random vibration theory (Cartwrite & Longuett-Higgins, 1956), non-linear equivalence is used to adjust stiffness modulus and damping factor by means of relations such as the ones showed in figure 2.

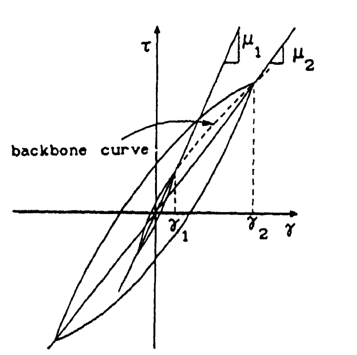

Figure 3. Stress-strain paths for two different viscoelastic solids.

4 NUMERICAL TESTING OF 1D RESULTS

In order to compare results from both techniques, the seismic response of a single soft layer was studied. Shear wave velocity and failure strength are 72 m/s and 10^5 dyn/cm^2, respectively. For the elastic half-space, shear wave velocity is 200 m/s. Density contrast is neglected. Thickness is assumed as h = 18, 36 and 54 m. Therefore, respective site periods are T_H = 1, 2 and 3 s. Input motion corresponds to an earthquake recorded at hill zone in Mexico City (M_s=6.9 with peak acceleration a_{max} = 15 cm/s^2). Its amplitude was scaled to study different strain levels by the factors F = 0.25, 0.5, 1.0, 2.0, 4.0, 8.0. Figure 4 shows acceleration time histories at the surface of the layer with period equal to 1 s for scale factors F = 1.0, 2.0, 4.0 and 8.0 . Figures 5 and 6 show the same results for layers with periods T_H = 2 and 3, respectively. Note that frequency content varies and relative reductions of amplitude increases when the scale factor grows. Figure 7 shows spectral ratios and transfer functions, obtained from finite-difference method and from the linear equivalent one, respectively, for studied site periods and scale input factors. Note that both methods reduce the response in almost the same amount. Besides, they predict a shifting of the dominant frequency to lower values. The differences in these plots appear for frequencies greater than the fundamental one. Fortunately, for most practical purposes, seismic response is strongly controlled by the dominant frequency where both methods coincide.

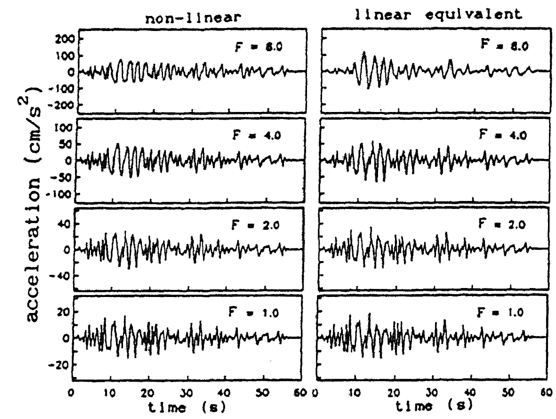

Figure 4. Computed accelerations at the surface of 1 s period site for several input factors. Both discussed method were applied.

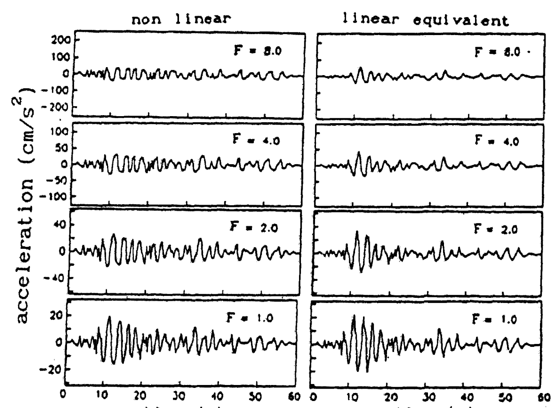

Figure 5. Computed accelerations at the surface of 2 s period site for several input factors. Both discussed method were applied.

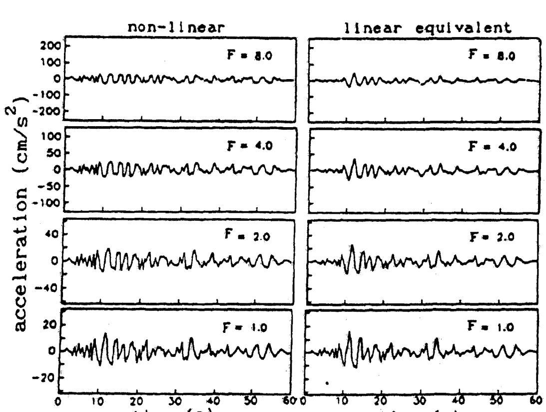

Figure 6. Computed accelerations at the surface of 3 s period site for several input factors. Both discussed method were applied.

