Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Radiation damping during in-plane building-soil interaction

Maria I. Todorovska
University of Southern California, Los Angeles, Calif., USA

ABSTRACT: In-plane soil-structure interaction has been investigated for buildings on embedded foundations. The system damping, the system frequency, the relative building response and the base rocking have been studied as those depend on the building mass and height, the flexibility of the soil, the embedment depth, the structural damping, the type of incident waves and their angle of incidence. A two-dimensional analytical model has been used to calculate the frequency depenedant compliances of the soil and the foundation driving forces. The coupling of the vertical motions with the horizontal motions and the rotation has been ignored. Both the kinematic and the dynamic interaction effects have been included. Excitations consisting of plane P- and SV-waves have been considered. It has been concluded that the soil-structure interaction under sertain conditions can be an efficient mechanism for dissipation of the energy in buildings excited by earthquake motions.

INTRODUCTION

The soil-structure interaction is a natural mechanism for dissipation of the energy in buildings excited by strong earthquake shaking. The scattering of the incident waves from the building foundation, and the radiation of the energy of the vibrating building into the soil introduce additional damping into the system response, referred to as radiation damping. The first mode is most affected by the interaction. The first system frequency is always lower than the first fixed-base frequency, and the reduction of the corresponding peak in the transfer-function between the relative building response and the incident wave motion is the most significant. It is of practical importance to understand well the soil-structure interaction phenomenon, so that advantage could be taken of its beneficial effects for natural control of response of structures. It is the purpose of this paper to show on a simple two-dimensional analytical model how the system response depends on the relative stiffness of the building compared with the soil, on the building properties, on the foundation depth, and on the type of incident waves and angle of incidence.

The system damping during dynamic building-soil interaction has been studied for three-dimensional (3D) models (Bielak 1971, 1975, 1976; Tsai 1974; Rainer 1975; Luco 1980), and for 2D models (Todorovska and Trifunac 1992a,b) excited by in-plane incident waves. Analogy with a single or a multi degree-of-freedom (SDOF and MDOF) fixed-base oscillator has been used to measure (Tsai 1974; Rainer 1975; Todorovska and Trifunac 1992a,b) or analytically determine (Bielak 1971, 1975, 1976; Luco 1980) the system damping ratios and the system frequencies in those studies. In Todorovska and Trifunac (1992a,b), the effects of the kinematic interaction (wave passage effects) on the system damping ratio have also been included. The effects of the depth of the embedment on the system response in 2D for in-plane excitation have been studied by Todorovska and Trifunac (1992b) for circular rigid foundations, and in 3D by Bielak (1975) for rectangular prismatic foundations.

THE MODEL

The two-dimensional building model (Fig. 1) is an equivalent single degree-of-freedom (SDOF) oscillator, consisting of a concentrated mass connected by a rod to the foundation (at point O) through a "rotational" spring with stiffness K_b and a rotational dashpot with damping C_b. The oscillator has mass per unit length (in the y-direction) m_b, and radius of gyration r_b. The height of the oscillator is H. The foundation is rigid, and has circular shape, with width $2a$ and depth h. The mass per unit length (in the y-direction) of the foundation is m_f. The foundation is embedded into a homogeneous elastic half-space with shear-wave velocity β, shear modulus μ and Poisson's ratio ν. The damping in the soil is neglected. The foundation motion is defined with respect to the inertial coordinate system $x-O-z$ (Fig. 1). The horizontal translation Δ is in the positive x-direction, the vertical translation V is in the positive z-direction, and the rotation φ is clockwise. The deformation of the oscillator is described by the rocking angle ψ^{rel}, measured clockwise from the axis ξ, which is always perpendicular to the foundation.

