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Seismic response of equipment in resilient-friction base isolated structures

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ABSTRACT: The response of equipment in structures supported on resilient-friction base isolators (R-FBI) subjected to earthquake ground motion is investigated. The R-FBI is composed of layers of teflon coated steel plates with a central rubber core to provide a restoring force. A SDOF subsystem model is considered for the equipment-structure-base isolator system. An efficient semi-analytical solution procedure for the response of the equipment is developed. The solution approach is extremely accurate as well as computationally efficient and avoids the necessity for a standard numerical integration of the combined system equations. The effect of friction coeffient, mass ratio, subsystem period (equipment, structure, isolator), and subsystem damping on the equipment response is investigated for several earthquake ground motions. The results indicate that, in general, the R-FBI is extremely effective in protecting internal equipment when the isolated structure is subjected to severe earthquake ground motions.

1 INTRODUCTION

Aseismic base isolation has been shown to be an effective means of protecting structures from the damaging effects of strong ground motions. Typically such devices fall into two broad categories: the laminated rubber bearing systems and friction systems. The laminated rubber bearing systems shift the fundamental periods of the structure away from the frequencies of the ground motion that contain significant energy. additional devices are used to provide energy dissipation. In the friction type systems, the maximum force transmitted to the superstructure is limited and energy dissipation is achieved through friction.

Rational techniques for determining the seismic response of lightweight equipment in fixed base structures has also been an area of significant recent research (Sackman and Kelly, 1979; Hernried and Sackman, 1984; Igusa and Der Kiureghian, 1985; and Singh and Suarez, 1987). Generally these methods only require the fixed base dynamic properties of the subsystems, the manner the subsystems are connected, and a pseudo-acceleration response spectrum associated with the excitation. The important case of tuning (when one natural frequency of each subsystem is equal or close to one another) and interaction effects between the equipment and the structure are considered in the proposed methodologies.

In this paper, the response of equipment in structures supported on a particular friction-type base isolation device (the resilient-friction base isolator, R-FBI) is considered. The R-FBI is composed of layers of teflon coated steel plates with a central rubber core to provide a restoring force (Mostaghel and Khodaverdian, 1987; Clark and Kelly, 1990). It is known that friction type isolation systems can generate high frequency effects in the superstructure which may be damaging to sensitive internal equipment. This issue will be specifically investigated. The subsystems of the equipment-structure-isolator system are considered to be SDOF models (Fig. 1). A semi-analytical procedure that avoids the necessity for a standard numerical integration of the combined systems equations is developed. The method is highly accurate and computationally efficient. Parameter studies which indicate the effect of friction coefficient, subsystem period, subsystem damping, and mass ratio on the response of the equipment when the R-FBI is subjected to several earthquake ground motions are presented and discussed.

2 GOVERNING EQUATIONS

The equations of motion governing the response of the superstructure (primary-secondary system) and the base isolator (see Fig. 1) are (Fan, Ahmadi and Tadjbakhsh, 1990)

$$\mathbf{m}\ddot{\mathbf{x}} + \mathbf{c}\dot{\mathbf{x}} + \mathbf{k}\mathbf{x} = -(\ddot{s} + \ddot{x}_g)\mathbf{m}\mathbf{r} \qquad (1)$$

$$\ddot{s} + 2\zeta_b\omega_b\dot{s} + \omega_b^2 s + \mu g\,\widehat{\mathrm{sgn}}(\dot{s}) + \sum_{i=1}^{2}\alpha_i\ddot{x}_i = -\ddot{x}_g \qquad (2)$$

where s is the relative displacement between the base of the structure and the ground, $\mathbf{x} = [x_i]$ where x_i is the relative displacement of the structure and the equipment with respect to the base, $\ddot{x}_g$ is the horizontal ground acceleration, μ is the friction coefficient, g is the acceleration of gravity, and $\mathbf{m} = [m_{ij}]$, $\mathbf{c} = [c_{ij}]$, $\mathbf{k} = [k_{ij}]$, i, j = 1, 2, represent the mass matrix, damping matrix, and stiffness matrix of the superstructure, respectively. In these equations, $\mathbf{r}$ is a unit vector, $\widehat{\mathrm{sgn}}(\dot{s})$ is a sign function which is equal to +1 when $\dot{s}$ is positive, and -1 when $\dot{s}$ is negative. The mass ratio α_i is defined as

