Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Optimum damping in base-isolated structures

Jose A. Inaudi & James M. Kelly
University of California at Berkeley, Calif., USA

ABSTRACT: Optimum viscous isolation damping for minimum acceleration response of base-isolated structures subjected to stationary random excitation is investigated. Two linear models are considered to account for the energy dissipation mechanism of the isolation system: a viscous linear element, and a Maxwell element, which are commonly used models for viscodampers. The criterion selected for optimality is the minimization of the peak floor acceleration response. The effects of frequency content of the excitation and superstructure properties on the optimum damping and minimum peak acceleration response are addressed.

1. INTRODUCTION

Base isolation is today an accepted design alternative for earthquake hazard mitigation for structures on firm soil. Superior seismic performance can be achieved by means of the introduction of a flexible set of isolators between a stiff superstructure and its foundation. The benefits of this design approach are not only complete preservation of the structural system but also equipment protection during moderate and strong ground motions. While the failure of the structural system is prevented by guaranteeing a maximum deformation demand on the isolation system, failure of sensitive equipment can be prevented by ensuring low enough levels of floor accelerations.

Ground motion and structural system characteristics determine the deformation demand on the isolation system and the floor acceleration response of the structure subjected to ground excitation. Intensity of the ground shaking, frequency content as well as maximum ground velocity are factors of crucial importance. Natural frequencies of the base-isolated structure and energy dissipation capability in the isolation system are controlling factors for the response. The relative displacement at the isolator level is dominated by the response of the system in its first mode of vibration. The superstructure damping capability has a negligible effect on the damping of the first mode of vibration of a base-isolated structure and consequently, the isolator deformation can not be controlled by an increase of the superstructure damping capability. However, significant damping can be introduced in the first mode of the structure by increasing the energy dissipation capability of the isolation system. Damping, although not essential in the isolation phenomenon, is needed to keep the isolator dispacements within limits in case of low frequency ground motion. High-damping rubbers, lead plugs and or added viscous or frictional dampers can give the desired energy dissipation capability to the isolation system and with that, a reduction of the demand on the isolation system can be attained. On the other hand, the energy dissipation mechanism of the isolation system has a significant effect on the floor acceleration response.

Tsai and Kelly (1988) have shown that the response of internal equipment on base-isolated structures in which the damping matrix is non-classical can not be accurately determined by the classical mode method. The high frequency content is distorted by the classical mode method and the use of complex modes is recommended to find equipment response. Constantinou and Tadjbakhsh (1985) have developed studies on the optimum fundamental period of base-isolated structures under random excitation. The work reported herein aims at determining optimal levels of damping induced by viscodampers which will render minimal acceleration response in the structure subjected to ground excitation. A statistical approach and a deterministic approach are followed. For the statistical approach, the ground motion is modelled as a stationary Gaussian random process and optimum damping is defined as that which renders mimumum peak floor acceleration. For the deterministic approach, the maximum floor acceleration of base isolated structures subjected to recorded ground motions is evaluated as a function of the isolation energy dissipation capacity by numerical simulation. Simple structural models were analyzed aiming at identifying the main controlling parameters.

2. STRUCTURAL MODELS

The dynamic response of a n-story symmetric isolated building subjected to unidirectional ground excitation can be described using a floor lumped-mass model in terms of relative coordinates by the following equation

$$M\ddot{y} + C\dot{y} + K y + l f = -M r w \quad (1)$$

where M, C and K are, respectively, the mass, damping and stiffness matrices of the superstructure; y is a vector which contains the relative displacements of basement and floors with respect to the ground, w represents the ground acceleration, $l^T = [\,1\ 0\ \cdots\ 0\,]$ and $r^T = [\,1\ 1\ \cdots\ 1\,]$. f represents the force that the isolation system applies on the basement of the structure. This term includes both forces resulting from deformation of the isolator and of the energy dissipation devices acting in the isolation system. The floor accelerations can be easily expressed as

$$\ddot{x} = -M^{-1} K y - M^{-1} C \dot{y} - M^{-1} l f \quad (2)$$

Shear building models are used for the analyses in this study. Figure 1 describes a typical structure used in this study. A parametric definition of the structure allows the assessment of the effect of the different parameters in the phenomenon under study. The superstructure stiffness matrix K is characterized by means of a single parameter k which is selected to give a desired frequency to the first fixed-base undamped mode of the superstructure T_{fb}. The flexibility of the superstructure will be varied in this study by changing the value of T_{fb}. The superstructure damping matrix C is defined by assuming modal dampings in the superstructure. In order to simplify this study all modal damping ratios of the superstructure are assumed equal ξ_{ss} except for the the free-body mode which is taken as zero.