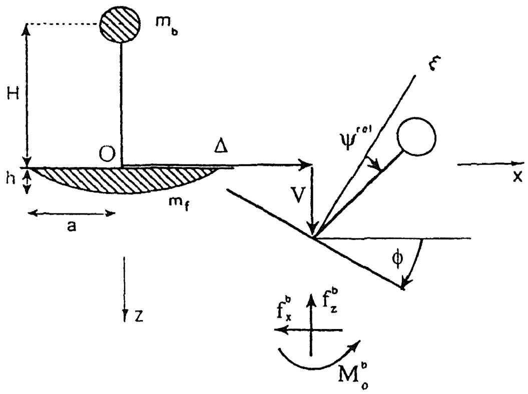

Fig. 1. The model

The linear analysis neglects the coupling between the horizontal and the vertical motions of the building and of the foundation. Then, for a harmonic ground acceleration with time dependence $e^{-i\omega t}$, the system response will be

$$\begin{aligned} \psi^{\text{rel}} &= \psi_0^{\text{rel}} e^{-i\omega t} \\ \boldsymbol{\Delta} &= \boldsymbol{\Delta}_0 e^{-i\omega t}, \end{aligned} \tag{1}$$

where $\boldsymbol{\Delta} = \{V, \Delta, \varphi H\}^T$ is a generalized displacement vector and $\boldsymbol{\Delta}_0 = \{V_0, \Delta_0, \varphi_0 H\}^T$ is its complex amplitude.

The equation of motion of the equivalent oscillator (equilibrium of moments about the point $\xi = 0$, Fig. 1) implies

$$\begin{aligned} &\ddot{\psi}^{\text{rel}} + 2\omega_N \varsigma \dot{\psi}^{\text{rel}} + \omega_N^2 \psi^{\text{rel}} \\ &\quad - \frac{m_b H^2}{I_0} \frac{g}{\omega_N^2 a} \frac{a}{H} \omega_N^2 \psi^{\text{rel}} \\ &= \frac{-m_b H^2}{I_0} \frac{\ddot{\Delta}}{H} - \ddot{\varphi} + \frac{m_b H^2}{I_0} \frac{g}{\omega_N^2 a} \frac{a}{H} \omega_N^2 \varphi \end{aligned} \tag{2}$$

where g is the acceleration due to gravity, $\frac{K_b}{I_0} = \omega_N^2$, and $\frac{C_b}{I_0} = 2\omega_N \varsigma$. $I_0 = m_b H^2 \left[1 + \left(\frac{r_b}{H}\right)^2\right]$ is the moment of inertia of the building about $\xi = 0$, and ω_N and ς are the fixed-base natural frequency and the ratio of critical damping. The term $g/\omega_N^2 a$ is a dimensionless parameter involving the acceleration due to gravity. For low buildings this ratio is very small ($\sim 10^{-4}$), while for the higher buildings, it is of the order 10^{-1}. The geometric parameters of the lumped mass building model, H and r_b, could be related to the geometric parameters of a shear beam building model with height H_{sb}, width W_{sb}, and same mass (per unit length in the y-direction) above the foundation level as the oscillator, as $H = \frac{H_{sb}}{\sqrt{3}}$ and $r_b = \frac{W_{sb}}{\sqrt{12}}$.

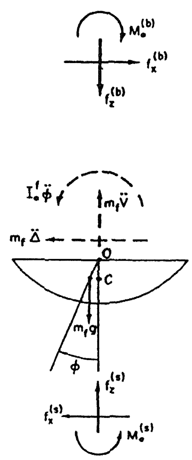

Fig. 2. Equilibrium of forces acting on the foundation at point O

For a harmonic motion of the foundation, $\boldsymbol{\Delta} = \boldsymbol{\Delta}_0 e^{-i\omega t}$; from Eq. (2), the relative rocking angle ψ^{rel} is

$$\psi_0^{\text{rel}} = \frac{\frac{m_b H^2}{I_0}\left(\frac{\omega}{\omega_N}\right)^2 \frac{\Delta_0}{H} + \left[\left(\frac{\omega}{\omega_N}\right)^2 + \frac{g}{\omega_N^2 H}\frac{m_b H^2}{I_0}\right]\varphi_0}{1 - 2i\varsigma\frac{\omega}{\omega_N} - \left(\frac{\omega}{\omega_N}\right)^2 - \frac{g}{\omega_N^2 H}\frac{m_b H^2}{I_0}}. \tag{3}$$