$$\alpha_1 = \frac{m_s}{M}, \quad \alpha_2 = \frac{m_e}{M}, \quad M = m_e + m_s + m_b \quad (3)$$

where m_e, m_s, m_b represent the mass of equipment, structure, and base, respectively. The natural circular frequency of the base, and its effective damping ratio are defined as

$$\omega_b^2 = \frac{K}{M}, \quad \zeta_b = \frac{C}{2M\omega_b} \quad (4)$$

where C and K are the damping and the horizontal stiffness of the isolator.

Equations (1) and (2) are a set of coupled nonlinear differential equations. There are two phases of motion: a non-sliding phase and a sliding phase. In each phase the system is linear. The transition from sliding to non-sliding is determined numerically. In the sliding phase, equations (1) and (2) govern the motion of the superstructure and its isolator. When the system is not sliding,

$$\dot{s} = \ddot{s} = 0 \quad (5)$$

and the motion is governed by equation (1) with $\dot{s} = 0$ only.

In a non-sliding phase the frictional force is greater than the total inertia force generated from the superstructure-base mat system and therefore the friction plates are sticking to each other. The criteria for transition from a non-sliding phase to a sliding phase therefore depends on the relative values of the friction force and the inertia force of the superstructure-base mat system. Therefore, the non-sliding phase continues as long as

$$\left| \ddot{x}_g + \omega_b^2 s + \sum_{i=1}^{2} \alpha_i \ddot{x}_i \right| < \mu g \quad (6)$$

As soon as the condition,

$$\left| \ddot{x}_g + \omega_b^2 s + \sum_{i=1}^{2} \alpha_i \ddot{x}_i \right| = \mu g \quad (7)$$

is satisfied, sliding occurs and the direction of sliding at this moment of transition is given by

$$s\hat{g}n\,(\dot{s}) = -\frac{\ddot{x}_g + \omega_b^2 s + \sum_{i=1}^{2} \alpha_i \ddot{x}_i}{\mu g} \quad (8)$$

The sliding of the system in one direction may be terminated by a transition to a stick condition, or it may continue sliding in the reverse direction. During the sliding phase, whenever s becomes zero, the criterion,

$$|\ddot{s}| \geq 2\mu g \frac{M}{m_b} \quad (9)$$

is checked to determine the subsequent behavior. If equation (9) holds, the sliding continues with sgn(s) taking the sign of s at the moment of transition; otherwise, the stick condition occurs and equations (1) and (5) for a non-sliding phase control the motion.

The precise evaluation of the time of phase transitions is very important to the accuracy of the response. A successive time-step-cutting procedure is used to accurately determine each of these transitions. If the difference between the left and right hand side of equation (7) is less than 10^{-10}cm/sec^2, then equation (7) is assumed to be satisfied. Similarly, if $\dot{s} < 10^{-10}$cm/sec then it is assumed that $\dot{s} = 0$, and equation (9) is checked for the subsequent behavior. In this manner, the time of transition is accurately determined.

3 SOLUTION METHODOLOGY

In this section, the solution procedure used to analyze the equipment-structure-base isolator system is developed. Since the earthquake excitation is a known piecewise linear function and the system is linear in each phase, a closed form solution for the response in each digitized interval of ground motion can be obtained. The formulations for two phases (non-sliding and sliding) need to be considered separately.

3.1 Non-sliding phase

The governing equations for non-sliding phase are equations (1) and (5). The earthquake excitation is assumed to be of the form

$$\ddot{x}_{g\tau} = \ddot{x}_{gi} + a\tau \quad (10)$$

where $\ddot{x}_{g\tau}$ is the earthquake acceleration at time τ, $0 \leq \tau \leq t_{i+1} - t_i$, $\ddot{x}_{gi}$ is the earthquake acceleration at time t_i and a is the slope between two consecutive digitized points of the earthquake motion at times t_i and t_{i+1}. Introducing the excitation into equation (1) and changing the time variables, one obtains

$$\mathbf{m}\ddot{\mathbf{x}}(\tau) + \mathbf{c}\dot{\mathbf{x}}(\tau) + \mathbf{k}\mathbf{x}(\tau) = -\mathbf{mr}\,(\ddot{x}_{gi} + a\tau) \quad (11)$$

