Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

On the use of free vibration procedures for dynamic properties evaluation

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ABSTRACT In this article the evaluation of time domain free vibration procedures is presented, the use of the Fourier Transform is analysed and several techniques for the determination of structural dynamic properties are proposed. It also evaluates the influence of the variables defining modal interference: transducer's location, load applying device's location, variable to be measured and structural typology. In the second part of the article, free vibration methodology is employed for the evaluation of the dynamic properties of a full-scale building.

1 TIME DOMAIN FREE VIBRATION ANALYSIS

Modal contributions to the structural response can be stated as:

$$u_i(t) = \sigma_i.\exp(-\xi_i\omega_i t).\cos(\omega_i t-\varphi_i) \quad (1)$$

For MDOF systems T_1 and $\underline{\phi}_i$ (modal shape) can be obtained by direct observation and ξ_1 by applying the logarithmic decay formula. This response will be, in general, governed by the first mode, but it will always contain fast decaying higher modes' contributions. Their influence will depend on the following experimental variables <Genatios et al 1989>:

(a) Structure's typology: larger frequencies separations reduce higher modes' contributions, facilitating first mode's identification. As an example, a beam-column building (shear behavior) will have closer frequencies than a tall shear-wall building (flexural behavior), so the contribution of the higher modes of the beam-column building will be stronger, interfering lower modes.

(b) Load applying device's location: in order to evaluate a chosen mode, loads must be applied in such a way that the initial structural deformation resembles it. For $\underline{\phi}_1$, the force must be applied in the higher level, so the initial deformation will be similar to the first mode's shape. For the higher modes, load must be applied in the lower levels, producing a strong modal interference, making the time domain identification process very difficult.

(c) Transducer's location: for the evaluation of a chosen mode, registers should be obtained from the locations that better magnify its relative contribution. For $\underline{\phi}_1$ transducers shall be located mainly in the higher levels. For higher modes, transducers shall be specially located in the lower levels. Nodal points present high interference due to the very low response level. Higher modes' contribution will produce strong interference for first mode's identification.

(d) Measured variable: acceleration contributions, compared to desplacements, are multiplied by ω_i^2, amplifying the contribution of the higher modes. Displacement registers shall then be employed to evaluate $\underline{\phi}_1$.

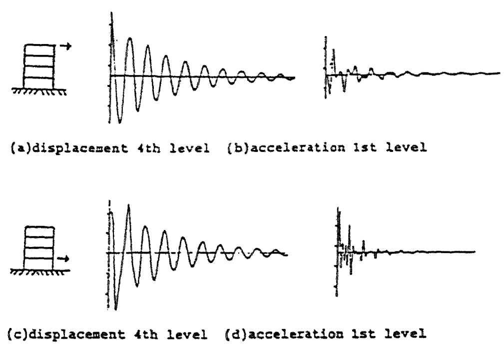

fig.1 influence of experimental variables

Fig. 1 shows different free vibration registers of a 4 story building. Properties of $\underline{\phi}_1$ can be obtained from 1.a. For the acceleration records, the higher modes conbributions make it difficult to obtain convenient results.

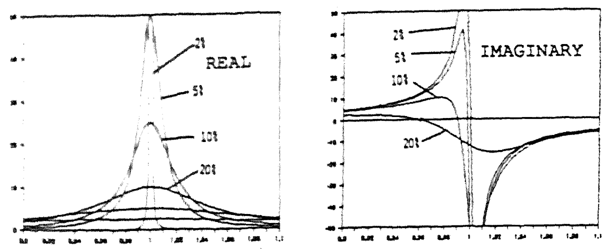

fig.2 Fourier transform components

2 FOURIER TRANSFORM ANALYSIS

Time records only allow first mode's evaluation. In order to obtain higher modes' properties, Fourier analysis can be carried out. Displacement records: <case 1.> eq.2 can also be stated as (for SDOF):

$$\eta=\sigma.\exp(-\xi\omega t)<\exp<i(+\omega_d t-\alpha)> + \exp<-i(\omega_d t-\alpha)>> \quad (2)$$

($i=\sqrt{-1}$). When the Fourier Transform is applied, eq.2 becomes:

$$\eta(p) = \frac{\sigma}{2} < \frac{\exp(-i\alpha)}{i(p-\omega_d)+\xi\omega} + \frac{\exp(+i\alpha)}{i(p+\omega_d)+\xi\omega} > \quad (3)$$

