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A multiple shear mechanism model for sand

S. Iai, Y. Matsunaga, T. Kameoka & T. Inatomi
Port and Harbour Research Institute, Ministry of Transport, Japan

ABSTRACT: The model proposed in this paper is of a strain space plasticity - multiple mechanism type. Each shear mechanism consists of the partial stress contribution from a class of contact forces within an assembly of sand particles. The model has a reasonable applicability in the analysis of anisotropically consolidated sand. Analyses of embankments, quay walls and buried structures indicate that the model can be a useful tool in practice.

1 INTRODUCTION

Many approaches have been proposed for modeling of cyclic behavior of sand but, up to the present, none of the proposals seem to be so successful as to be readily applicable in practice of soil dynamics and earthquake engineering. The main problems widely seen in the previous studies are (1) inadequate modeling of initially anisotropically consolidated soil and (2) numerical vulnerability associated with the stress path in the vicinity of the shear failure line when liquefaction and cyclic mobility of sand are of major concern.

In order to solve these problems, the first author has proposed a set of constitutive equations within the framework of plasticity theory defined in the strain space with the concept of multiple mechanism (Iai, 1991). This paper discusses the physical background of the model and the capability of the model in the practice of earthquake engineering.

2 REVIEW ON THE STRAIN SPACE MULTIPLE MECHANISM MODEL

The model as presented previously for the plane strain deformation of sand (Iai, 1991) was given as a relationship between the effective stress and strain defined in terms of such vectors as

$$\{\sigma'\}^T = \{\ \sigma_x' \quad \sigma_y' \quad \tau_{xy}\ \} \tag{1}$$
$$\{\epsilon\}^T = \{\ \epsilon_x \quad \epsilon_y \quad \gamma_{xy}\} \tag{2}$$

in which compressive stress and contractive strain are assumed negative and γ_{xy} is the shear strain in the engineering definition. The constitutive relation was given by

$$\{d\sigma'\} = K\{n^{(0)}\}\{n^{(0)}\}^T(\{d\epsilon\} - \{d\epsilon_p\}) + \sum_{i=1}^{I} R_{L/U}^{(i)}\{n^{(i)}\}\{n^{(i)}\}^T\{d\epsilon\} \tag{3}$$

in which volumetric strain increment $\{d\epsilon_p\}$ due to dilatancy was given by

$$\{d\epsilon_p\}^T = d\{\epsilon_p/2 \quad \epsilon_p/2 \quad 0\} \tag{4}$$

and the direction vectors were given by

$$\{n^{(0)}\}^T = \{\ 1 \quad 1 \quad 0\ \} \tag{5}$$
$$\{n^{(i)}\}^T = \{\ \cos\theta_i \quad -\cos\theta_i \quad \sin\theta_i\ \} \quad (\text{for } i = 1,\ldots, I)$$

with the angle θ_i given by

$$\theta_i = (i-1)\Delta\theta \quad (\text{for } i = 1,\ldots,I) \tag{6}$$

in which $\Delta\theta = \pi/I$.

The first term in Eq.(3) represents the volumetric mechanism with the rebound modulus K. The second term represents the shear mechanism with the tangent shear moduli $R_{L/U}^{(i)}$ specified for virtual simple shear mechanisms $i = 1,\ldots,I$. The modulus $R_{L/U}^{(i)}$ is defined as a function of the present value and the history of the virtual shear strain given by

$$\gamma^{(i)} = \{n^{(i)}\}^T\{\epsilon\} \tag{7}$$

The loading and unloading of shear

mechanism, indicated by the subscript L/U for the modulus $R^{(i)}$, are separately defined for each mechanism by the sign of $d\gamma^{(i)}$.

The modulus $R_{L/U}^{(i)}$ is defined so that each virtual simple shear mechanism follows the hyperbolic stress-strain relation with the hysteresis given by a rule similar to Masing's rule.

The volumetric strain representing the dilatancy ϵ_p is defined as a function of cumulative shear work and the deviatoric stress ratio.

The model has 10 parameters; two of which specify elastic properties of soil, other two specify plastic shear behavior, and the rest specify dilatancy. More details can be found in a paper by Iai (1991).

