

## An empirical method for identifying non-linear soil amplification of strong earthquake motions

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**Abstract:** Strong earthquake ground motions are influenced remarkably by local soil conditions. Among various factors of local soil conditions contributing to motions, the effect owing mainly to the non-linear stress-strain relationship of surface soils is studied in this paper with the help of statistical analyses. A statistical model is derived so as to obtain soil amplification due to the material non-linearity of surface soils using proportionality of soil strain to particle velocity. The model is applied to 228 strong-motion records observed at 26 sites in Japan, providing site-specific soil amplification dependent on the motion level which is called the "non-linear soil amplification" in this paper. The non-linear soil amplifications identified by the statistical analysis show quite period-dependent characteristics as well as site-dependent. The comparison between the non-linear soil amplification and their corresponding soil profiles reveals that the non-linearity of local site response is well correlated with the softness and formation of soils, particularly, with the N-value distribution at each observation site.

### 1 INTRODUCTION

As a result of many efforts made in the past years, it is nowadays clear that strong ground motions result principally from the three factors: seismic source, propagation path and local site conditions. Of the three factors, the third one is the most important in the short-period range which engineers are concerned with. There may be various sub-factors related to the third effect: rigidity, density and viscosity of surface soils, topographical irregularities on and under the ground surface, non-linear relations between strain and stress of soil material and so on. Even though these sub-factors bring about local ground motions peculiar to each site as some united effects, the material non-linearity would play an important role, especially, in soft soil layers which are found mostly in large cities in Japan. The purpose of this paper is to identify amplification factors due to the material non-linearity of local soil layers by using a statistical analysis of strong-motion spectra. The non-linear amplification factors identified statistically are also discussed in connection with the soil profiles at each observation site.

### 2 REGRESSION MODEL OF STRONG-MOTION SPECTRA TO IDENTIFY NON-LINEAR AMPLIFICATION DUE TO SURFACE SOIL LAYERS

The author presented a method for obtaining separately the effects due to the three factors from the strong motion spectra observed on the ground surface of many sites (Kamiyama and Yanagisawa, 1986, Kamiyama 1987). The method was based on a regression analysis of spectra in which amplification factors due to local site conditions at each observation

site were made clear with the aid of the concept of the dummy variables. The regression model was given by

$$\log_{10} V(T) = a(T)M^2 + b(T)M + c(T) \log_{10}(\Delta + 30) + d(T)D + e(T) + \sum_{i=1}^{N-1} A_i(T)S_i \quad \text{--- (1)}$$

where  $V(T)$ : spectra of ground motions,  $M$ : earthquake magnitude,  $\Delta$ : epicentral distance (km),  $D$ : focal depth (km),  $N$ : total number of observation sites,  $S_i$ : dummy variables,  $a(T), b(T), c(T), d(T), e(T)$  and  $A_i(T)$ : regression coefficients, and  $T$ : period.

According to Eq.(1), the amplification factor  $AMP_i(T)$  specific to the  $i$ -th site against a reference site is determined as

$$AMP_i(T) = 2 \cdot 10^{A_i(T)} \quad \text{--- (2)}$$

Even though it is quite useful to obtain simply the effects due to seismic source, propagation path and local site conditions, Eq.(1) has such an insufficient point that amplification factors are determined as a peculiar parameter to each observation site in disregard of their corresponding motion level, as definitely shown in Eq.(2). In other words, Eq.(1) implicitly assumes that there exists no difference in the amplification factors between strong motions and weak ones.

Theoretically speaking, as far as the soil materials have linear stress-strain relationship, their amplification factors are determined uniquely as a frequency response function irrespective of motion levels. So we might be able to derive the soil amplification as a parameter unique to a site from the

observed strong motion spectra on the condition that the soils show little material non-linearity. In case of some conspicuous non-linearity of soil materials, however, we can no longer expect amplification factors peculiar to only site conditions and it would be natural that amplification factors undergo a change in response to earthquake motion levels. They may depend markedly on magnitude of time-varying strain or something induced in soils and have some non-stationary characteristics. Though one needs a method faithful to such complicated non-linearity of soil material, it is almost impossible to extract soil amplification applicable to non-stationary behavior of soil material merely from some limited strong-motion spectra observed at the ground surface. Accordingly, a simplified definition of soil amplification resulting from the material non-linearity of soils is attempted in this paper so as to derive easily from only the strong-motion spectra at the ground surface. That is, in reference to the "equivalent linear approximation" used in earthquake response analysis (Erdik, 1987), we herein derive the non-linear soil amplification whose factors depend on the motion level, by extending Eq.(1).

