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Mathematical modelling of non-linear viscous damping of dashpots used in vibration base isolation

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ABSTRACT: A three-bay five-storey steel frame model, base isolated with spring and dashpot elements, was intensively tested on the biaxial shaking table at the Dynamic Testing Laboratory of the University in Skopje. According to the experimental results, it was discovered that the viscous damping of the dashpots was continuously decreasing by increasing the number and intensity of the testing. The kinetic energy, which is transmitted through the pistons to the viscous media, is converted into thermal energy by which the viscosity is decreased.

1 INTRODUCTION

Base isolation is an antiseismic strategy by means of which damaging earthquake motion reduces structural responses through a mechanism built-into the structural system. Such a system, consisting of helical springs and viscous dashpots, is developed by the GERB Company in Germany. For the needs of determination of its effectiveness, intensive experimental research has been performed at the Dynamic Testing Laboratory of the Institute of Earthquake Engineering and Engineering Seismology, University of Skopje, the Republic of Macedonia. A five-storey three-bay steel frame structural model, isolated by GERB vibration base isolation elements, was intensively tested applying a set of different earthquake motions on a biaxial seismic shaking table. During the testing it was observed that the viscous damping of the dashpots was continuously decreasing. Namely, with the increase of the number of oscillations of the cylinders into the viscous mass, increased the temperature of the fluid and, consequently, decreased the damping coefficients.

On the basis of this experimental conclusion, research was performed (Jovanovic) in order to determine the relationship of the change in the damping depending on the path passed by the cylinder. It means, physically, that the kinetic energy of the cylinder, when it moves in a viscous medium, turns into thermal one. With the increase of the fluid temperature, decreases its viscosity, which in turn causes decrease of the damping capacity. To determine this relationship, experimental results obtained by seismic shaking table testing and the technique of parameter system identification have been applied.

2 EXPERIMENTAL RESULTS

In Fig. 1 the test model is presented. It is a five-storey three-bay steel moment resistant frame, mounted on two heavy girders, supported by four sets of spring-dashpot elements manufactured by the GERB Company, for simulation of a base isolated model. The dead load is provided by steel blocks, tied down to the frame, thus simulating a mass of 4700 kg at each floor level. The total mass of the dead load is approximately 23.5 tons, while the mass of the steel frame is 2.3 tons. Additionally, 6.0 tons of mass are added at the base level, thus a comprehensive force of approximately 80 kN is produced in each of the springs. The dead load, provided by the steel blocks, produced stress levels proportional to those in a full-scale structure. The length scale factor is 1/4 while the time scale is $1/\sqrt{4}$.

The position of the set, consisting of a spring and dashpot elements, as a detail, is shown in Fig. 2. The dashpot axis and the column axis are overlapping. The spring and dashpots are designed for the real dead load of the model, which is approximately 80 kN.

The experimental programme was planned in a way to ensure collection of maximum useful experimental data. So, the displacement and acceleration time histories were recorded for various sets of earthquakes of different excitation levels at each floor level. Besides strain measurements of the external columns, the displacement time histories were recorded in vertical direction at the point between the spring and the dashpot, relatively to the shaking table. Also, the frequency response curves were obtained after the application of various excitation levels, generated on the shaking table, using forced or ambient vibration measurements.