3 PHYSICAL BACKGROUND OF THE STRAIN SPACE MULTIPLE SHEAR MECHANISM MODEL

A physical background for the multiple shear mechanism may be given as follows. Let us begin by regarding sand as an assembly of sand particles. Stress in sand as defined for continuum will then be given as a certain average of contact forces between the sand particles. Before taking the average over all the contact forces in a representative volume element, let us classify the contact forces according to the direction. Let us think of a class of contact forces of which direction is at an angle $\theta_i/2$ relative to the x axis and take the appropriate average to form a partial contribution to the stress. The stress can then be given by combining all the partial stress contributions over the entire angles of θ_i.

When deformation is induced in the sand, the partial stress contribution will be likely to change its magnitude in accordance with the change in a representative length along the direction of the contact forces. Let us take a normal strain component along the direction of the contact forces as a measure of the change in the representative length.

Let us think of a pair of the partial stress contributions associated with the contact forces which are at right angle. Let us also think of a pair of normal strain components along the directions of those contact forces. If the volumetric components are removed from the pairs of the stress contributions and the strain components, the relation between these pairs will be very similar to the stress strain

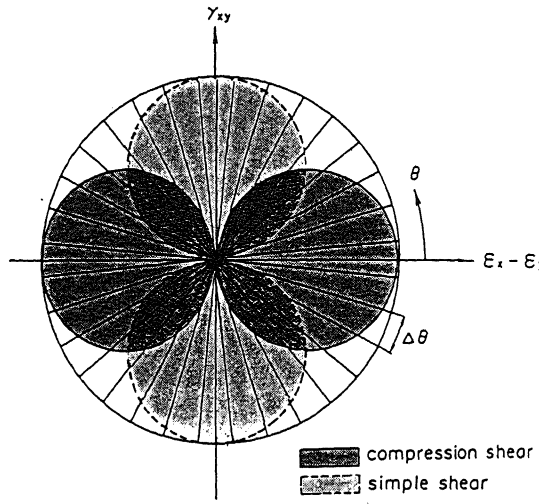


Figure 1. Multiple shear mechanism (after Iai, 1991)

relation in compression shear. If the angle is rotated in $\pi/4$, the compression shear becomes simple shear. Thus the relation between those pairs represents the virtual simple shear mechanism in the present model.

According to the multiple shear mechanism, actual simple shear mobilizes most of the virtual simple shear mechanisms but the extent of mobilization represented by the value of $\gamma^{(i)}$ depends on the direction of each mechanism; for example, a compression shear along the y axis (i.e. $(\epsilon_x-\epsilon_y) > 0$ and $\gamma_{xy} = 0$) mobilizes the virtual simple shear strains (i.e. $\gamma^{(i)} = (\epsilon_x-\epsilon_y)\cos\theta_i$), being indicated by a circle with darker hatching on the right hand side in Fig. 1, in which each strip represents each virtual simple shear mechanism. Extension shear similarly mobilizes the ones indicated by a circle with darker hatching on the left hand side whereas simple shear does the ones indicated by either of circles with lighter hatching shown in the same figure.

4 UNDRAINED CYCLIC BEHAVIOR OF SAND UNDER ANISOTROPIC CONSOLIDATION

Two types of cyclic behavior of sand have been identified through the laboratory studies on undrained cyclic simple shearing of anisotropically consolidated sand. When the lateral normal strain of sand specimen is constrained during cyclic shearing, behavior of sand is not affected by the anisotropic consolidation condition provided that initial effective mean stress is kept the same (Ishihara et al, 1977). When lateral normal strain is not constrained but axial stress difference is kept the same, axial strain difference gradually becomes large, causing settlement and lateral bulging in the sand specimen (Ishihara and Li, 1972).

This behavior of sand, though commonly encountered in earthquake engineering, was not easy to analyze with the conventional plasticity models because it involves effects of rotation of principal stress axis directions. The present model offers a prospect for solving the difficulty as shown below.

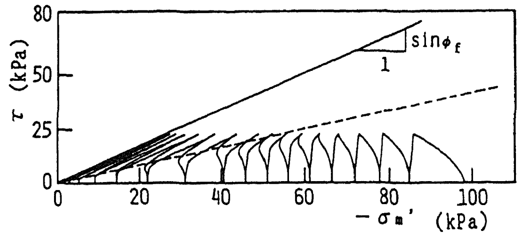
4.1 Analysis with constraining lateral normal strain

First of all, the soil behavior under undrained cyclic simple shearing is analyzed with constraining lateral normal strain of soil as in the level ground. The parameters for the loose sand shown in Table 1, being the same as those used in the previous study for analyzing isotropically consolidated sand (Iai, 1991), are used in the computation with single element of finite element approximation. The soil is initially consolidated at the earth pressure coefficient of K_0 = 0.5. The results are shown together with those for isotropic condition.