For the initial displacement condition <case 1.1.> $\xi<<1$, and $(p/\omega)=z$

$$\eta(z) = \frac{u_0}{\omega}\ \frac{(\xi + iz)}{((1-z^2)^2 + (2\xi z)^2)} \exp(-i\theta) \quad (4)$$

with:

$$tg(\theta) = (2\xi z) / (1-z^2) \quad (5)$$

modulus (absolute value) <case 1.1.1>:

$$\eta(z) = \frac{u^\circ}{\omega}\ \frac{z}{\sqrt{(1-z^2)^2 + (2\xi z)^2}} \quad (6)$$

The maximal value corresponds to z=1 (p=w), allowing the determination of ω by direct observation. ξ, can be obtained by applying the bandwidth method <Clough 1975>

$$\xi = (p_2-p_1) / 2\omega \quad (7)$$

A similar process can be derived for the *real component* η^r of the Fourier Transform of the displacement response <case 1.1.2>.

$$\eta^r(z) = \frac{u^\circ}{w}\ \frac{\xi(1+z^2)}{(1-z^2)^2 + (2\xi z)^2} \quad (8)$$

The maximal (peak) value corresponds to z=1:

$$\eta^r(z)_{max} = u^\circ / (2\omega\xi) \quad (9)$$

so w can also be directly observed. To obtain ξ, $z_1=(1-\omega)$ and $z_2=(1+\omega)$ are choosen so $(z_2-z_1)=\xi$; leading to $\eta^r(z_1) = \eta^r(z_2) = u^\circ/(4\omega\xi)$. It can be noticed that $\eta^r(z_1) = \eta^r(z_2) = \eta^r(z)_{max}/2$, so ξ can be obtained by a **modified bandwidth method** that evaluates the frequencies from two values $\eta^r(z_1)$ and $\eta^r(z_2)$ equals to 1/2 the peak value. This expression leads to a 3 to 5% error for 2 to 10% damping values. The *imaginary component* can also be employed <case 1.1.3>:

$$\eta^i(z) = \frac{u^\circ}{w}\ i\frac{z(1-2\xi^2) - z^3}{(1-z^2)^2 + (2\xi^2)^2} \quad (10)$$

for $\xi<<1$ and z=1, $\eta^i(z) = 0$, allowing the direct observation of ω (z=1). The exact value of $\eta^i(z)=0$ corresponds to $z=\sqrt{1-2\xi^2}$. If $\xi=0$, z= 1 corresponds to a singularity, with a left hand side limit equal to $+\infty$ and a right hand side limit of $-\infty$. Whenever $\xi<<1$ a maximum can be found at $z_1=1-\xi$ and a minimum can be found at $z_2=1+\xi$, allowing damping evaluation by the bandwidth formula (eq.7). We have called this case **opposite components bandwidth method**

Initial velocity condition <case 1.2.> eq.9 becomes

$$\eta(z) = \frac{v^\circ}{\omega^2}\ \frac{1}{\sqrt{(1-z^2)^2 + (2\xi z)}} \cdot \exp(-i\theta) \quad (11)$$

modulus (absolute value) <case 1.2>

$$\eta(z) = \frac{v^\circ}{\omega^2} \cdot \frac{1}{\sqrt{(1-z^2)^2 + (2\xi z)_2}} \quad (12)$$

Eq. (12) allows the direct observation of w because z=1 (for $\xi<<1$) is a maximum. ξ can be obtained by applying of the bandwidth formula <Genatios et al 1989>.

The *real component* <case 1.2.2>:

$$\eta^r(z) = \frac{v^\circ}{w^2} \cdot \frac{(1-z^2)}{(1-z^2)^2 + (2\xi z)^2} \quad (13)$$

this expression becomes singular for $\xi=0$ and z=1, with left hand side limit of $+\infty$ and right hand side limit of $-\infty$. This case is

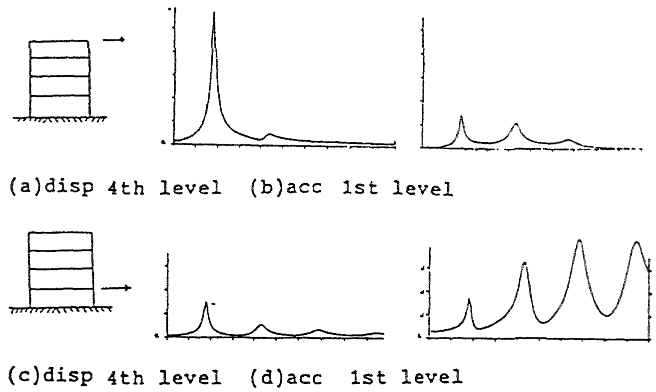

fig.3 influence of experimental variables (Fourier Trans.)