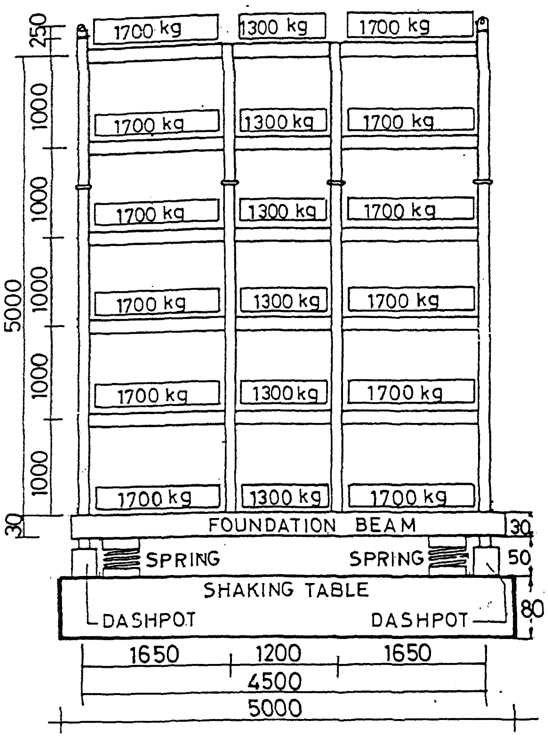

Figure 1. Structural model on the shaking table with vibration base isolation elements

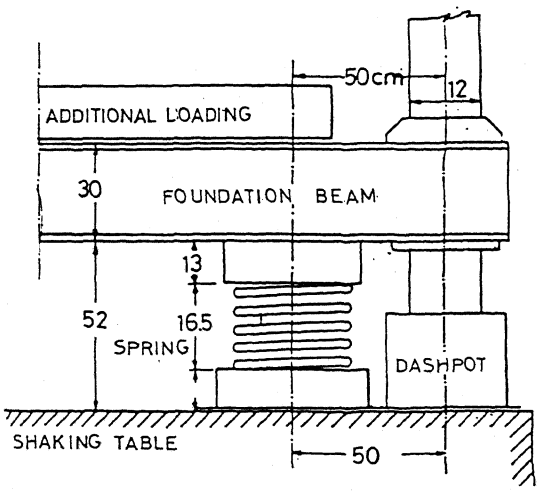

Figure 2. Detail of spring and dashpot element

3 MATHEMATICAL MODELLING

The considered structure was in elastic range for all the simulated earthquakes. The supporting spring remained, also, in elastic range, while non-linearity was observed only in the behaviour of the dashpots. According to these experimental results and using the available test data, two models (O. Jovanovic) have been considered. The first model (Fig. 3) have the super-structure idealized by 72 degrees of freedom, and masses concentrated at the joints. Each mass has three degrees of freedom, two translations and one rotation. The model is, basically, supported by a system of springs and nonlinear viscous dashpots.

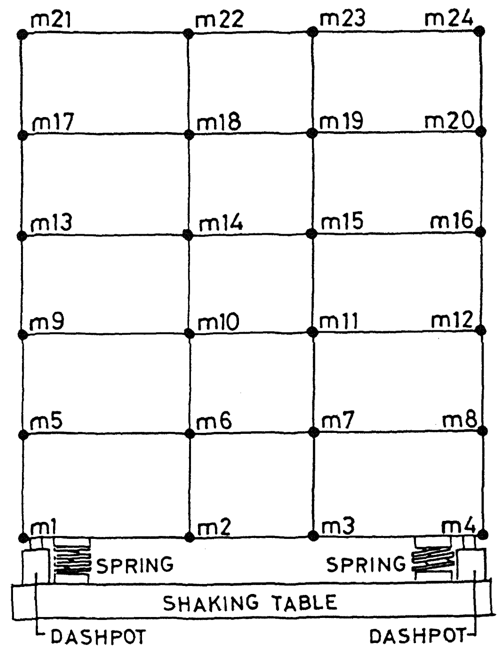

Figure 3. Mathematical model of the upper structure with vibration base isolation elements

The second model (Fig. 4) has the supper-structure idealized as a rigid body which is elastically supported by elastic springs and nonlinear viscous dashpots. The analysis showed that the simplified model with only three degrees of freedom, two translations and one rotation provides very good results.

Having in mind that, in the considered case, the mechanical properties of the dashpots, having non-linear relationship, are not known, the strategy of the formulation of the mathematical model for the given physical problem was directed towards the identification of the change in the viscous damping of the dashpots depending on the temperature. Literature offers many relationships experimentally defined as different empirical and theoretical models, however they always apply to known fluids. Thus, Fig. 5 shows the relationship of the viscosity of different industrial oils and their temperature. It is obvious from the same figure that for the measured temperature interval the decrease in the viscosity is of very high order.