Taking the Laplace transform of both sides leads to

$$p^2\mathbf{m}\bar{\mathbf{x}} + p\mathbf{c}\bar{\mathbf{x}} + \mathbf{k}\bar{\mathbf{x}} - (\mathbf{c}+p\mathbf{m})\mathbf{x}^* - \mathbf{m}\dot{\mathbf{x}}^* = -\mathbf{mr}\left(\frac{\ddot{x}_{gi}}{p} + \frac{a}{p^2}\right) \quad (12)$$

where

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}, \qquad \mathbf{x}^* = \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix}, \qquad \dot{\mathbf{x}}^* = \begin{bmatrix} \dot{x}_{1i} \\ \dot{x}_{2i} \end{bmatrix} \quad (13)$$

$\bar{\mathbf{x}}$ is the Laplace transform of $\mathbf{x}$ and p is the Laplace variable, $\mathbf{x}^*$ and $\dot{\mathbf{x}}^*$ are the initial displacement and the initial velocity at time t_i. Solving equations (12) for the transform response leads to

$$\bar{x}_1 = \frac{A(m_e p^2 + c_{22}p + k_{22}) - B(c_{12}p + k_{12})}{D} \quad (14)$$

$$\bar{x}_2 = \frac{B(m_s p^2 + c_{11}p + k_{11}) - A(c_{21}p + k_{21})}{D} \quad (15)$$

where

$$A = [(m_s p + c_{11})x_{1i} + m_s\dot{x}_{1i} + c_{12}x_{2i}]p^2 - m_s(\ddot{x}_{gi}p + a) \quad (16)$$

$$B = [(m_e p + c_{22})x_{2i} + m_e\dot{x}_{2i} + c_{21}x_{1i}]p^2 - m_e(\ddot{x}_{gi}p + a) \quad (17)$$

$$D = p^2[(m_s p^2 + c_{11}p + k_{11})(m_e p^2 + c_{22}p + k_{22}) - (c_{12}p + k_{12})(c_{21}p + k_{21})] \quad (18)$$

The responses in Laplace domain are given by equations (14) and (15). In order to obtain the responses in the time domain, one must take the inverse Laplace transform of equations (14) and (15). Using the residue theorem one can write

$$x_1 = \sum \mathrm{Res}\,[\bar{x}_1(p)e^{p\tau}] = \sum_k R_{1k} \quad (19)$$

$$x_2 = \sum \text{Res}\,[\bar{x}_2(p)e^{pt}] = \sum_k R_{2k} \tag{20}$$

In order to evaluate the residues in equations (19) and (20), one must obtain the zeroes of the denominators in equations (14) and (15) - a sixth degree polynomial in the transform variable p. There are two zero roots. The remaining roots of the polynomial are determined numerically using the routine from IMSL library. It is noted that only five residues need to be evaluated (four for the distinct roots and one for repeated root). Therefore the summation in equations (19), (20) is from k=1 to 5. After the zeroes of the denominator are obtained, the residues are calculated as follows:

a) for the distinct roots p_k (k=1,2,3,4)

$$R_{1k} = \lim_{p\to p_k} (p-p_k)\frac{A(m_e p^2+c_{22}p+k_{22}) - B(c_{12}p+k_{12})}{m_s m_e p^2(p-p_1)(p-p_2)(p-p_3)(p-p_4)}e^{pt} \tag{21}$$

$$R_{2k} = \lim_{p\to p_k} (p-p_k)\frac{B(m_s p^2+c_{11}p+k_{11}) - A(c_{21}p+k_{21})}{m_s m_e p^2(p-p_1)(p-p_2)(p-p_3)(p-p_4)}e^{pt} \tag{22}$$

b) for the repeated root

$$R_{15} = \lim_{p\to p_5}\frac{d}{dp}[(p-p_5)^2\bar{x}_1(p)e^{pt}] \tag{23}$$

$$R_{25} = \lim_{p\to p_5}\frac{d}{dp}[(p-p_5)^2\bar{x}_2(p)e^{pt}] \tag{24}$$

The responses obtained from equations (19) and (20) are the relative displacements of the structure and the equipment. The relative velocity responses of the structure and equipment are calculated by

$$\dot{x}_1 = \sum \text{Res}\,[p\bar{x}_1(p)e^{pt}] = \sum_k R^*_{1k} \tag{25}$$

$$\dot{x}_2 = \sum \text{Res}\,[p\bar{x}_2(p)e^{pt}] = \sum_k R^*_{2k} \tag{26}$$