As known from various soil dynamic tests in laboratories and fields, soil strain induced during earthquake is the most important parameter controlling the non-linear behavior of ground motions. In spite of such importance, there has been almost no observation of soil strain because of the technical difficulty, and it is thus impossible to introduce such parameter into the regression model of Eq.(1). On the other hand, it is shown from the wave theory having one dimensional propagation that strain is proportional to its corresponding particle velocity (Kolsky, 1963). Referring to the theory, we assume in this study that the variations of non-linear soil amplification are controlled mainly by the maximum particle velocity at the ground surface which is relatively easily obtained by numerical calculation from the observed surface accelerograms or by direct observation, and introduce it into Eq.(1) as a replacement of the strain. When introducing the maximum particle velocity at the ground surface into Eq.(1), it is needed to treat it as a site-dependent variable because it varies remarkably depending on local site conditions. In this point, Eq.(1) already gives us a clue for such treatment, that is, the concept of the dummy variable is also applicable to the site-dependency of maximum particle velocity. This idea leads to building the following regression model by adding new terms, which include the dummy variables as well as the maximum particle velocity at the ground surface, into Eq.(1) (Kamiyama 1992):

$$\log_{10} v(T) = a(T)M^2 + b(TM) + c(T)\log_{10}(\Delta + 30) + d(T)D + e(T) + \sum_{i=1}^{N-1} A_i(T)S_i + \sum_{j=1}^N B_j(T)VS_j \quad (3)$$

Where  $v$ : maximum particle velocity at the ground surface (cm/sec),  $S_j$ : dummy variables, and  $B_j(T)$ : regression coefficient.

The maximum velocity is included in Eq.(3) as a kind of explanatory variable, but it is not only site-dependent but also depends on earthquake factors such

as magnitude  $M$ , epicentral distance  $\Delta$  and focal depth  $D$ . Therefore, a similar regression expression to Eq.(1) is satisfied about the maximum velocity  $v$ , namely,

$$\log_{10} v = a'M^2 + b'M + c'\log_{10}(\Delta + 30) + d'D + e' + \sum_{i=1}^{N-1} A_i' S_i \quad (4)$$

In Eq.(4),  $10^{A_i'}$  means the amplification factors of the maximum velocity at the  $i$ -th site against a reference site. In other words, when the maximum particle velocity at a reference site during an earthquake is given by  $v$ , the maximum particle velocity  $v_i$  at the  $i$ -th site for the same earthquake condition is expected statistically to become

$$v_i = 10^{A_i'} \cdot v \quad (5)$$

Using Eqs.(3) to (5), we eventually obtain the amplification factor of spectra at the  $i$ -th site  $AMP_i(T)$ , as follows, which is dependent on site as well as on the maximum velocity at a reference site.

$$AMP_i(T) = 2 \cdot 10^{A_i(T) + B_i(T) \log_{10} v} / 10^{B_B(T) \log_{10} v} \quad (6)$$

where  $B_B(T)$  corresponds to the  $B_i(T)$  for a reference site.

Although the reference site stated above can be picked up arbitrarily from any of the observation sites, the most desirable is to select a site where there exists outcrop hard enough to constitute the seismic bed rock for the other sites. Under such selection of a reference site, the variable  $v$  in Eq.(6) is interpreted equivalently as the maximum particle velocity on the seismic bed rock which is virtually underlain at each observation site as well as being associated with the outcrop rock of the reference site. Thus Eq.(6) becomes equivalent physically to the amplification factors varying with the maximum particle velocity given on the virtual underlying seismic bed rock of each site, namely, it finally follows that we can obtain our target "non-linear soil amplification" through Eq.(6).

### 3 REGRESSION ANALYSIS OF STRONG-SPECTRA OBSERVED IN JAPAN

The regression models of Eqs.(3) and (4) were applied to strong-motion accelerograms observed in Japan. The accelerogram data are the same as the ones in Kamiyama and Yanagisawa (1986) which consist of 228 horizontal accelerograms observed at 26 sites. The earthquake of the data are shown in Fig.1 together with the observation sites. We computed velocity response spectra with no damping from the accelerograms and used them as  $V(T)$  in Eq.(3). The reason for using such velocity response spectra was already explained in Kamiyama (1987). Also, the maximum velocity  $v$  in Eqs.(3) and (4) was estimated by obtaining numerical velocity records from the acceleration records according to Iai et al.(1978). In