Then, the forces that the foundation exerts onto the building (the vertical force $f_z^{(b)}$, the horizontal force $f_x^{(b)}$ and the moment about O, $M_0^{(b)}$) can be calculated in terms of the displacement of the foundation, using the dynamic equilibrium equations of the structure. $f_x^{(b)}$ is positive in the negative x direction, $f_z^{(b)}$ is positive up and $M_0^{(b)}$ is positive counterclockwise. Let $\mathbf{F}^{(b)} = \{f_z^{(b)}, f_x^{(b)}, M_0^{(b)}/H\}^T$ be a generalized force vector. Then, in matrix form, one can write

$$\mathbf{F}^{(b)} = m_b\omega^2 \left[[K^{(b)}] + [C_g^{(b)}]\right] \Delta_0 e^{-i\omega t} \quad (4)$$

where

$$[K^{(b)}] = \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & k_{23} \\ 0 & k_{32} & k_{33} \end{bmatrix}, \quad (5)$$

is the complex stiffness matrix for the building, and

$$[C_g^{(b)}] = \frac{g}{\omega^2 H} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & c_{23} \\ 0 & c_{23} & c_{33} \end{bmatrix}, \quad (6)$$

is the impedance matrix associated with the gravity forces acting on the building. The sum of $[K^{(b)}]$ and $[C_g^{(b)}]$ gives the impedance matrix for the building.

Fig. 2 shows the free body diagram of the foundation where $f_x^{(b)}$, $f_z^{(b)}$ and $M_0^{(b)}$ are the horizontal and vertical forces and the moment that the building exerts onto the foundation at point O; $f_x^{(s)}$, $f_z^{(s)}$ and $M_0^{(s)}$ are the horizontal and vertical forces and the moment applied onto the foundation at point O by the elastic half-space; $m_f\ddot{\Delta}$, $m_f\ddot{V}$ and $I_0^{(f)}\ddot{\varphi}$ are the D'Alambert forces of the foundation acting also at point O ($I_0^{(f)}$ is the mass moment of inertia of the foundation about point O); $m_f g$ and point C are respectively the gravity force acting on the foundation and center of gravity. Let us define a generalized force vector $\mathbf{F}^{(s)} = \{f_x^{(s)},\ f_z^{(s)},\ M_0^{(s)}/H\}^T$. $\mathbf{F}^{(s)} = \mathbf{F}_0^{(s)} + \mathbf{F}_\Delta^{(s)}$, where $\mathbf{F}_0^{(s)}$ and $\mathbf{F}_\Delta^{(s)}$ are the generalized force vectors representing the foundation driving forces (forces acting on the foundation at rest and due to the free-field motion) and the forces induced in the half-space due to the deformations caused by the moving foundation, in the absence of incident waves. $\mathbf{F}_0^{(s)}$ is equal in magnitude and of opposite direction to the force that must be applied to the foundation to keep it at rest, while it is forced to move by the free-field waves. Consequently, $\mathbf{F}_0^{(s)}$ depends only on the characteristics of the free-field motion (type of incident waves, angle of incidence and their amplitude), $\mathbf{F}_\Delta^{(s)}$ depends on the imposed motion $\mathbf{\Delta}$, and both depend on the shape of the foundation and on the frequency of excitation. $\mathbf{F}_\Delta^{(s)}$ can be written as

$$\mathbf{F}_\Delta^{(s)} = -2\mu[Q]\mathbf{\Delta} \quad (7)$$

where $2\mu[Q]$ is the impedance matrix for the foundation and μ is the shear modulus of the half-space. The expressions for $\mathbf{F}_0^{(s)}$, for incident plane P- and SV- and surface Rayleigh waves, for $[Q]$, and for the coefficients k_{ij} and c_{ij} in Eqs. (5) and (6), can be found in Todorovska and Trifunac (1990).