From equations (25) and (26), it is observed that there are only five distinct roots in the denominator. The residues are given by the following formulas

$$R^*_{1k} = \lim_{p\to p_k} (p-p_k)\frac{A(m_e p^2+c_{22}p+k_{22}) - B(c_{12}p+k_{12})}{m_s m_e(p-p_1)(p-p_2)(p-p_3)(p-p_4)(p-p_5)}e^{pt} \tag{27}$$

$$R^*_{2k} = \lim_{p\to p_k} (p-p_k)\frac{B(m_s p^2+c_{11}p+k_{11}) - A(c_{21}p+k_{21})}{m_s m_e(p-p_1)(p-p_2)(p-p_3)(p-p_4)(p-p_5)}e^{pt} \tag{28}$$

where k = 1, 2, 3, 4, 5.

The relative acceleration responses of the structure and equipment are given by

$$\ddot{x}_1 = \sum \text{Res}\,[p^2\bar{x}_1(p)e^{pt}] = \sum_k R^{**}_{1k} \tag{29}$$

$$\ddot{x}_2 = \sum \text{Res}\,[p^2\bar{x}_2(p)e^{pt}] = \sum_k R^{**}_{2k} \tag{30}$$

Since there are only four distinct roots in the denominator of equations (29) and (30), only four residues need to be calculated;

$$R^{**}_{1k} = \lim_{p\to p_k} (p-p_k)\frac{A(m_e p^2+c_{22}p+k_{22}) - B(c_{12}p+k_{12})}{m_s m_e(p-p_1)(p-p_2)(p-p_3)(p-p_4)}e^{pt} \tag{31}$$

$$R^{**}_{2k} = \lim_{p\to p_k} (p-p_k)\frac{B(m_s p^2+c_{11}p+k_{11}) - A(c_{21}p+k_{21})}{m_s m_e(p-p_1)(p-p_2)(p-p_3)(p-p_4)}e^{pt} \tag{32}$$

where k = 1, 2, 3, 4.

3.2 Sliding phase

The motion of the sliding phase is governed by equations (1) and (2). Introducing the earthquake excitation and taking the Laplace transform of both sides leads to

$$(p^2+2\zeta_b\omega_b p+\omega_b^2)\bar{s} + \alpha_1 p^2\bar{x}_1 + \alpha_2 p^2\bar{x}_2 = (p+2\zeta_b\omega_b)s_i$$
$$+ \dot{s}_i + \alpha_1(px_{1i}+\dot{x}_{1i}) + \alpha_2(px_{2i}+\dot{x}_{2i}) + \frac{F}{p} - \frac{\ddot{x}_{gi}p+a}{p^2} \tag{33}$$

$$p^2 m\bar{x}+pc\bar{x}+k\bar{x} - (c+pm)x^* - m\dot{x}^* = - mr\left(\frac{\ddot{x}_{gi}}{p} + \frac{a}{p^2}\right) \tag{34}$$

where $F = -\mu g\,\widehat{\text{sgn}}(\dot{s})$ and $\bar{s}$ is the Laplace transform of s. It should be noted that $\widehat{\text{sgn}}(\dot{s})$ doesn't change sign in the sliding phase. The initial displacement and the initial velocity of the base at time t_i are s_i and $\dot{s}_i$, respectively. Solving equations (33) and (34) leads to

$$\bar{s} = \frac{N_1}{D^*}, \quad \bar{x}_1 = \frac{N_2}{D^*}, \quad \bar{x}_2 = \frac{N_3}{D^*} \tag{35}$$