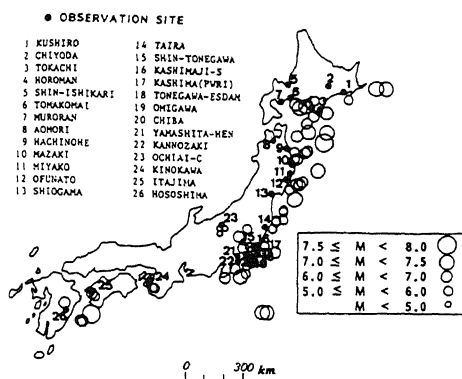


Fig.1 Earthquake origins and observation sites of the strong-motion records used in this study. M means earthquake magnitude. The records were observed under the conditions of earthquake magnitude ranging from 4.1 to 7.9, focal depth from 0 to 130 km and epicentral distance from 10 to 310 km.

Table 1 Results of the multiple correlation coefficients and standard deviations obtained by the analyses of Eq.(1) and Eq.(3). The comparisons are made only for representative periods.

Period (sec)	MODEL EQ.1		MODEL EQ.3	
	Standard Deviation	Multiple Correlation Coefficient	Standard Deviation	Multiple Correlation Coefficient
0.10	0.242	0.794	0.231	0.841
0.20	0.245	0.797	0.219	0.863
0.30	0.243	0.747	0.208	0.848
0.50	0.245	0.825	0.197	0.906
0.70	0.255	0.847	0.196	0.925
1.00	0.278	0.849	0.227	0.915
1.50	0.261	0.848	0.195	0.930
2.00	0.234	0.863	0.169	0.941
3.00	0.207	0.872	0.128	0.959
4.00	0.199	0.860	0.125	0.955

addition to these data, the reference site described in the preceding section was assigned to the Ofunato site labelled 12 in Fig.1. The outcrop of the Ofunato site is composed of a hard slate whose S wave velocity is supposed to be 1~2 km/sec. Accordingly, the "bed rock" implied in this study means a rock with such a rigidity.

We analyzed Eqs.(3) and (4) by using the above conditions and the least square technique. First the results due to Eq.(3) are compared with the ones of Eq.(1) in order to check their reliabilities of analyses. Table 1 shows the multiple correlation coefficients and standard deviation for representative periods obtained from both analyses of Eqs.(1) and (3). We can see from the Table that Eq.(3) gives far more reasonable results than Eq.(1) because of providing additional variables. On the other hand, the regression coefficients  $a(T)$ ,  $b(T)$ ,  $c(T)$ ,  $d(T)$  and  $e(T)$  in Eq.(3) were obtained as values depending on period in a similar way to the corresponding coefficients in Eq.(1). In this paper, we are mainly interested in the amplification factors due to each local soil condition,

so the results of these regression coefficients are not shown here.

We can estimate amplification factors of each site by the use of the coefficients  $A_i(T)$ ,  $B_i(T)$ ,  $A_i^*(T)$  and  $B_i^*(T)$  obtained from the regression analysis and the maximum velocity  $v$  given arbitrarily on the virtual seismic bed rock, according to Eq.(6). On account of space consideration, we here show the amplification factors estimated only for the five representative sites of Shiogama, Miyako, Tomakomai, Yamashita-hen, and Hososhima. The amplification factors for these sites are drawn in Figs.2 to 6 which are expressed so as to be responsive to the variation of the maximum velocity of the bed rock motions. In these figures, the maximum velocity is varied in an identical manner. The maximum velocities of 1, 4, 8, 12 cm/sec correspond, respectively, to the regressed values for  $M=5.0, 6.5, 7.2$  and  $7.5$  with  $\Delta=80$  km and  $D=0$  km according to Eq.(4). We can see from these figures that the amplification factors statistically obtained here are not only quite specific to each site but remarkably dependent on period as well. It is also shown in these figures that the variations of the amplification factors with the maximum velocity are greatly characteristic to each site. Meanwhile, Fig.7 is a comparison of amplification factors determined from Eq.(1) and Eq.(3) for the Shiogama site. We may conclude from the comparison in Fig.7 that Eq.(1) which ignores the effects due to motion levels presents a kind of averaged amplification factors against Eq.(3).