A large variety of devices with a wide range of mechanical behaviors have been proposed to be used as part of the isolation system. Natural rubber bearings made of different rubber compounds, rubber bearings with lead plugs, rubber bearings in combination with viscous dampers, friction dampers or elastoplastic dampers are among the most commonly proposed devices. In the present paper, visco-dampers modelled by linear models will be considered. Extrapolating these results and conclusions to highly non-linear devices is not a recommendable scheme. Series and parallel combinations of linear springs and dashpots can be used then to obtain different viscoelastic models for the mechanical elements under study. At the expense of greater complexity the number of springs and dashpots can be increased seeking a more accurate agreement between the response of the model and the response of the real mechanical device. In order to keep this study as simple as possible the models we will consider include at most three parameters.

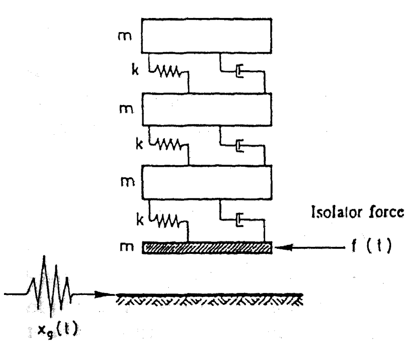

Figure 1. Structural model used in this study.

2.1. Kelvin element and Maxwell element

A Kelvin element, a linear spring in parallel with a pure viscous damper constitutes the first model for this study. The force in a Kelvin element f^I satisfies

$$f^I = k_o\, y_1 + c_o\, \dot{y}_1 \quad (3)$$

The dissipation of energy in harmonic excitation is linearly proportional to the frequency of excitation for this model. The parameter c_o will be varied to study the effect of viscous damping in the acceleration response of base-isolated structures. k_o will be selected to give a certain first natural frequency ω_1 to the isolated structure.

The second model for the isolation system to be used in the present study consists of a linear spring in parallel with a of Maxwell element, so called standard solid in mechanics of solids. The element force f^{II} is governed in this case by

$$f^{II} = k_o\, y_1 + f_e \quad (4)$$

$$\dot{f}_e + \frac{1}{\tau} f_e = g\, \dot{y}_1 \quad (5)$$

The main mechanical characteristic of a Maxwell model is its relaxation time τ. The energy dissipated in a cycle in this model increases with frequency for frequencies less than $1/\tau$ and monotonically decreases with frequency for frequencies larger than $1/\tau$.

3. ACCELERATION RESPONSE FOR MODEL I

A formulation of the equations of motion in state space form is convenient for developing the analysis. The set of n second order differential equations (1) is converted into a set of $2n$ first order differential equations by defining a suitable state vector $z^T = [y^T\ \dot{y}^T\,]$. Considering eq.(3), eq.(1) can be expressed in state space as

$$\dot{z} = A^I z + B^I_w w \qquad \ddot{x} = D^I z \quad (6)$$

$$A^I = \begin{bmatrix} O & I \\ -M^{-1}\bar{K} & -M^{-1}\bar{C} \end{bmatrix} \qquad D^I = \begin{bmatrix} -M^{-1}\bar{K} & -M^{-1}\bar{C} \end{bmatrix}$$

$$\bar{C} = C + diag(c_o,0,...,0) \qquad \bar{K} = K + diag(k_o,0,...,0) \qquad B^I_w = \begin{bmatrix} O \\ -r \end{bmatrix}$$