where

$$D^* = p^2\Big[(p^2+2\zeta_b\omega_b p+\omega_b^2)(m_s p^2+c_{11}p+k_{11})(m_e p^2+c_{22}p$$
$$+k_{22}) + \alpha_2 m_s p^4(c_{21}p+k_{21}) + \alpha_1 m_e p^4(c_{12}p+k_{12})$$
$$- \alpha_2 m_e p^4(m_s p^2+c_{11}p+k_{11}) - \alpha_1 m_s p^4(m_e p^2+c_{22}p$$
$$+k_{22}) - (c_{12}p+k_{12})(c_{21}p+k_{21})(p^2+2\zeta_b\omega_b p+\omega_b^2)\Big] \tag{36}$$

$$N_1 = \bar{A}^*\Big[(m_s p^2+c_{11}p+k_{11})(m_e p^2+c_{22}p+k_{22})$$
$$- (c_{21}p+k_{21})(c_{12}p+k_{12})\Big]$$
$$+ \bar{B}^* p^2\Big[\alpha_2(c_{21}p+k_{21}) - \alpha_1(m_e p^2+c_{22}p+k_{22})\Big]$$
$$+ \bar{C}^* p^2\Big[\alpha_1(c_{12}p+k_{12}) - \alpha_2(m_s p^2+c_{11}p+k_{11})\Big] \tag{37}$$

$$N_2 = \bar{A}^* p^2\Big[m_e(c_{12}p+k_{12}) - m_s(m_e p^2+c_{22}p+k_{22})\Big] +$$
$$\bar{B}^*\Big[(p^2+2\zeta_b\omega_b p+\omega_b^2)(m_e p^2+c_{22}p+k_{22}) - \alpha_2 m_e p^4\Big]$$
$$+ \bar{C}^*\Big[\alpha_2 m_s p^4 - (p^2+2\zeta_b\omega_b p+\omega_b^2)(c_{12}p+k_{12})\Big] \tag{38}$$

$$N_3 = \overline{A}^* p^2[m_s(c_{21}p+k_{21}) - m_e(m_s p^2+c_{11}p+k_{11})] \quad (39)$$

$$+ \overline{B}^*[\alpha_1 m_e p^4 - (p^2+2\zeta_b\omega_b p+\omega_b^2)(c_{21}p+k_{21})] +$$

$$\overline{C}^*[(p^2+2\zeta_b\omega_b p+\omega_b^2)(m_s p^2+c_{11}p+k_{11}) - \alpha_1 m_s p^4]$$

and

$$\overline{A}^* = [(p + 2\zeta_b\omega_b)s_i + \dot{s}_i + \alpha_1(px_{1i} + \dot{x}_{1i})$$

$$+ \alpha_2(px_{2i} + \dot{x}_{2i})]\, p^2 + Fp - (\ddot{x}_{gi}p + a) \quad (40)$$

$$\overline{B}^* = [m_s(p\, s_i + \dot{s}_i) + (m_s p + c_{11})x_{1i} + m_s\dot{x}_{1i}$$

$$+ c_{12}x_{2i}]\, p^2 - m_s(\ddot{x}_{gi}p + a) \quad (41)$$

$$\overline{C}^* = [m_e(p\, s_i + \dot{s}_i) + (m_e p + c_{22})x_{2i} + m_e\dot{x}_{2i}$$

$$+ c_{21}x_{1i}]\, p^2 - m_e(\ddot{x}_{gi}p + a) \quad (42)$$

Applying the residue theorem to equation (35), the relative displacements of the base, structure and equipment are given by

$$s = \sum \text{Res}\,[\overline{s}(p)e^{pt}] = \sum_k R_{sk} \quad (43)$$

$$x_1 = \sum \text{Res}\,[\overline{x}_1(p)e^{pt}] = \sum_k R_{1k} \quad (44)$$

$$x_2 = \sum \text{Res}\,[\overline{x}_2(p)e^{pt}] = \sum_k R_{2k} \quad (45)$$

The zeroes of the eighth degree polynomial [equation (36)] must be calculated to obtain the residues in equations (43), (44) and (45). There are six distinct roots and one repeated root and therefore seven residues need to be calculated. The residues are obtained by the following formulas:

a) for the distinct roots p_k (k=1,2,3,4,5,6)