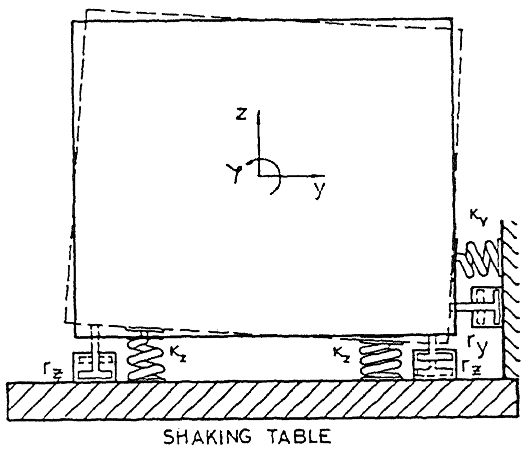

Figure 4. Simplified mathematical model of the upper structure as a rigid body

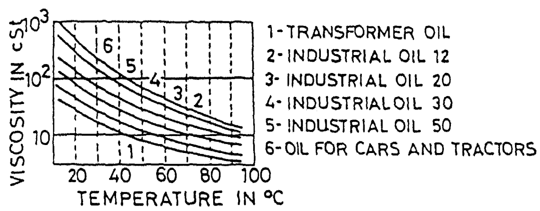

Figure 5. Relationship between viscosity and temperature for different oil

The viscous material of the GERB dashpots, in this case as a physical medium, is considered to have unknown properties. Therefore, the technique, applied for formulation of the mathematical model, is based on the parameter system identification.

In a relative coordinate system, the differential equations of the dynamic behaviour of the physical system in a matrix form are expressed in Eq. 1, as:

$$[M]\{\ddot{u}(\beta)\}+[R(\dot{u})]\{\dot{u}(\beta)\}+[K]\{u(\beta)\}=-[M]\{\ddot{n}(t)\} \quad (1)$$

with:

$$[M]=\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_c \end{bmatrix} \text{ the mass matrix} \quad (2)$$

$$[R]=\begin{bmatrix} r_z & 0 & r_{z\varphi} \\ 0 & r_y & r_{y\varphi} \\ r_{\varphi z} & r_{\varphi y} & r_\varphi \end{bmatrix} \text{ the damping matrix} \quad (3)$$

$$[K]=\begin{bmatrix} k_z & 0 & k_{z\varphi} \\ 0 & k_y & k_{y\varphi} \\ k_{\varphi z} & k_{\varphi y} & k_\varphi \end{bmatrix} \text{ the stiffness matrix} \quad (4)$$

The vector of motion $\{u(\beta)\}=\{z(\beta), y(\beta), \varphi(\beta)\}$ indicates the total motion for the two displacement degrees of freedom and one rotational degree of freedom for the mathematical model. Vectors $\{\ddot{u}(\beta)\}$ and $\{\dot{u}(\beta)\}$ are the vectors of acceleration and velocity as a function of an unknown vector. In accordance with the vector of excitation $\{\ddot{n}(t)\}=\{0,\ddot{u}(t),0\}$ only horizontal acceleration was used.

The damping matrix is versus of the dimensions of the system and ratio of the vertical damping. Damping is given through three unknown parameters in the function of the path of the body in the cylinder.

$$r_z = b + (a-b)\exp\left(-c^{-1}\int |v|dt\right) \quad (5)$$

$\{\beta\} = \{a,b,c\}$ - vector of unknown parameters.

A number of experiments have shown that the damping caused by constant displacement of the body in the cylinder decreases in the physically allowable limits, which brings to the conclusion that it is a question of an exponentially decreasing function which is the reason why the damping is expressed through the functions, as shown in Fig. 6. The unknown parameters (a, b, c) will be determined by the parameter system identification procedure.