The equilibrium equations of the foundation are

$$[M_f]\ddot{\mathbf{\Delta}} = \mathbf{F}^{(b)} - \mathbf{F}^{(s)} - \mathbf{F}_g^{(f)} \quad (8)$$

where $[M_f] = \text{diag}\{m_f,\ m_f,\ I_0^{(f)}/H^2\}$ is the mass matrix of the foundation, and $\mathbf{F}_g^{(f)} = \{0, 0,\ m_f g c\varphi/H\}^T$ is the generalized force vector of the gravity forces of the foundation (point $\xi = -c$ is the center of gravity). Then, for given characteristics of the structure and for various types of excitation, Eq. (8) can be solved (Todorovska and Trifunac 1990) for $\mathbf{\Delta}$.

RESULTS AND ANALYSIS

The dimensionless parameters used in the analysis are: the dimensionless frequency $\eta = \frac{2a}{\beta T} = \frac{\omega a}{\pi\beta}$, the dimensionless stiffness parameter $\eta_N = \frac{2a}{\beta T_N} = \frac{\omega_N a}{\pi\beta}$, the mass ratios $\frac{m_b}{m_f}$ and $\frac{m_f}{m_s}$ (m_s is the mass per unit length in the y-direction of the soil replaced by the foundation), the dimensionless height of the equivalent oscillator, H/a, and the dimensionless depth of the foundation, h/a. η equals the number of shear-wavelengths in the soil, with frequency ω, contained in length equal to the width of the foundation.

For example, for a 10-story building (natural frequency $f_N = 1$ Hz and base half-width $a = 15$ m), situated on medium soft soil with shear wave velocity $\beta = 400$ m/s, $H/a \approx 1.9$ and $\eta_N \approx 0.076$. The same building in Los Angeles ($\beta \approx 250$ m/s) would have $\eta_N \approx 0.15$; in Mexico City, where the shear wave velocity in the soil can be as low as $\beta = 50$ m/s, $\eta_N \approx 0.3$. Small η_N means a flexible building and/or stiff soil, and large η_N means a stiff building and/or very flexible soil. Limiting value $\eta_N \to 0$ corresponds to a fixed-base building model excited by horizontal motion at the base. $\eta_N \to \infty$ corresponds to a rigid building welded to the foundation, and oscillating as a rigid body with translational and rocking degrees of freedom.

In the presented examples, the Poissin's ratio $\nu = 0.3333$, the gravity forces are neglected, and the foundation and the building have same density. A typical value of the ratio of the densities of the building and of the soil is taken to be $\frac{\rho_b}{\rho_s} = 0.2$. Then, $\frac{m_b}{m_f} \approx 2.2\frac{H}{a}$ when the foundation is semi-circular, and $\frac{m_b}{m_f} \approx 4.9\frac{H}{a}$ when the depth-to-half-width ratio $h/a = 0.5$.

The system frequency, η^{sys}, and the system damping ratio, ς^{sys}, are determined from the amplitude spectrum of the transfer function between the relative oscillator response $u_b^{rel} = \psi^{rel}H$, and the incident wave motion, using the standard measure of the width of the peak in the transfer-function of a

SDOF oscillator. The system frequency, η^{sys}, is the frequency of the peak in the transfer-function. If η_1 and η_2 are the frequencies to the left and to the right of η^{sys} for which $u_b^{rel} = u_b^{rel}(\eta^{sys})/\sqrt{2}$, then the system damping ratio, ζ^{sys}, is

$$\zeta^{sys} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} \approx \frac{\eta_2 - \eta_1}{2\eta^{sys}}. \tag{9}$$

The system response (the peak relative building displacement amplitude, the peak base rocking amplitude, and the system frequency and system damping ratio), as it depends on the relative stiffness parameter η_N, the mass ratios m_b/m_s, the building damping ratio ζ, and the building height, for the model in Fig. 1 was studied in detail by Todorovska and Trifunac (1992a), for vertically incident plane SV waves. The results of their study can be summarized as follows.