#### 4 CORRELATION BETWEEN THE NON-LINEAR AMPLIFICATION FACTORS AND THE LOCAL SOIL CONDITIONS

The soil profiles of each site in Figs. 2 to 6 are shown, respectively, in Figs.8 to 12 in order to investigate how the non-linear amplification factors correlate to their local soil conditions. Referring to the standard penetration tests, that is, N values shown in these figures, one can see that each site is rich in variety of softness of soils and has various soil formations. For example the Shiogama site consists of extremely soft silty soils underlain by a hard rock, whereas rather stiff gravel layers overlying on a rock constitute the Miyako site. Both of the Shiogama and Miyako sites have rock layers at a comparatively shallow depth, particularly, the rock layer for the latter site is found at the shallowest depth among all the sites in Figs.8 to 12. The Hososhima site also has several layers changing alternately from soft to hard overlying on a hard rock. Contrary to these sites, the other sites of Tomakomai and Yamashita-hen have no rock within the depths of boring log, and there seems to be a rock at a greater depth. Concerning the average softness of shallow layers less than a depth of about 15 m, the Tomakomai site appears to have harder layers in comparison with Yamashita-hen. The characteristic point to the Tomakomai site is that it is formed with a clearer low-rigidity layer of sandy loam overlain by rather firm sandy gravel.

A close look at the non-linear soil amplifications in Figs.2 through 6 and at the soil profiles in Figs.8 through 12 reveals some interesting correlations between them. It may be summarized for each site as follows.

Shiogama site:

It is clear in Fig.2 that the predominant periods lengthen and at the same time the amplification, especially for short-periods, decreases as the maximum velocity increases. These phenomena may be closely connected with the extremely soft silty layer on the hard rock. That is, they can be clearly explained by the material non-linearity of the soft soil in which the strain-dependent rigidity and damping, respectively, decreases and increases with increasing strain, i.e., with increasing maximum velocity. From this example, one can emphasize that monotonous linear response is not expected but highly non-linear behavior becomes apparent during strong motions at a site of soft surface layers such as Shiogama.

Miyako:

This site has a clear predominant period in relatively short-period region in comparison with the other sites. This can be explained by the presence of the shallowest hard rock layer among all the shown sites. Different from the Shiogama site, this site has a tendency to demonstrate rather larger amplification around the predominant period with the increase of maximum velocity, showing little lengthening of the predominant period. This may be attributable to the stiff gravel layer near the ground surface whose non-linear hysteretic stress-strain relationship differs considerably from that of the soft soils in the Shiogama site.

Tomakomai site:

At this site, the variation of the amplifications are slight in the periods shorter than about 1.0 sec, and on the contrary the amplification beyond the period become greater with increasing maximum velocity. For instance, a clear predominant period appears at around 2.5 sec in the amplification at a large maximum velocity level of 12 cm/sec. This may be associated with the existence of the underlying low-rigidity layer of loam as well as with a deep rock deposit which is not recognized in Fig.10. Perhaps the "vibration impedance" between the low-rigidity loam and the supposed rock becomes remarkable in compliance with decreasing rigidity of the loam, and as a result the long predominant period becomes apparent.

Yamashita-hen:

This site shows a similar variation of amplifications to the Tomakomai site except for the remarkable decrease of amplifications in short-periods. The short-period behaviors may be connected with the rather soft silty layer deposited near the surface. Namely, the sandy silt layers whose  $N$  values are distributed ranging less than 10 may have a hysteretic characteristics similar to Shiogama's soft silty soils.

Hososhima:

The amplifications of this site vary in a similar way to the Shiogama site. That is, the predominant periods increase and the amplifications in short-periods become smaller in proportion to maximum velocity. This site is composed of several layers which change alternately their softness, but their averaged  $N$  value is rather small and this site is regarded, as a whole, as one soft layer on a hard rock deposit. This may be the reason that the Hososhima site is similar to the Shiogama site in the variation of amplification.

As a result of the above comparisons, it would be concluded that the non-linear soil amplifications of each site correlate intimately with their soil formations

and the dynamic material non-linearity has a close relation with the rigidity parameter of the soil material like  $N$  value.