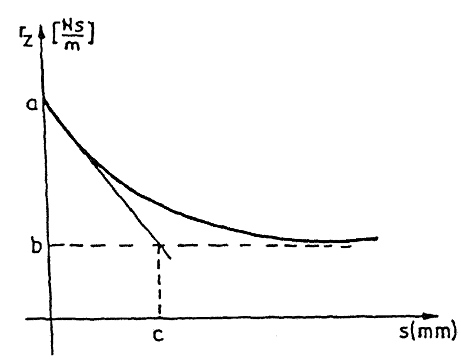

Figure 6. Relationship between damping and the passed path

Unknown parameters a and b show initial and ultimate value of the damping ratio of the material in the cylinders and are of a dimension [Ns/m].

The next step is the selection of a criterion function or the error function. It is expressed as the integrated least squares error between the theoretical and experimental displacement time histories:

$$J(\beta) = \int_{o}^{T} W_k \left[u(\beta,t) - u(t) \right] \left[u(\beta,t) - u(t) \right]^T dt \tag{6}$$

where T are time intervals defining the sensitive part of the model responses, while $u(\beta,t)$ is the vector of displacement obtained from the mathematical model, and $u(t)$ is the vector of displacement experimentally recorded from the physical variable influences. W_k has the value of one, since the error function is formed only by the displacement.

The third step of the identification is very important and it represents selection of the algorithm for parameters adjustment to minimize the criterion function (6). There is a large number of methods in the mathematical optimization theory which can be used in the identification process. Most of them are based on the iterative technique.

The gradient methods are the most suitable for determination of the function minimum in a multidimensional space.

4 DISCUSSION OF THE RESULTS

The algorithm for parameters adjustment is based on the modified Gauss-Newton method which usually provides convergence. Using this developed algorithm at first the model presented in Fig. 4 was analysed.

Following the parameter system identification technology for the given mathematical model and developed algorithm for parameters adjustment, it was necessary that the initial values of the parameters *a*, *b* and *c* be estimated. Using the empirical relations, the initial values were estimated to be

$$\{\beta_o\} = \{57000;40000;6\}$$

After the forth iteration, vector $\{\beta\}$ has the following values:

$$\{\beta_4\} = \{93000;63000;5.8\}$$

Using the above identified values of $\{\beta\}$, and given earthquake time histories, the structural responses were calculated. In Fig. 7, analytical and experimental displacement time histories calculated from Eq. (1) and experimentally measured at the center of mass for the considered model are presented for El Centro 1940, N-S component, simulated on the shaking table with SPAN 600 (PGA on the shaking table 0.42 g). From that figure, visually can be seen a good correlation between the experimental and analytical time histories. For that case, the error function is 3.59 cm^2 for time duration of 18.0 s. Similar to this, in Fig. 8, the experimental and analytical displacement time histories are presented for Petrovac earthquake 1979, N-S component, simulated on the shaking table with SPAN 500 (PGA on the shaking table 0.51 g). For this case, the error function is 2.16 cm^2.

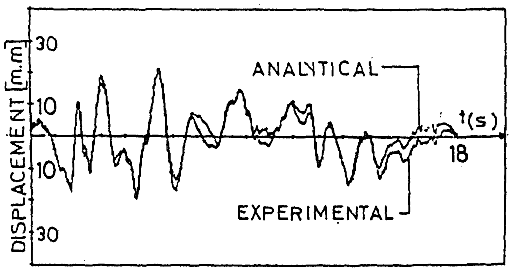

Figure 7. Analytical and experimental displacement time histories at the center of the model of Fig. 5 for El Centro 1940 N-S component earthquake SPAN 600

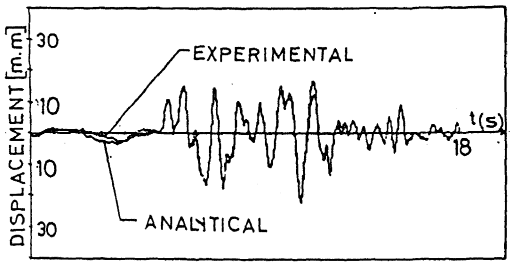

Figure 8. Analytical and experimental displacement time histories at the center of the model of Fig. 5 for Petrovac 1979 N-S component earthquake SPAN 600

The same analyses have been performed for the model presented in Fig. 3, and the correlation of the analytical and experimental displacement time histories on the fifth floor for both earthquakes (El Centro 1940, and Petrovac 1979) is shown in Fig. 9 and Fig. 10. For the second model, which is more complex than the previous one, the error function calculated for one storey is less than the error function calculated for the simplest model, but the effort made for the second model is much higher compared to the effort made for the analysis of the first model.