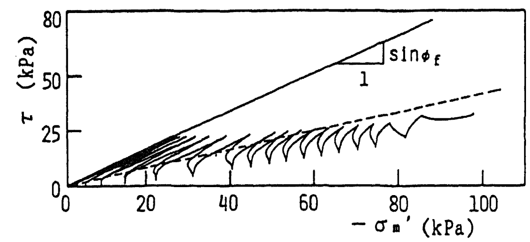
The computed stress path, shown in Fig.2 plotted as a relation between deviatoric stress $\tau = \sqrt{[(\sigma_x'-\sigma_y')/2]^2 + \tau_{xy}^2}$ and effective mean stress ($-\sigma_m'$), indicates that the initial deviatoric stress due to K_0 consolidation is gradually released as the cyclic loading goes on. This is consistent

Table 1. Parameters of sand used for the analysis

Parameters	Loose Sand (Dr = 47%)	Dense Sand (Dr = 75%)
G_{ma} (kPa)	103700	140700
K_a (kPa)	270500	366800
$-\sigma_{ma}'$ (kPa)	98	98
$\sin\Phi_f$	0.87	0.91
$\sin\Phi_p$	0.42	0.42
p_1	0.45	0.40
p_2	1.4	0.72
w_1	2.0	2.85
S_1	0.0035	0.005
c_1	1.0	1.0

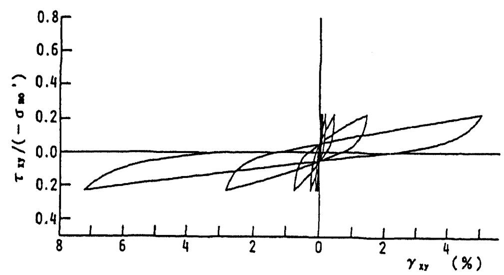


(a) Initially K_0 = 1.0

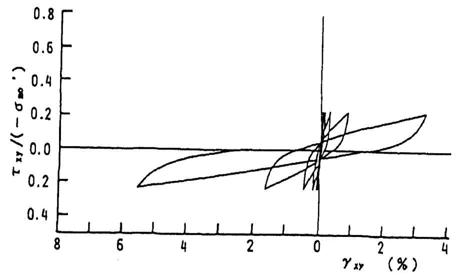


(b) Initially K_0 = 0.5

Figure 2. Stress path when lateral normal strain is constrained



(a) Initially K_0 = 1.0



(b) Initially K_0 = 0.5

Figure 3. Stress strain curve when lateral normal strain is constrained

with the laboratory findings by Ishihara and Li (1972).

The stress strain relation with respect to τ_{xy} and γ_{xy} in the initially anisotropically consolidated sand, as shown in Fig.3, is quite similar to those in the initially isotropically consolidated sand.

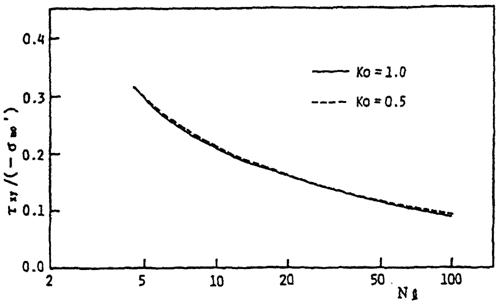


Figure 4. Liquefaction resistance of isotropically and anisotropically consolidated sands

If the shear stress ratio, i.e. the ratio of the cyclic shear stress over the initial confining pressure, is plotted, as shown in Fig.4, with number of the cyclic loading required to cause shear strain of 5 percent in the double amplitude, the computed result for the isotropically consolidated sand agrees with that for the K_0 consolidated sand. This is consistent with the results identified through the laboratory study by Ishihara et al (1977).

The mechanism involved in this type of soil behavior is understood as follows. In the present approach, effect of anisotropic consolidation is to induce such a compression shear as those indicated by the right hand half of the small circles shown by darker hatching in Fig. 1; i.e. this type of shear mainly affects the virtual simple shear mechanisms oriented to or close to $\pi/4$ direction relative to x axis. It does not significantly affect the virtual simple shear mechanisms oriented to or close to x axis. The cyclic simple shear induced after the initial anisotropic consolidation involves only those mechanisms indicated by the broken lines with lighter hatching in Fig. 1. Thus, the virtual simple shear mechanisms mobilized during the cyclic simple shear are mainly those which were unaffected by the initial anisotropic consolidation. This naturally explains that the behavior of sand under cyclic simple shear loading is not affected by the initial anisotropic consolidation condition if the initial mean effective stress remains the same.