The floor acceleration frequency response can be obtained from eqs.(8 and 9) as

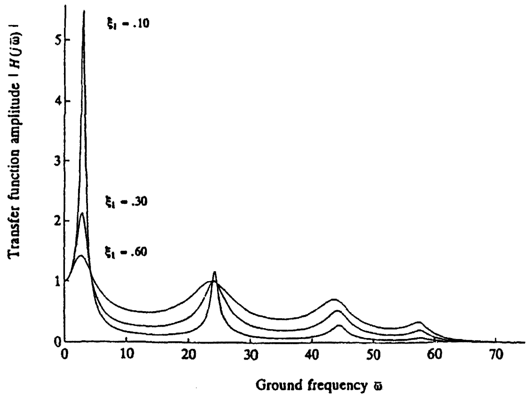

Figure 2. Acceleration frequency response: Model I.

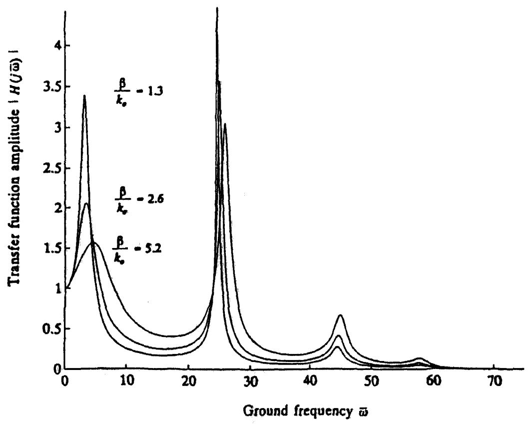

Figure 3. Acceleration frequency response: Model II.

$$\mathbf{H}^{I}(j\bar{\omega}) = \frac{\ddot{\mathbf{X}}(j\bar{\omega})}{W(j\bar{\omega})} = \mathbf{D}^{I}(j\bar{\omega}\,\mathbf{I} - \mathbf{A}^{I})^{-1}\,\mathbf{B}^{I}_{w} \tag{7}$$

Figure 2 describes the acceleration frequency response for the top floor of a 4DOF structure ($\omega_1 = \pi\ rad/s$, $T_{fb} = .6\ s$) for different values of c_o. The three curves correspond to induced damping ratios in the first mode of $\xi_1 = .10$, .30 and .60. It is worth noticing that only resonant responses are suppressed by means of an increase in the isolation damping. While the transfer function amplitude decreases for higher damping at low frequency, it increases at high frequencies. This effect is very important in the understanding of the phenomenon under consideration. If the ground motion presents dominant low frequency content heavily damped isolation systems will improve the performance of the structure in terms of acceleration response and isolation deformation, however if high frequency is dominant in the ground acceleration signal, a heavily damped isolation system will reduce isolation deformation but will tend to increase the floor acceleration response of the structure.

4. ACCELERATION RESPONSE FOR MODEL II

In order to study the effect that Maxwell viscous dampers introduced in the isolation system have in the floor acceleration response of the structure a state space formulation is developed assuming that a certain number N_e of such devices with identical mechanical characteristics are connected in parallel with the isolators. The force in the i-th Maxwell element is denoted as f_e^i. From eqs.(4 and 5) the force in the isolation system will then satisfy

$$f^{III} = k_o\, y_1 + f_e \qquad f_e = \sum_{i=1}^{N_e} f_e^i \tag{8}$$

$$\dot{f}_e + \frac{1}{\tau} f_e = \beta\, \dot{y}_1 \qquad \beta = N_e\, g$$

Since for the Maxwell model f_e satisfies a first order differential equation, a suitable definition of an extended state $\mathbf{z}_e^T = [\mathbf{z}\ \ f_e]$ allows us to put the system in state form

$$\dot{\mathbf{z}}_e = \mathbf{A}_e^{II}\,\mathbf{z} + \mathbf{B}_e^{II}\, w \qquad \ddot{\mathbf{x}} = \mathbf{D}_e^{II}\,\mathbf{z}_e \tag{9}$$