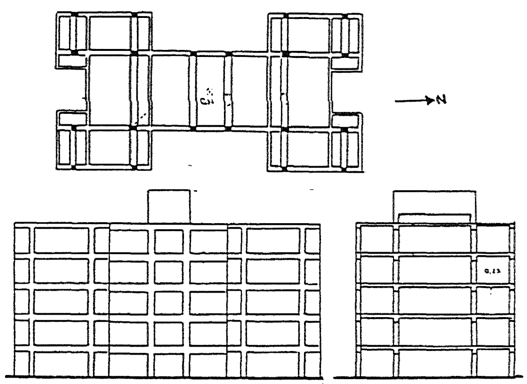

fig. 4. 5 story building

similar to the imaginary component with initial displacement condition, so case 1.2.2 is similar to case 1.1.3 , and it can be shown that the w can be obtained by direct observation and ξ can be obtained by applying the **opposite components bandwidth method.**

Imaginary component <case 1.2.3>:

$$\eta^{i}(z) = \frac{v^{\circ}}{w^{2}} \cdot i \cdot \frac{(-2\xi z)}{(1-z^{2})^{2} + (2\xi z)^{2}} \qquad (14)$$

a maximum corresponding to $z = 1-(\xi^{2}/2)$; this case is similar to 1.1.2 ; ω can be obtained by direct observation and ξ by applying the **modified bandwidth method.**

Acceleration Records: <case 2.>:

$$\mu(z) = \frac{\langle u^{\circ}\omega(-\xi+2\xi^{2}) - (v^{\circ})\rangle - i z \langle u^{\circ} w (1+2\xi^{2}) + v^{\circ} 2\xi\rangle}{\langle 1-z^{2} + i 2\xi z\rangle} \qquad (15)$$

The application of the exposed methodologies allows the modal properties' determination. The evaluation of the damping coefficients can be made as follows:

Initial displacement condition:
- modulus: original bandwidth method
- real component: modified bandwidth method
- imaginary component: opposite components bandwidth method

Initial velocity condition:
- modulus: original bandwidth method
- real component: opposite components bandwidth method
- imaginary component: modified bandwidth method

3 FOURIER TECHNIQUE APPLICATION

Frequency domain analysis allows the determination of higher modes' properties. The structural response will be conditioned by the same four experimental variables:

(a) Structure's typology: the more separated the modal frequencies are, the easiest it is to determine the structural properties, because the modal interference will be lower. Higher modes' properties can then be obtained.

(b) Load applying device's location: when the initial condition imposed to the structure, contains the contribution of all modes, the structural response will present all this contributions. For the evaluation of the superior modes, the load shall be applied in the lower floors of a building; in the study of a dam or a bridge, loads shall be applied in a region around the supports; this allows the excitation of all modes.

(c) Transducer's location: to obtain the strongest contribution of the higher modes, the registers must be taken in the same regions that have been recommended for the load application.

(d) Measured variable: the acceleration registers will magnify the contribution of the higher modes, due to the presence of ω_i^2

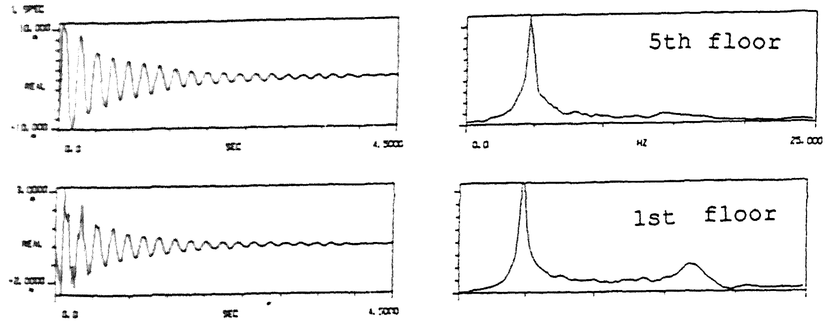

fig. 5. free vibration registers

Table 1 frequencies

		frequencies		
Φ_i	Test	E-O	N-S	Torsion
1	Free Vib.	3.88	4.69	-
	Vib Harm.	4.00	4.75	5.00
	Environ.	4.00	5.00	5.00
2	Free Vib	13.46	17.28	-
	Environ	14.00	17.33 *	-