It is difficult to give a correct answer to the question which model to be applied in the analysis or design of a real structure, although by the system identification technique it is possible to quantify some answers. But, practically speaking, it seems that the physics of the problem should be of primary importance for decision making on which model should be applied.

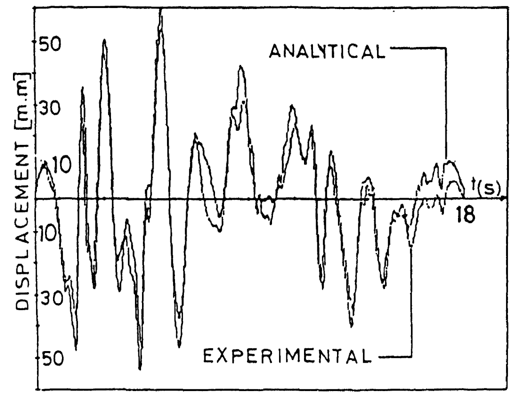

Figure 9. Analytical and experimental displacement time histories at the fifth floor, El Centro earthquake 1940 N-S component SPAN 600

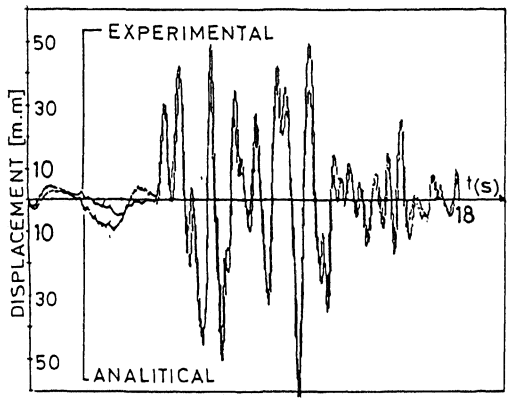

Figure 10. Analytical and experimental displacement time histories at the fifth floor, Petrovac 1979 earthquake N-S component SPAN 500

5 CONSLUSION

Helical springs and dashpots are durable and less sensitive to ambient temperature changes and to physical conditions of the air. The viscous dashpots are sensitive to the value of converted kinetic energy into the thermal, by which the viscosity of the fluid is reducing. The reduction of the viscosity or damping capacity of the dashpots for the considered case is in the order of 30-40%, which mainly depends on the passed path of the cylinder moving into the fluid of the dashpots.

REFERENCES

Bekey, A.G. 1970. System identification - an introduction and survey. *Simulation*, Vol. 4: 151-166

Distefano, N. and B. Pena-Prado 1975. System identification of frames under seismic loads, Meeting preprints 2446, ASE National structural engineering convention, New Orleans, Louisiana

Jovanovic, O. 1988. *Mathematical modelling of steel frame structure base isolated using parameter system identification* (original in Serbo-Croat), Ph.D., Institute of Earthquake Engineering and Engineering Seismology, University "Kiril and Metodij", Skopje

Jovanovic, O. and D. Jurukovski 1989. Parameter identification in mathematical modelling of steel frame with base isolation, *Theoretical and Applied Mechanics 15*: 45-53.

Jurukovski, D. 1980. Mathematical model formulation of a two-storey steel frame structure using parameter system identification and shaking table experiments, 7WCEE, Istanbul

Jurukovski, D. 1987. *Mathematical modelling of system identification technique. Engineering aspects of earthquake phenomena.* Omega Scientific, England

Jurukovski, D. et al. 1984. Shaking table test of a five-storey steel frame isolated by GERB vibration isolation elements. Report IZIIS 84/128. University of Skopje

Jurukovski, D. and O. Jovanovic 1990. System identification as a tool for mathematical modelling in earthquake engineering, *Structural dynamics*, Kratzig et al. (eds.), Rotterdam: Balkema