$$\mathbf{A}_e^{II} = \begin{bmatrix} \mathbf{O} & \mathbf{I} & 0 \\ -\mathbf{M}^{-1}\overline{\mathbf{K}} & -\mathbf{M}^{-1}\mathbf{C} & \mathbf{M}^{-1}\mathbf{l} \\ [0..0] & [-\beta\ 0..0] & -\frac{1}{\tau} \end{bmatrix} \qquad \mathbf{B}_e^{II} = \begin{bmatrix} \mathbf{O} \\ -\mathbf{r} \\ 0 \end{bmatrix}$$

$$\mathbf{D}_e^{II} = [\,-\mathbf{M}^{-1}\overline{\mathbf{K}}\ \ -\mathbf{M}^{-1}\mathbf{C}\ \ -\mathbf{M}^{-1}\mathbf{l}\,]$$

The poles of the system as a function of β and the relaxation time τ can be obtained by solving for the eigenvalues of $\mathbf{A}_e^{II}(\beta,\tau)$. It is interesting to note that the most significant effect in the modal damping ratios is introduced by τ. The smaller the τ the most effectively the poles can be moved into the left half of the complex plane. A change in β can produce a redistribution of negative real parts between the poles but no change in the trace of $\mathbf{A}_e^{III}$ and consequently no change in the center of gravity of the poles. An other important characteristic of this model is that it introduces an stiffening effect on all the modes of vibration increasing their natural frequencies.

The acceleration frequency response of the system under Maxwell-type damping can be obtained as

$$\mathbf{H}^{II}(j\bar{\omega}) = \mathbf{D}_e^{II}\,(j\bar{\omega}\,\mathbf{I} - \mathbf{A}_e^{II})^{-1}\,\mathbf{B}_e^{II} \tag{10}$$

Figure 3 describes the acceleration frequency response of the top floor of a 4DOF structure ($\omega_1 = \pi\ rad/s$, $T_{fb} = .6\ s$) for $\tau = .10$ and different values of β. The stiffening effect of the Maxwell elements appears clear in the shifting of the peaks of $|H(j\bar{\omega})|$. While for low frequency range the transfer function magnitude decreases for higher β, for high frequency it increases.

5. RESPONSE TO STATIONARY EXCITATION

The effect of damping in the acceleration response of the base-isolated structure subjected to random excitation is addressed in this section for both models. Optimum damping levels are defined as those which render a minimal peak acceleration response of the base-isolated structure.

Given a linear system as that of eq.(6 or 9) subjected to stationary excitation $w(t)$ with zero mean and power spectral density $S_g(\bar{\omega})$, the power spectral density of the floor acceleration response $S_{\ddot{x}}(\bar{\omega})$ can be obtained as

$$S_{\ddot{x}}(\bar{\omega}) = H(j\bar{\omega})\, S_g(\bar{\omega})\, H^*(j\bar{\omega}) \tag{11}$$

The ground acceleration variance σ_g^2 is given by

$$\sigma_g^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_g(\bar{\omega})\, d\bar{\omega} \tag{12}$$

The floor acceleration covariance matrix can be obtained as

$$E[\ddot{x}\,\ddot{x}^T] = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\ddot{x}}(\bar{\omega})\, d\bar{\omega} \tag{13}$$

The diagonal of the acceleration covariance matrix contains the floor acceleration variances. If we assume that the ground motion is a Gaussian random process the estimation of extreme values of an output signal v of the excited linear system over a certain period of time T can be done according to the Poisson model for the barrier crossings

$$V_{max} = \sigma_v \left[2 \ln \left(- \frac{T\, \sigma_{\dot{v}}}{\pi \ln \gamma\; \sigma_v} \right) \right]^{\frac{1}{2}} \tag{14}$$

where γ represents the probability that V_{max} will not be exceeded during an interval of duration T of the random process, and σ_v and $\sigma_{\dot{v}}$ are the standard deviations of the signal v and its time derivative (Vanmarke, 1883). Since we are interested in the floor acceleration response, the evaluation of floor jerks **e**, time derivative of the floor acceleration, becomes necessary. The floor jerk covariance matrix $E[e\, e^T]$ can be obtained as

$$E[e\, e^T] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{\omega}^2\, H(j\bar{\omega})\, H^*(j\bar{\omega})\, S_g(\bar{\omega})\, d\bar{\omega} \tag{15}$$

The diagonal of the floor jerk covariance matrix contains the floor jerks variances.