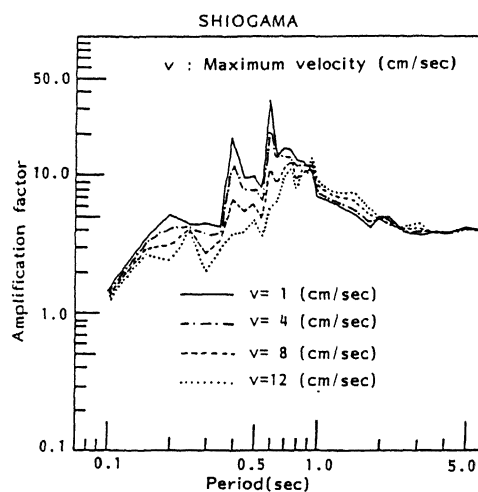


Fig.2 Non-linear soil amplification estimated at the Shiogama site according to Eq.(6).  $v$  means the maximum particle velocity which can be given arbitrarily on the virtual seismic bed rock. The values of 1, 4, 8 and 12 cm/sec correspond, respectively, to the maximum particle velocities expected for  $M=5.0, 6.5, 7.2$  and  $7.5$  with  $\Delta=80$  km and  $D=0$  km according to Eq.(4).

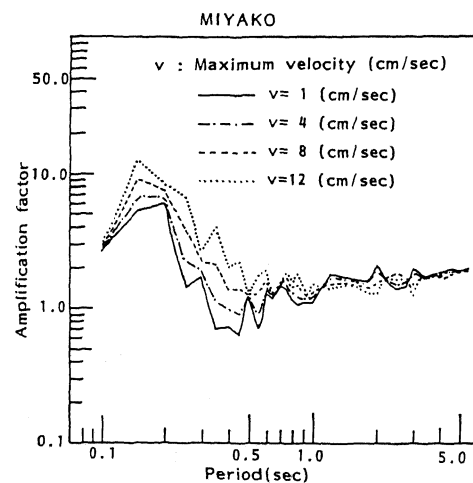


Fig.3 Non-linear soil amplification estimated at the Miyako site according to Eq.(6). The legends are the same as in Fig.2.

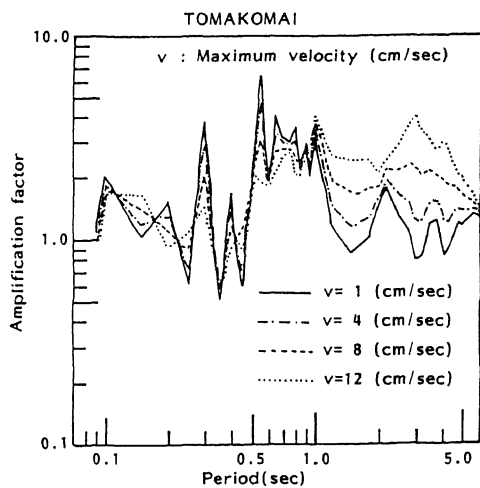


Fig.4 Non-linear soil amplification estimated at the Tomakomai site according to Eq.(6). The legends are the same as in Fig.2.

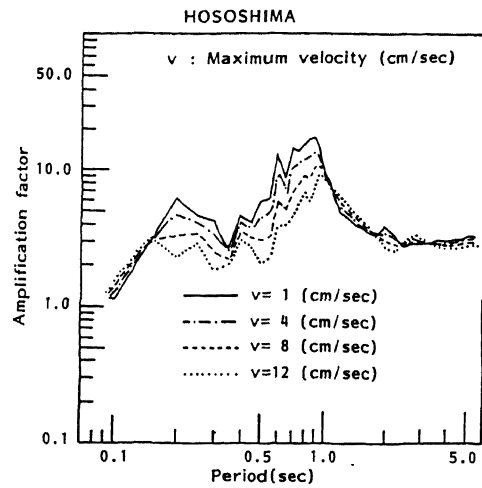


Fig.6 Non-linear soil amplification estimated at the Hososhima site according to Eq.(6). The legends are the same as in Fig.2.

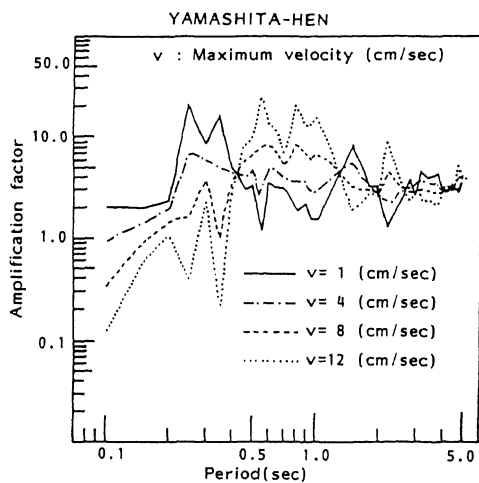


Fig.5 Non-linear soil amplification estimated at the Yamashita-hen site according to Eq.(6). The legends are the same as in Fig.2.