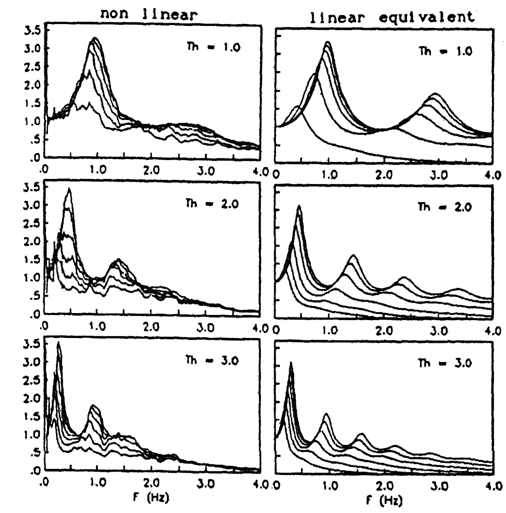

Figure 7. Spectral ratios and transfer functions for the studied sites and for input factors F = 1/4, 1/2, 1, 2, 4 and 8.

5 HYBRID NON-LINEAR RESPONSE

In order to study the influence of large scale lateral irregularities on the non-linear seismic response of soft soils, we use a hybrid model which consist in a 2D viscoelastic valley underlaying a non-linear stratified medium. This kind of modeling was presented in a previous work (Sánchez-Sesma et al, 1988). The purpose was to study the 2D effects in the linear seismic response of soft soils. They used a 2D model which is based on the grouping of rays on bands where one-dimensional wave propagation occurs, and solution is obtained by superposing folded bands and neglecting diffraction. Authors found that this can be done for a class of symmetric triangular valleys which dip angle is of the form $\theta = \pi/2N$, $N = 1,3,5...$ Its simplicity allows to consider approximately the effects of deformable basin boundaries, arbitrary angles of incident SH waves and realistic waveforms with very low computational effort.

As in that work, we use the output motion at any site on the surface of the valley as input to the non-linear layered medium. The main assumption is to neglect the interaction between the uppermost non-linear stratigraphy and the viscoelastic alluvial valley. This is a reasonable approximation if the impedance contrast between both media is sufficiently high. Since under this conditions, propagation wave through soft layers is nearly vertical. On the other hand, radiation to the higher stiffness medium is very low. In our work, linear equivalent method is applied to account for non-linearity of the superficial layers.

6 NUMERICAL RESULTS

Numerical example was performed in order to compare different 2D effects on the non-linear seismic response of superficial soft soils. The model is depicted in figure 8 and it consists of a viscoelastic triangular alluvial valley embedded in an elastic half-space. Shear wave velocities, mass densities and quality factors are indicated in the figure for both, valley and half-space. Uppermost structure is the same along the surface of the valley and it is composed by four layers with the following properties given from the top to the bottom:

thickness (m)	shear wave velocity (m/s)	mass density (gr/cm^3)	damping factor	failure strength (dyn/cm^2)
7	100	1.5	0.05	linear
15	40	1.2	0.05	1.75×10^5
18	70	1.2	0.05	5.30×10^5
18	180	1.5	0.05	linear

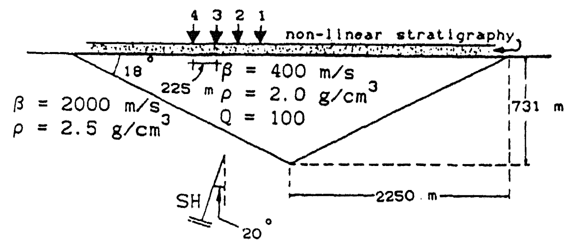

Figure 8. Viscoelastic valley underlying a non-linear stratigraphy. Excitation is given by the oblique incidence of SH waves. Results are computed at sites 1, 2, 3 and 4.

For 20 degrees incidence angle, we scaled a record of the 1985 Michoacan earthquake at hill zone site in Mexico city (Ms = 8.1, peak acceleration a_{max} = 28 cm/s^2) by several factors in order to reach different non-linear strain levels in the superficial response. Figures 9-11 show acceleration time series computed at the sites indicated in figure 8 for scale factors F = 1/2, 1 and 2, respectively. As it is expected, non-linear features are clearer in the response as the scale factor increases. However, it is also clear that frequency content has strong dependency on the space location. It is due to the 2D effects such as focusing and trapping of energy, since at any site in the surface uppermost layers are the same. For comparison, figures 12 and 13 show transfer functions at sites 1 and 4, respectively. Linear case and the ones related to the scale factors F = 1/8, 1/4, 1/2, 1, 2, 4 are displayed. In general, linear transfer function has the largest amplitudes, as in the 1D model alone. However, we found that for sites close to the center of the valley, like station 1, the fundanental frequency can be efficiently excited by non-linearity. This allows to say that largescale lateral irregularities can significantly affect the surface non- linear response.

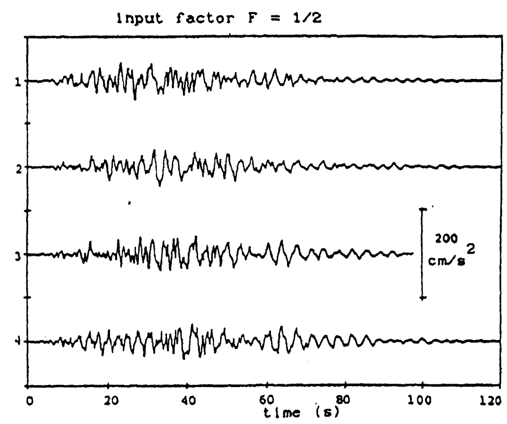

Figure 9. Computed accelerations at sites 1, 2, 3 and 4 for input factor F = 1/2.