Equations (13), (14) and (15) along with the expressions for the acceleration frequency response of the system can be used then to estimate the peak floor acceleration $\ddot{x}_{peak}$ of a base isolated structure as a function of the different damping models in the isolation system. The optimum value of damping parameter (co_{opt} or β_{opt}) can now be estimated by minimizing the peak floor acceleration value over the values of the damping parameter.

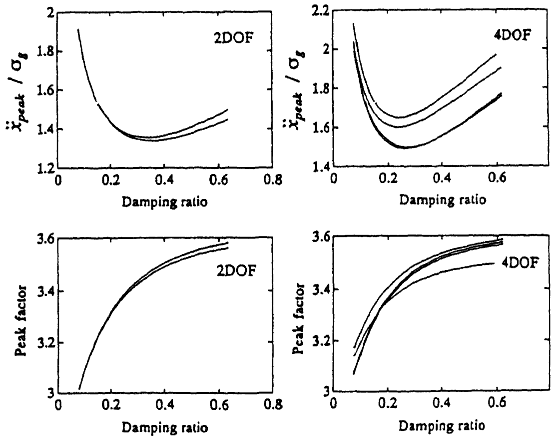

Figure 4. Peak floor acceleration: Model I.

5.1. MDOF systems under band limited excitation

The effect of the number of degrees of freedom of the structural system, the flexibility and damping of the superstructure on the optimum damping is investigated in this section. Two structural models are considered to analyze the effect of number of degrees of freedom: a 2DOF system and a 4DOF system. Their fixed base periods T_{fb} are taken as .15 s and .45 s respectively. The value values of the first natural frequency of the undamped base-isolated structure is taken as $\omega_1 = 3.1415\ rad/s$. The power spectral density of the ground motion is assumed as

$$S_g(\bar{\omega}) = S_o \qquad -\omega_o < \bar{\omega} < \omega_o \tag{16}$$

with $\omega_o = 80\ rad/s$. Figures 4 and 5 show the results obtained from for both models. The peak floor acceleration ($\gamma = .5$, $T = 40\ s$) for the 2DOF and the 4DOF systems are plotted at the top of the figure as a function of the induced first mode damping ratio (Models I and II). As we can notice, the optimum damping decreases with an increase in the number of degrees of freedom for model I while for the maxwell model it does not present high sensitivity to a change in the number of degrees of freedom. Peak acceleration values corresponding to optimum damping levels augment with the number of degrees of freedom. The peak factors (quotient between the peak acceleration and its standard deviation) are shown in the figures for comparison. The peak factor slightly increases with the increase in the damping parameter. The results shown for model II correspond to $\tau = .10s$. Larger values of τ render larger peak acceleration response.

The effect of the superstructure flexibility T_{fb} and superstructure damping ξ_{ss} is analyzed in figures 6 and 7 for models I and II. A 2DOF system is used in the analyses and the optimum damping values are obtained for different

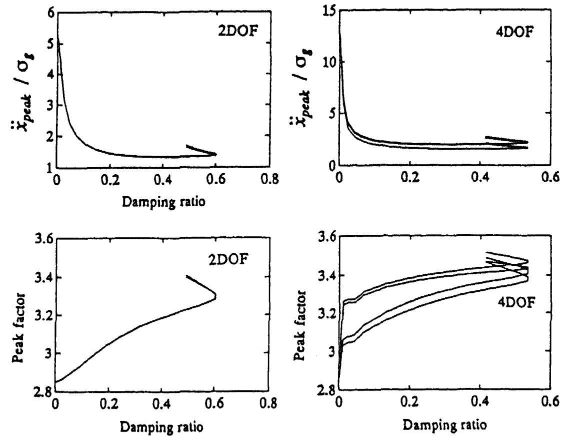

Figure 5. Peak floor acceleration: Model II.