In the limit when the stiffness of the soil is infinitely large compared with the stiffness of the building, the system response approaches the fixed-base model response. Then the system damping approaches the damping in the building, the system frequency approaches the fixed-base natural frequency. In the limit when the building is infinitely stiff compared with the soil, the system damping and the system frequency asymptotically approach the system damping ζ^{rig} and the system frequency η^{rig} of a rigid building welded to the foundation. ζ^{rig} is smaller and η^{rig} is lower when the building mass is larger. When the foundation is deeper, ζ^{rig} is lower. ζ^{rid} does not depend on the damping in the building.

When both the soil and the building are flexible, depending on the value of η_N, two regimes of the system behavior have been noticed. In Regime I, when the soil is stiffer (smaller η_N), the system response is more sensitive to the changes in the oscillator damping ratio and natural frequency. In Regime II, when the building is stiffer (larger η_N), the system response is more affected by the value of the oscillator mass and height. The system frequency is always lower than the fixed-base natural frequency of the building; the reduction is larger when the soil is softer (larger η_N) and when the building is heavier (larger m_b/m_s). The system damping is larger when the soil is softer; in Regime I it is larger when the oscillator mass is larger, while in Regime II it is the opposite. The relative building response decreases as the flexibility of the soil increases, and it is smaller for larger values of m_b/m_s. In Regime I, the base rocking is larger when the soil is more flexible. It is larger when the building damping is smaller, and when m_b/m_s is larger.

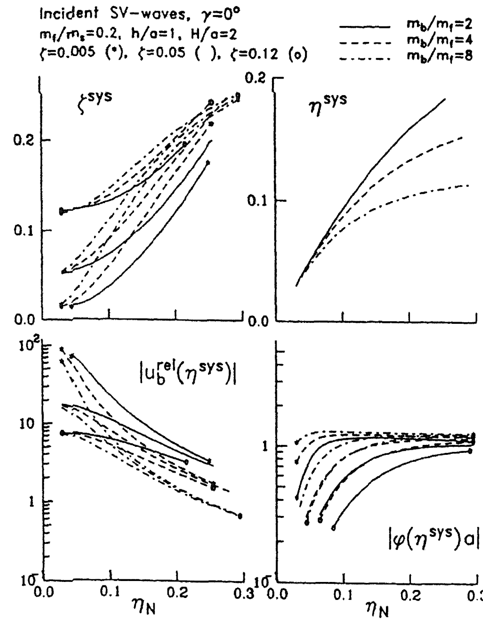

Fig. 3. The system response versus the relative stiffness parameter, η_N, for a building on a semi-circular foundation

Fig. 3, redrawn from Todorovska and Trifunac (1992a), illustrates some of these effects. There, the system damping, ζ^{sys}, the system frequency, η^{sys}, the amplitude of the peak relative building response, $|u_b^{rel}(\eta^{sys})|$, and the amplitude of the peak base rotation, $|\varphi(\eta^{sys})a|$, have been plotted versus the "relative stiffness" parameter η_N for a typical building ($H/a = 2$), with damping ratios $\zeta = 0.005$, 0.05 and 0.12, and mass ratios $m_b/m_f = 2$, 4 and 8 ($m_b/m_f = 4$ corresponds to the typical value of ρ_b/ρ_s). It can be seen that, as $\eta_N \rightarrow 0$, the curves corresponding to same value of ζ merge together, while, for larger η_N, the curves with same value of m_b/m_f merge together.