4.2 Analysis without constraint on normal strain

When two dimensional analysis is

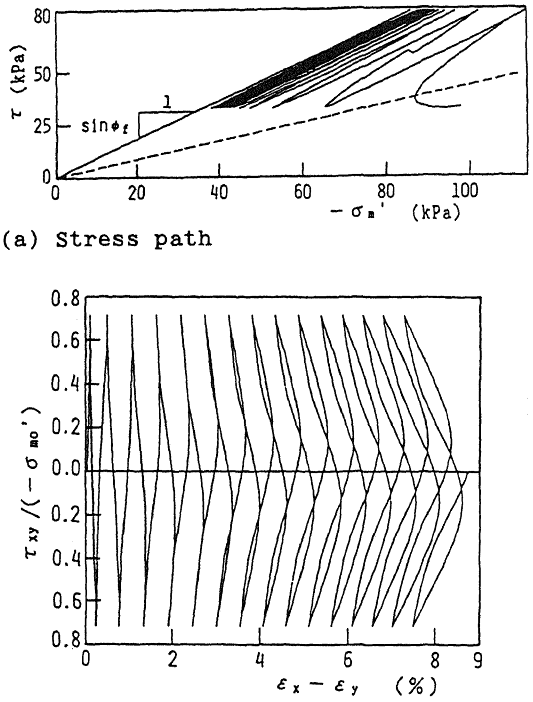


Figure 5. Computed results without constraining lateral normal strain at K_0 = 0.5

conducted for such soil structures as embankments, initial stress due to gravity will act as a driving force for gradual settlement and lateral bulging. In order to examine the ability to analyze this type of behavior, undrained cyclic simple shearing of initially anisotropically consolidated sand is analyzed with keeping the initial axial stress difference unchanged as the stress boundary condition. The same parameters for the dense sand and the same initial confining pressure and the same shear stress as used in the previous study (Iai, 1991), shown in Table 1, are used in the computation.

The computed stress path in τ - ($-\sigma_m'$) plane, shown in Fig.5 (a), indicates gradual approach of effective stress path toward the failure line. In accordance with this, the axial strain difference ($\epsilon_x-\epsilon_y$), shown in Fig.5 (b), exhibits cumulative increase. These results are consistent with the previous laboratory study by Ishihara and Li (1972), suggesting reasonable applicability of the present model in the two dimensional analysis.

In the present approach, the

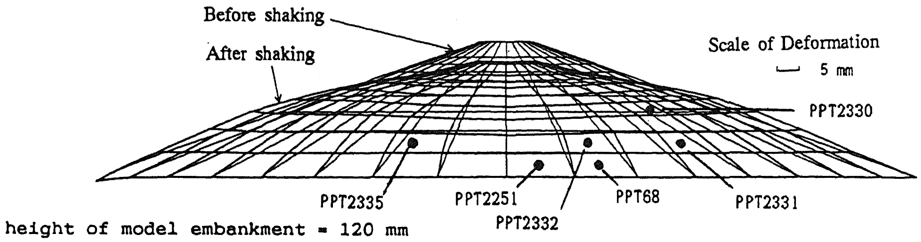


Figure 6. Computed deformation of model embankment in centrifuge

mechanism involved in this type of soil behavior is understood as follows. As mentioned earlier, the effect of the anisotropic consolidation is to induce such compression shear as indicated by the right hand half of the small circles shown by the solid line with darker hatching in Fig. 1. The cyclic torsion shear further mobilizes those virtual simple shear strains indicated by the broken lines with lighter hatching in Fig. 1. While the virtual simple shear strains indicated by the broken lines with lighter hatching repeat the change from positive to negative direction and vice versa, the virtual simple shear strains indicated by the solid line with darker hatching grows toward the positive direction, resulting in the lateral bulging with the settlement of the soil element. The mechanism for inducing cumulative axial strain difference is thus very easy to understand.