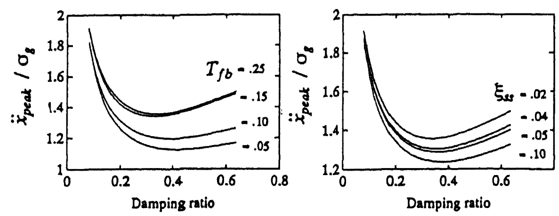

Figure 6. Effect of flexibility and damping: Model I.

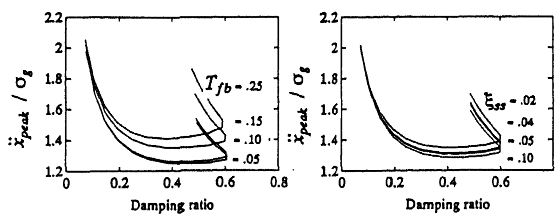

Figure 7. Effect of flexibility and damping: Model II.

Table 1. Recorded signals used for dynamic simulation

Station name	Direction	PGA $[g]$	PGV $[cm/s]$
El Centro	S90W	0.21	36.92
Taft	N21E	0.15	15.72
James Road	S40E	0.52	43.99

values of superstructure flexibility $T_{fb} = .05, .10, .15, .25\ s$ keeping constant superstructure damping $\xi_{ss} = .02$ and for different values of superstructure damping $\xi_{ss} = .02, .04, .05, .10$ keeping $T_{fb} = .15$ constant. An increase in the flexibility of the superstructure causes a decrease in the value of the optimum damping as well as an increase in the peak acceleration. This effect is particularly noticeable in the viscous model. The effect of superstructure damping has the opposite sense: the higher ξ_{ss} the higher the value of optimum damping and the lower the peak acceleration response.

6. RESPONSE TO RECORDED GROUND MOTIONS

In this section, the effect of isolation damping in the dynamic behavior of base isolated structures is evaluated by numerical simulation of the response of the structure to recorded ground motions. The acceleration response of the structure is computed for several recorded signals under different levels of isolation damping. Optimal levels of damping are defined as those that render minimum maximum floor acceleration response for a particular ground motion. The isolator deformation is also computed. The effects of the type of isolation system, the number of degrees of freedom, the superstructure flexibility, the frequency content of the excitation as well as the frequency content of the excitation are evaluated. The signals used in the study are listed in the following table and correspond to the earthquakes of Imperial Valley (May 18, 1940), Kern County (July 21, 1952) and Imperial Valley (October 15, 1979) respectively.

6.1. Results for Model I and Model II

The effect of isolation damping in the floor acceleration response is basically to reduce it up to a certain value of damping from which an increase in damping determines an increase in acceleration response. For a given structural system and a given ground motion there exist a damping value which minimizes the floor acceleration response while satisfying the constraint imposed by maximum deformability in the isolation system. Deformability of the isolation system determines some bounds on the allowable deformation demand. Once this safety requirement is met, the criterion for defining the optimum energy dissipation capacity of the isolation system should be the minimization of maximum floor accelerations. In order to evaluate the effect of number of degrees of freedom, superstructure flexibility in the optimum damping value, maximum acceleration response spectra were generated for different ground motions and different structural systems ($\omega_1 = \pi\ rad/s$ and $\xi_{ss} = .02$). Tables (2) and (3) summarize the results. There is significant variability of the optimum damping values which it is mainly caused by the different characteristics of the ground motions selected for the analysis. This leads us to the first obvious conclusion: the optimum damping value crucially depends on the ground motion characteristics, basically in its frequency content in relation to the fundamental frequency of the base-isolated structure. There exist however some trends that coincide with those obtained in previous sections for the random model of the ground excitation. Those are basically the following: the optimum damping value decreases with an increase in the fundamental period of the base-isolated structure and with an increase in the superstructure flexibility. The higher the frequency content of the ground motion the lower the value of optimum damping and the

Table 2. Optimum damping for model I.

Signal	*NDOF*	T_{fb} [*s*]	ξ_1^{opt}	max($\ddot{x}$) [cm/s^2]	max(y_1) [*cm*]
El Centro	8	.7	.33	174	9.6
El Centro	4	.6	.27	153	10.6
Taft	8	.7	.19	77	5.1
Taft	4	.6	.18	69	5.2
James R.	8	.7	.15	281	19.4
James R.	4	.6	.16	262	18.3

Table 3. Optimum damping for model II.