The system response for different foundation depths of the model in Fig. 1 has been studied in detail by Todorovska and Trifunac (1992b), for vertically incident SV waves. They concluded that the depth of the embedment may significantly affect the system damping and the system frequency. The soil acts as "stiffer" when the foundation is deeper. For the 2D model studied, for a building on a shallow foundation, the system frequency is lower, and the system damping ratio is larger even though the radiation damping coefficients, related to the imaginary parts of the foundation stiffness matrix, are smaller for smaller foundation depth. Then, the relative building response is smaller for smaller foundation depth. The opposite has been concluded for em-

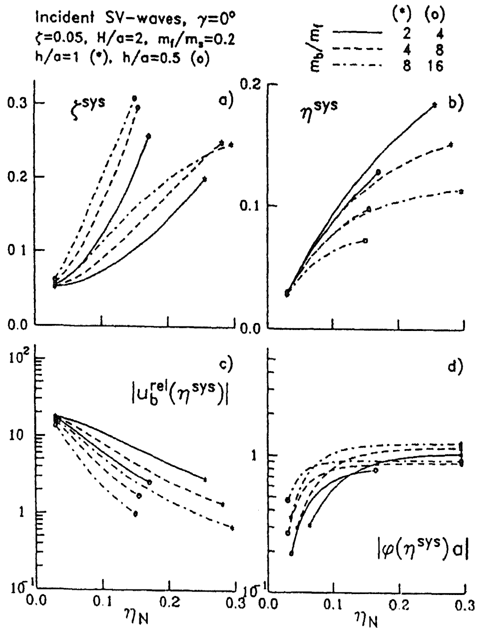

Fig. 4. The system response versus η_N, for a buildings on a semi-circular and a shallow foundation

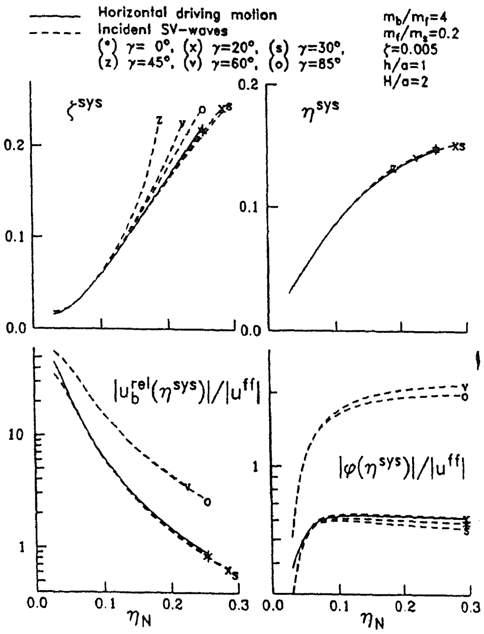

Fig. 5. Comparison between the system response for incident plane SV-waves and for horizontal input driving motion

bedded 3D prismatic foundations in previous studies (Bielak, 1975). The reasons for these discrepancies, as well as some similarities in trends of the system response with other soil-structure interaction models (Trifunac, 1972; Trifunac and Wong, 1974; and Lee, 1979) have been discussed in Todorovska and Trifunac (1992b). In general, when the soil is stiffer, the base rocking is smaller for buildings on deeper foundations, and when the soil is softer, it is larger for buildings on deeper foundations.

These effects are illustrated in Fig. 4, redrawn from the same paper, where ζ^{sys}, η^{sys}, $|u_b^{rel}(\eta^{sys})|$, and $|\varphi(\eta^{sys})a|$, have been plotted versus the "relative stiffness" parameter η_N, for buildings on a deep and on a shallow foundation ($h/a = 1$ and 0.5). The different type of lines correspond to the same values of m_b/m_s as in Fig. 3. Different symbols are used for different foundation depths. The foundation mass is such that $m_f/m_s = 0.2$.