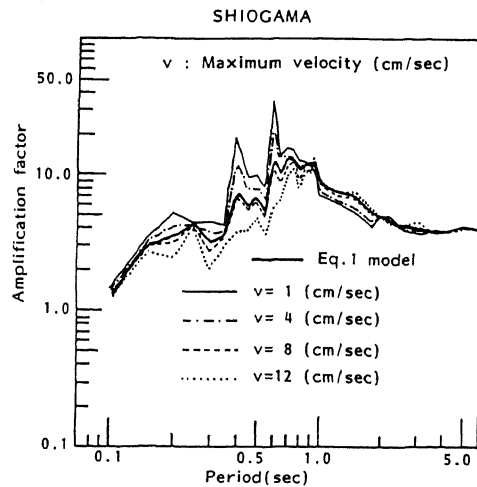


Fig.7 Comparison of the non-linear soil amplification factors obtained by Eq.(3) against the ones by Eq.(1). The thick solid line means the soil amplification by Eq.(1). The soil amplification due to Eq.(1) is given as a parameter peculiar to site while the ones by Eq.(3) vary depending on the maximum particle velocity. The comparison is shown for the Shiogama site.

## 5 CONCLUDING REMARKS

The regression model of Eq.(3) is useful to identify non-linear amplifications of strong-motion spectra caused by surface layers. The non-linear soil amplification obtained empirically here are quite specific to each observation site, in addition to being dependent remarkably on period. They are also greatly sensitive to the type of soil formations as well as the softness of soils. In particular, the non-linear characteristics of amplification factors are correlated

well with N value distribution of soil profile. At a site where there exist of soft soils of 10 m or more in thickness near the ground surface whose N values are distributed with a value less than 10 or 20, as one example, it is expected that the predominant periods of spectra increase and amplification factors in short-periods decrease together with increasing motion level. In contrast to such a soft soil layer site, a site having surface layers, especially, sandy gravel layers of 5 m

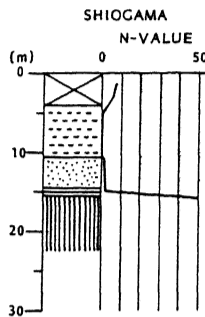
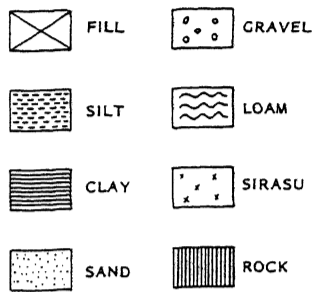


Fig.8 Soil profile (Shiogama site)

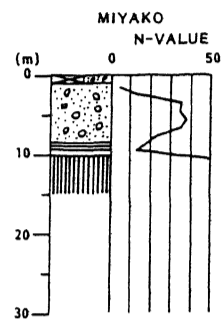


Fig.9 Soil profile (Miyako site)

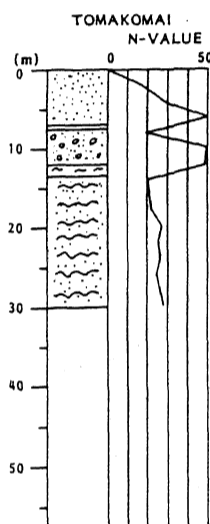


Fig.10 Soil profile (Tomakomai site)

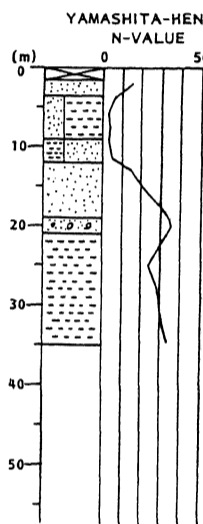


Fig.11 Soil profile (Yamashita-hen site)

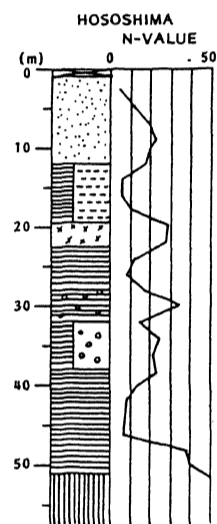


Fig.12 Soil profile (Hososhima site)

or thicker with a N value greater than about 30 tends not to show clearly detectable non-linearity of soil amplification, and therefore, a response analysis based on the linear theory is applicable to this kind of site.

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