Signal	*NDOF*	T_{fb} [*s*]	ξ_1^{opt}	max($\ddot{x}$) [cm/s^2]	max(y_1) [*cm*]
El Centro	4*	.6	.23	153	10.9
El Centro	8*	.7	.24	176	10.4
El Centro	4**	.6	.21	160	9.8
El Centro	8**	.7	.23	171	8.7
Taft	4*	.6	.09	91	6.3
Taft	8*	.7	.16	96	5.2
Taft	4**	.6	.08	100	6.8
Taft	8**	.7	.12	106	5.8
James R.	4*	.6	.17	279	17.0
James R.	8*	.7	.14	298	18.4
James R.	4**	.6	.17	273	15.7
James R.	8**	.7	.16	277	16.2

* $\tau = .10\ s$ ** $\tau = .20\ s$

better the performance of the base-isolated structure. This effect should be expected since both an increase in the flexibility of the superstructure will automatically reduce the efficacy of the isolation system in decoupling higher frequency dynamics. Furthermore, the force induced by the viscodampers on the basement of the structure is "felt" by all the modes of the structure and if the stiffness of those modes is reduced the excitation of those higher modes will increase. Although the results are not shown in the tables, it was found that the optimum damping value increases with an increase in the superstructure damping ξ_{st} and that the low-pass filter characteristics of a base-isolated structure are deteriorated in presence of a heavily damped isolation system.

7. CONCLUDING REMARKS

A procedure for defining optimum damping in linear isolation systems have been presented. The optimization scheme has as objective function the peak floor acceleration response for the base-isolated structure subjected to stationary Gaussian excitation. Two simple linear elements with different energy dissipation mechanisms have been analyzed. The same procedure can be easily extended to consider more elaborate linear viscoelastic models of viscodampers. Summarizing the most significant findings of this study we conclude the following points:

* The optimum damping has been obtained based on minimum peak acceleration response to Gaussian excitation. The minimization of acceleration variances renders very similar values for the optimum damping values. This is the case since the peak factor is not sensitive to changes in the isolation damping.
* The effect of high frequency content in the excitation is to decrease the optimum viscous damping. For both Kelvin and Maxwell models, superior acceleration reduction can be attained under high frequency excitation and optimum isolation damping design.
* The results have shown that the optimum isolation damping decreases with an increase in the number of degrees of freedom. An increase in the damping of the superstructure produces an increase in the optimum damping value while an increase in the flexibility of the superstructure tends to decrease the optimum damping and amplify the peak floor accelerations.
* The low-pass filter characteristics of a base-isolated structure are deteriorated by a heavily damped isolation system. Special care should be taken in defining the isolation damping when designing an isolated structure for sensitive equipment protection.

REFERENCES

Bhatti M.A. and Pister K.S., "A dual criteria approach for optimal design of earthquake-resistant structural systems", *Earthquake Engineering and Structural Dynamics, Vol. 9, 557-572 (1981).*

Constantinou, A.M., Tadjbakhsh A., "Optimum characteristics of isolated structures", *Journal of Structural Engineering, Vol. 111, No. 12, 2733-2750 (1985).*

Kelly J.M., Tsai H.C.,"Non-classical damping in dynamic analysis of base-isolated structures with internal equipment", *Earthquake Engineering and Structural Dynamics, Vol. 16, 29-43 (1988).*

Lai M.L., Soong T.T., "Seismic design considerations for secondary structural systems", *Journal of Structural Engineering, Vol. 117, No. 2, 459-472 (1991).*

Manolis G., Juhn G., Constantinou M., Reinhorn A., "Secondary systems in base-isolated structures: Experimental investigation, stochastic response and stochastic sensitivity", *Tech. Report NCEER-90-0013, July 1990,State University of New York at Buffalo.*

Vanmarke E., "Random Fields: Analysis and Synthesis", *The MIT Press, Cambridge, Massachussets, (1983).*