$$R_{sk} = \lim_{p\to p_k} \frac{N_1(p - p_k)}{D^{**}}, \quad R_{1k} = \lim_{p\to p_k} \frac{N_2(p - p_k)}{D^{**}},$$

$$R_{2k} = \lim_{p\to p_k} \frac{N_3(p - p_k)}{D^{**}} \quad (46)$$

$$D^{**} = m_s m_e(1-\alpha_1-\alpha_2)p^2(p-p_1)(p-p_2)(p-p_3)(p-p_4)$$

$$(p-p_5)(p-p_6) \quad (47)$$

b) for the repeated root

$$R_{s7} = \lim_{p\to p_7} \frac{d}{dp}[(p - p_7)^2\overline{s}(p)e^{pt}] \quad (48)$$

$$R_{17} = \lim_{p\to p_7} \frac{d}{dp}[(p - p_7)^2\overline{x}_1(p)e^{pt}] \quad (49)$$

$$R_{27} = \lim_{p\to p_7} \frac{d}{dp}[(p - p_7)^2\overline{x}_2(p)e^{pt}] \quad (50)$$

Finally, in order to calculate the relative velocity and acceleration response, the same procedure used for the non-sliding phase is applied to the sliding phase.

4 NUMERICAL STUDIES

The semi-analytical solution procedure discussed in the previous section is used to determine the peak absolute acceleration response of equipment in structures supported on resilient-friction base isolators subjected to several earthquake ground motions. In the sections that follow, the effect on the equipment response of varying various physical quantities of interest is determined. In all instances, the time step used to evaluate the closed form results is either the earthquake digitization interval (0.02 secs) or the minimum period of the system divided by twenty. The equipment response to the first 30 seconds of the El Centro S00E record is presented in this paper. The response to other earthquake ground motions will be presented at the conference.

In order to investigate the effect of the stick-slip friction action on the response of the equipment, the Fourier transform of the acceleration response of the structure alone is computed and the equipment is tuned to peaks in this spectra.

4.1 Effect of friction coefficient

The effects of friction coefficient of the base isolator on the response of the equipment is shown in Fig. 2, where the friction coefficient μ varies from 0.01 to 0.1. The natural period of the structure (T_s) and base (T_b) are 0.3 secs and 4.0 secs. The damping ratios and masses of the equipment, structure and base are $\zeta_e = 0.02$, $\zeta_s = 0.02$, $\zeta_b = 0.08$; and $m_e = 0.01$ kg, $m_s = 1.0$ kg, $m_b = 1.0$ kg, respectively. For purposes of comparison, also given in the figure is the response of the equipment when the structure is fixed to the ground.

From Fig. 2, significant amplifications in equipment response occur when the structure is fixed to the ground and the equipment is tuned to the structure natural frequency ($T_e/T_s = 1.0$). A dominant peak in the Fourier spectra of the structure response occurs at the structure natural frequency, indicating that significant energy from the ground motion is transmitted to the natural frequency of the structure. When the natural frequency of the equipment is tuned to the natural frequency of the structure, a large amplification in equipment response occurs due to a resonant effect.

Fig. 2 indicates that for a wide range of frequencies (including the tuned case $T_e/T_s = 1.0$), the R-FBI effectively reduces the response of the equipment as compared to the fixed base system. The smaller the friction coefficient the greater the reduction in equipment response. For $T_e/T_s > 10.0$, there is relatively little change in the response of the equipment in the isolated system as compared to the fixed base case.

In order to illustrate the magnitude of the reduction in equipment response, consider a typical friction coefficient $\mu = 0.04$. In the high frequency region ($T_e/T_s \leq 0.3$), the normalized response is reduced by a factor of five. At tuning ($T_e/T_s = 1.0$), the normalized response is reduced by a factor of six.

4.2 Effect of damping

Fig. 3 illustrates the effect of equipment damping on the response of secondary systems in R-FBI supported structures subjected to the El Centro S00E record. The damping ratios of the structure and base are 0.02 and 0.08, while the damping ratio of the equipment varies from light damping of 0.02 to relatively heavy damping of 0.1. Base friction coefficients of 0.01 and 0.1 representing two extremes of small and relatively large friction coefficients are considered. In addition, the natural period of the structure and base are 0.3 sec and 4.0 sec respectively while the natural period of the equipment varies. The masses of the equipment, structure and base are 0.01 kg, 1.0 kg, and 1.0 kg, respectively.