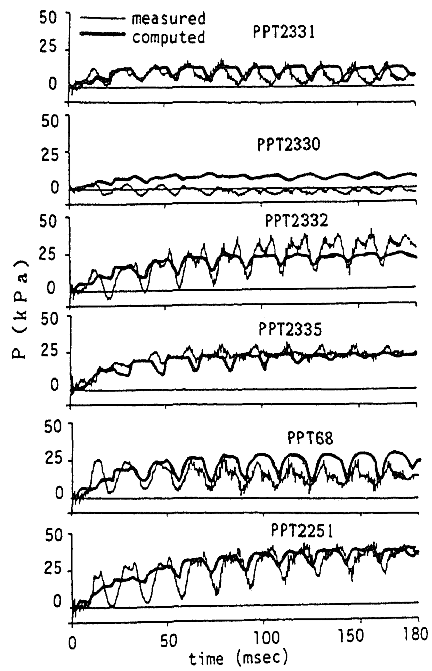


Figure 7. Measured and computed excess pore water pressures in model embankment

5 MODEL PERFORMANCE

In order to examine the overall applicability of the present model, effective stress analyses were conducted on a model soil embankment, a sheet pile quay wall, and a submerged tunnel. The results can be summarized as follows.

When the present model was applied for a centrifuge test result of an embankment conducted at Cambridge University (Dean, 1987), the results were obtained as shown in Figs. 6 and 7, indicating a reasonable applicability of the present model (Iai, 1989).

When the present model was applied for a quay wall at Akita Port in Japan, the result was obtained as shown in Fig. 8, being consistent with the observed behavior of the quay wall during the 1983 Nihonkai Chubu Earthquake (Iai and Kameoka, 1991).

When the present model was applied for a submerged tunnel, the result was obtained as shown in Fig. 9, indicating a reasonable applicability of the present model (Iai and Matsunaga, 1991).

Although these examples are of the preliminary nature and need more rigorous examination, they indicate the potential capability of the present model in predicting the effects of earthquakes on the soil-structure systems.

CONCLUSIONS

The strain space multiple shear mechanism model proposed in the present study is a promising approach to realistic modeling of cyclic

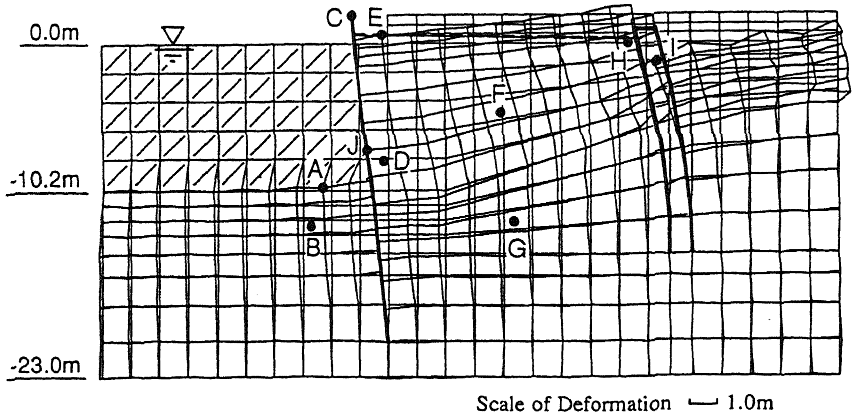


Figure 8. Computed deformation of anchored sheet pile quay wall

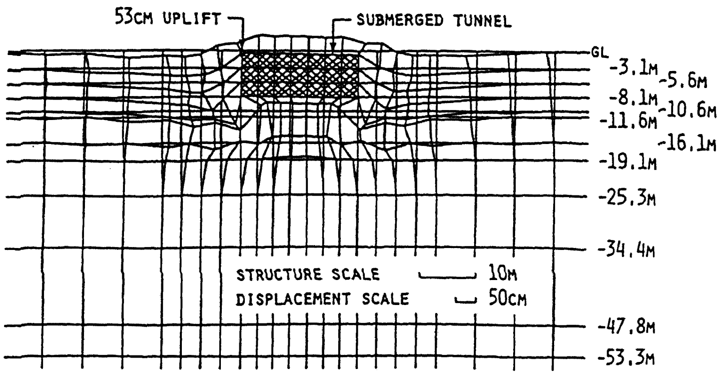


Figure 9. Computed uplift of submerged tunnel and deformation of soil

behavior of sand. The model achieves numerical robustness and realistic modeling of stress induced anisotropy and rotation of principal stress axis direction. The model can be a useful tool for assessing earthquake and liquefaction induced deformation of various soil- structure systems such as embankments, quay walls and buried structures.

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