The foundation acts as a scatterer and filter for the shorter wavelengths of the incident waves. Therefore, the foundation input motion (response of a massless foundation to the incident waves in the absence of the building) differs from the free-field motion. The free-field motion on the half-space surface, and, therefore, the foundation input motion amplitudes and phases, depend significantly on the type of incident waves and their angle of incidence (Trifunac, 1982; Todorovska and Trifunac, 1990). The horizontal component and the point rotation of the free-field motion may considerably differ in amplitude, depending on the incident angle and the type of incident waves. Moreover, for embedded foundations, the foundation input motion may have rotation even when the point rotation of the free-field motion is equal to zero (Todorovska nd Trifunac, 1990). However, in practice, the wave nature of the earthquake excitation of buildings is usually ignored. The actual excitation, which consists of translations and rotations, is approximated by a synchronous horizontal motion of the base, which corresponds to vertically incident, long compared with the foundation size, plane SV-waves.

To illustrate how different the system response would be if the wave passage effects (kinematic interaction) are not included, in Fig. 5 (redrawn from Todorovska and Trifunac, 1992a), ζ^{sys}, η^{sys}, $|u_b^{rel}(\eta^{sys})|/|u^{ff}|$ and $|\varphi(\eta^{sys})|/|u^{ff}|$ are plotted versus η_N when the foundation is driven only by a horizontal motion with amplitude $\Delta = 2$ (the solid lines), and when the excitation is an incident plane SV-wave with incident angle $\gamma = 0°$, 20°, 30°, 45°, 60° and 85° (the dashed lines). $|u^{ff}|$ is the amplitude of the horizontal component of the free-field motion on the ground surface. Different symbols are used to distinguish the lines for different incident angles. The building height is such that $H/a = 2$, the foundation is semi-circular ($h/a = 1$), the damping in the building is $\varsigma = 0.005$, and the mass ratios are $m_b/m_f = 4$ and $m_f/m_s = 0.2$. It can be seen from this figure that, when the wave passage effects are ignored, the system damping ratio is very close to the values for incident angles $\gamma \leq \gamma_{crit}$, but it is smaller than the values for $\gamma > \gamma_{crit}$ ($\gamma = 45°$, 60° and 85°). The building relative response, normalized by the amplitude of the driving displacement, Δ, also has very similar peak amplitudes to the ones corresponding to incidence below critical angle, but significantly smaller than the peak amplitudes for incidence beyond critical angle. $|u_b^{rel}(\eta^{sys})|/|u^{ff}|$ and $|\varphi(\eta^{sys})|/|u^{ff}|$ are not plotted for $\gamma = 45°$, because for this incident angle $|u^{ff}| = 0$. The peak rocking response, normalized by $|u^{ff}|$, also has similar amplitudes to the case for incidence below critical angle, but noticeably smaller amplitudes than for incidence beyond critical angle. This means that, for incident angles below critical angle, most of the base rotation comes from the inertia forces of the building. It can be concluded that, if the wave passage effects are excluded from the analyses, by assuming simplified excitation, the system damping ratio and the amplitudes of the system response may be underestimated. These effects appear to be caused by the inhomogeneous part of the free-field motion, for incident angles grater than the critical angle.

In Todorovska and Trifunac (1992a) the effect of the kinematic interaction on the system damping ratio and on the peak response amplitudes has been studied also for incident plane P-waves. For incident P-waves, the conclusions are similar as for incident plane SV-waves below critical angle. How much the relative response spectral amplitudes (in the interval $0 < \eta < 2$) can be underestimated, for the model in Fig. 1, by approximating the true foundation input motion by a synchronous horizontal motion of the base, has been studied in Todorovska and Trifunac (1992c).

CONCLUSIONS

It can be concluded from the analyses of this simple 2D model that the soil-structure interaction, under certain conditions, can be an efficient mechanism for dissipation of the energy in buildings excited by earthquake motions. Because of the many assumptions made (the real world is three dimensional and nonlinear, the real buildings vibrate in more than one mode, and the building foundation in not absolutely rigid and circular in shape), the results of this study are not directly applicable to real buildings. However, this initiates thinking along the lines how natural phenomena can be used for reduction of the response of buildings to strong earthquake shaking. Because it is a natural phenomenon, and does not require involvement of complicated equipment and large amount of additional energy (comparable to the earthquake energy) for control of the building response, it should be studied further, and ways should be found how advantage of it could be taken.

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