It is observed that, for both friction coefficients, the acceleration response of the equipment is reduced as the equipment damping increases. In particular, resonant peaks are significantly reduced by increasing the equipment damping. This indicates that damping in the equipment is an effective mechanism for reducing peaks in equipment response resulting from either the stick-slip friction action or tuning. Equipment damping has no effect on the equipment response in the low period (high frequency) region ($T_e/T_s \leq 0.2$) for the particular system considered.

Fig. 4 illustrates the effect of structural damping on the equipment response for the El Centro S00E ground motion. The damping ratio of the equipment is fixed at 0.02 and the damping ratio of the structure varies from 0.02 to 0.1. The remaining properties of the equipment, structure and base are those described above.

The acceleration response of the equipment is significantly reduced by increasing the damping ratio of the structure for period ratios $T_e/T_s \leq 2.0$. For $T_e/T_s \geq 2.0$, there is no noticeable change in the equipment response as the damping in the isolated structure varies.

4.3 Effect of mass ratio

The effect of equipment to structure mass ratio on the acceleration response of the equipment is examined in Fig. 5. The system is subjected to the El Centro S00E ground motion and three equipment natural periods of 1.0 sec, 0.3 sec and 0.04 sec representing relatively low to high frequency equipment are used. The natural period of the structure and base are 0.3 sec and 4.0 sec, respectively. The damping of the equipment, structure and base are 0.02, 0.02 and 0.08, respectively. In addition, two friction coefficients 0.01 and 0.10 are considered.

It is observed that for $T_e = 1.0$ and $T_e = 0.04$ (equipment natural frequency detuned from the natural frequency of either the structure or the base), and for both small and large friction coefficients, the mass ratio has no noticeable effect on the equipment response.

For $T_e = 0.3$ (equipment natural frequency tuned to the structure natural frequency) and a small friction coefficient ($\mu = 0.01$), varying the mass ratio has no effect, in general, on the equipment response (Fig. 5). For $T_e = 0.3$ and a relatively large friction coefficient ($\mu = 0.1$), the equipment response increases as the mass ratio decreases. From previous research on fixed base structures, when the system is tuned, the amplification in equipment response depends indirectly on the mass ratio (as the mass ratio decreases, the equipment response increases). The equipment-structure-R-FBI system would behave like a fixed base equipment-structure system if the friction coefficient in the R-FBI were infinitely large. For finite but relatively large values of the friction coefficient, the system behaves more like a fixed base system than if the friction coefficient is small. This explains the amplifications in equipment response when $\mu = 0.1$ that are absent when $\mu = 0.01$.

5 CONCLUSIONS

The dynamic response of equipment in structures supported on resilient -friction-base isolators (R-FBI) subjected to the El Centro S00E ground motion is investigated. The model considered is a SDOF equipment item attached to a SDOF structure which is attached to a base mat supported on the R-FBI. An efficient semi-analytical solution procedure for the determination of the equipment response is developed. Closed form expressions for the response in each phase, sliding and non-sliding, are derived. The expressions in each phase are exact, thus eliminating any numerical difficulties associated with a standard numerical integration scheme. The transition from sliding to non-sliding is determined numerically. A series of parametric studies, using the developed methodology, is performed to examine the effects of friction coefficient, equipment and structure damping as well as equipment to structure mass ratio on the peak response of the equipment. Based on the present results, the following conclusions may be drawn:

1. The acceleration response of the equipment in structures subjected to earthquake excitations is, in general, effectively reduced when the structure is supported by resilient-friction base isolators.
2. Varying the friction coefficient of the R-FBI affects the equipment response. In general, the smaller the friction coefficient, the greater the reduction in the response of the equipment.
3. Equipment and structure damping effectively reduce resonant effects. The response of the equipment is reduced as the damping of the equipment and/or the structure increases.
4. When the equipment natural frequency is tuned to the structural frequency, the equipment response increases as the mass ratio decreases for large friction coefficients. When the friction coefficient is small, however, there is no noticeable amplification in the equipment response as the mass ratio decreases. In addition, when the natural frequencies of the equipment and the structure are detuned, the mass ratio has no noticeable effect on the equipment response; in either isolated or fixed base structures.