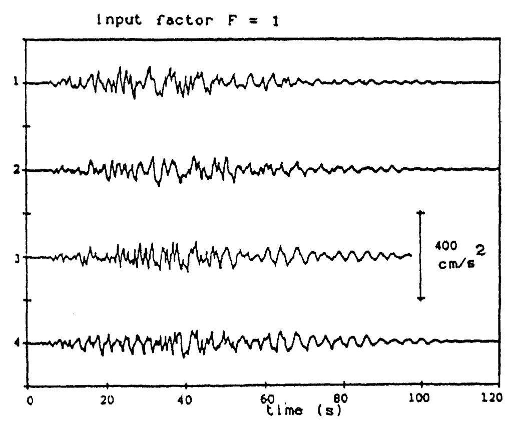

Figure 10. Computed accelerations at sites 1 2, 3 and 4 for input factor F = 1.

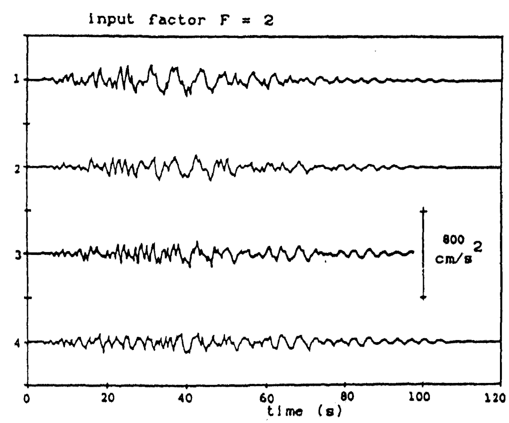

Figure 11. Computed accelerations at sites 1 2, 3 and 4 for input factor F = 2.

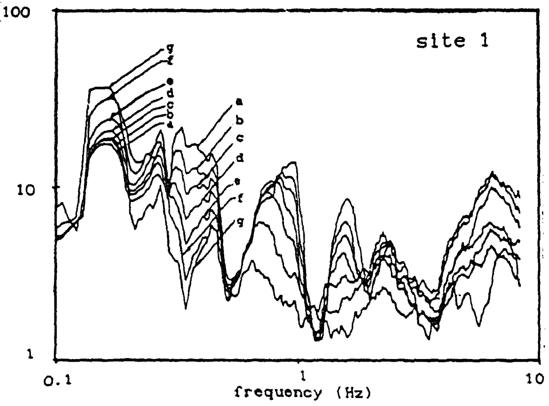

Figure 12. Transfer functions at site 1 for the linear case (a), and for input factor F = 1/8 (b), 1/4 (c), 1/2 (d), 1 (e), 2 (f) and 4 (g).

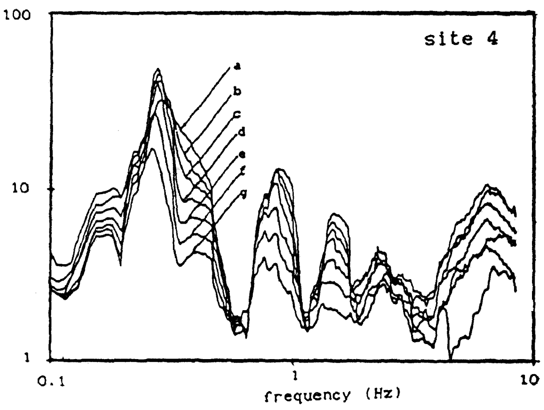

Figure 13. Transfer functions at site 4 for the linear case (a), and for input factor F = 1/8 (b), 1/4 (c), 1/2 (d), 1 (e), 2 (f) and 4 (g).

7 CONCLUSIONS

One-dimensional non-linear seismic response has been investigated with finite differences in space and time domains and with the linear equivalent method. The first one is assumed as rigorous, since it allows to follow prescribed non-linear stress-strain trajectories. The second approach is based on the adjustment of dynamic properties to represent non-linear behavior of an equivalent viscoelastic solid. Both methods confirm that principal features of non-linear seismic response are: a) relative reduction of response amplitudes with respect to the linear case and b) shift of the spectral ordinates to lower frequencies, that is to say, increase of the fundamental site period. The results also show that the linear equivalent method provides very good results with a relatively minor computational cost. Certainly, this is well known.

We presented a hybrid scheme to approximately consider the combination of large-scale 2D site effects with localized non-linear response of the uppermost softer layers. To illustrate this we used a simple triangular valley which allows to obtain results with reduced computer resources. From synthetic acceleration time series of the studied model we pointed out non-linear features in the response in terms of a scaling input factor. We found that frequency content is also strongly dependent on the space location. In any event, both aspects of the problem, non-linearity and 2D effects are mixed. By using linear equivalent method in a full wave formulation, the hybrid approach can be followed in a more rigorous way.

8 ACKNOWLEDGEMENTS

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