Table 2 Damping Coefficients

Φ_i	Tests	Methode	Direct. E-O			Direct. N-S			Torsion		
			ϵ_i %	σ*	N*	ϵ_i %	σ*	N*	ϵ_i %	σ*	N*
1	Free Vibr	Logarith.	2.5	0.2	4	4.1	0.7	4	1.8	0.7	4
		Modulus	3.6	0.1	6	3.9	0.2	6	-	-	-
		Real Imag	3.0	0.3	5	4.7	0.1	6	-	-	-
	Harm Vibr	Bandwidth	5.3	0.0	6	5.9	0.1	6	4.8	0.4	2
		Mean Sq	4.3	0.3	6	4.9	0.2	6	4.1	0.0	2
		Tangents	3.1	0.3	4	5.2	0.6	5	-	-	-
	Envi Vibr	Bandwidth	3.0	0.4	6	6.0	0.2	5	1.9	0.3	4
		Mean Sq	2.6	0.2	5	4.7	0.4	5	1.5	0.4	4
2	Free Vibr	Modulus	8.9	-	1	5.3	0.3	5	-	-	-
		Real Imag	3.5	0.7	4	4.8	0.4	5	-	-	-

4. EXAMPLE: 5 story framed building:

An actual 5 story framed building has been analysed (fig.4) <Genatios 1991 also Cascante 1985>. Modal properties have been evaluated for the first two modes on each direction and for the first torsional mode. Free vibration methodologie has been employed together with harmonic and environmental techniques. Evaluated properties are shown in tables 1 and 2. Vibration modes are shown in fig 6.From an experimental point of view free vibration obtained frequencies are equal to the ones obtained by other means. Damping coefficients: free vibration obtained results present lower values. This is due to the fact that the torsional frequency lies very close to the first frequencies in both directions, coupling its effects and producing a certain modal interference. The frequency domain results produce better results <Genatios 1991>. The modal shapes are very similar for all the three techniques and show an important soil-structure interaction.

5. CONCLUSIONS

- Time domain analysis of free vibration essays, allows in general, a realistic evaluation of the dynamic properties of the first mode. Higher modes cannot be evaluated by the means of this procedure.
- In order to emphasize the contribution of the first mode of vibration (for time domain analysis), the load must be applied on the highest levels of the building and displacement registers should be employed; the maximal relative contribution is obtained in the higher levels. Slender shear-wall buildings (flexural behavior) have more separated frequencies than beam-column buildings, allowing a more accurate evaluation of the dynamic properties.
- Fourier Transform techniques can be employed in order to obtain dynamic properties of the higher and lower modes. The corresponding equations have been evaluated for the displacement and acceleration registers subjected to initial displacement or initial velocity conditions. Procedures are discussed for the determination of the dynamic properties: direct observation for the frequencies; bandwidth method, modified bandwidth method and opposite components bandwidth method for the determination of the damping coefficients.

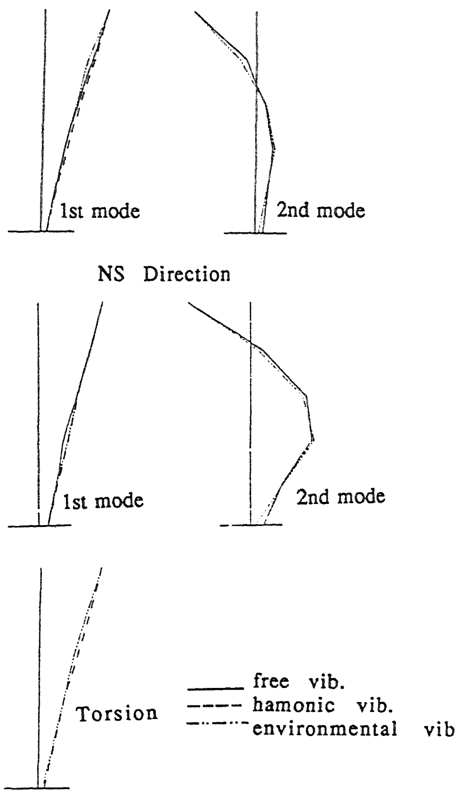

fig.6 Modal Shapes (framed building)

- In order to emphasize the contribution of the higher modes, loads must be applied in the lower levels of the buildings, acceleration records shall be employed and they shall be obtained in the lower levels.

- The evaluation of the dynamic properties of a 5 story framed building has been made. Free vibration results are very similar to the ones obtained by other technniques.

REFERENCES

Cascante,G."**Experimental evaluation of the dynamic properties of structures: Study of Two Cases**" (in spanish) M.Sc. Thesis. IMME Central University of Venezuela, Caracas, January 1985.

Clough,R.,Penzien,J. "Dynamics of Structures" Mc Graw-Hill New York, 1975.

Genatios,C.,Lopez,O.,Lorrain,M. "**Theoretical evaluation of experimental techniques for the determination of structural dynamic properties**" Modern Practice in Stress and Vibration Analysis. Ed. by E.Mottershead, Pentech Press, London, 1989.

Genatios,C."**Experimental techniques for structural dynamic properties determination**"(in french). Dr. Thesis INSA/Toulouse,1991.

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