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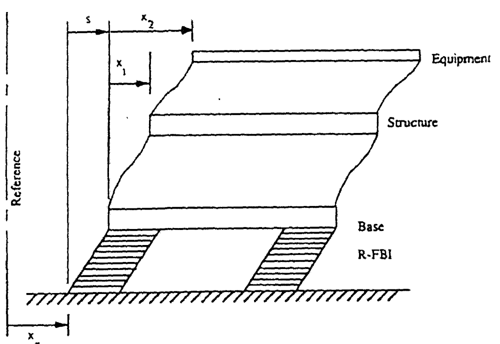

Figure 1. Equipment-structure system supported on resilient-friction base isolator subjected to earthquake excitation.

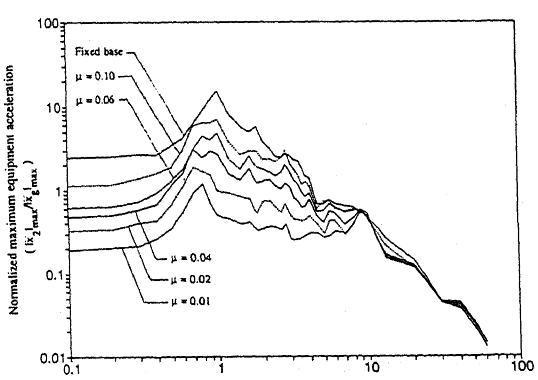

Figure 2. Effect of friction coefficient on equipment response.

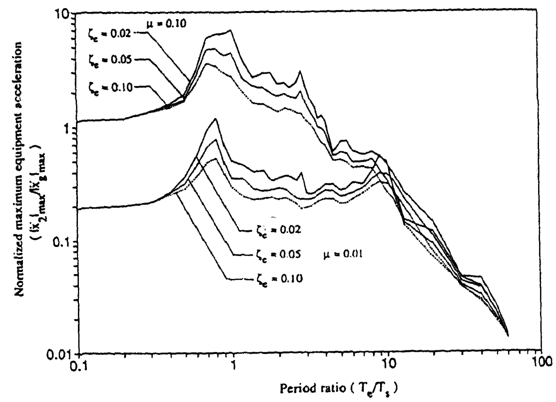

Figure 3. Effect of equipment damping on equipment response.

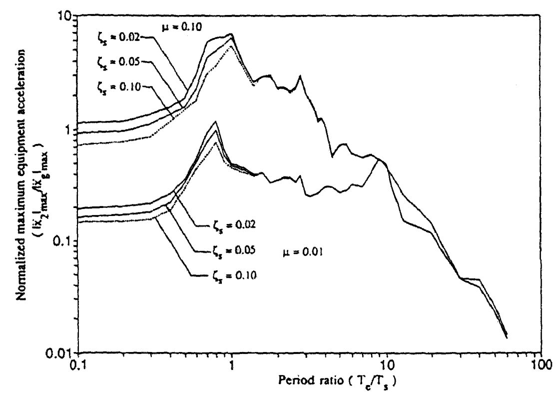

Figure 4. Effect of structure damping on equipment response.

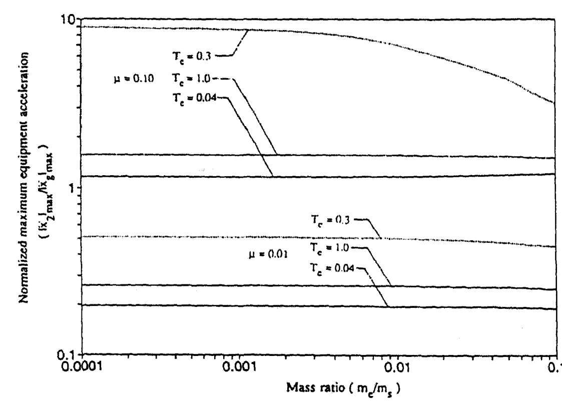

Figure 5. Effect of mass ratio on equipment response.