Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

A scaled-time test method

F.J. Molina
Universidad de Oviedo, ETSII Gijón, Spain

E. Alarcón
Universidad Politécnica de Madrid, ETSII Madrid, Spain

ABSTRACT: The Pseudo-Dynamic Test Method (PDTM) is being developed currently as an alternative to the shaking table testing of large size models. However, the stepped slow execution of the former type of test has been found to be the source of important errors arising from the stress relaxation. A new continuous test method, which allows the selection of a suitable time-scale factor in the response in order to control these errors, is proposed here. Such scaled-time response is theoretically obtained by simply augmenting the mass of the structure for which some practical solutions are proposed.

1. INTRODUCTION

The necesity of full-scale testing and the economic limitations existing for its execution in a shaking table at real time have motivated the development of the Pseudo-Dynamic Test Method (PDTM) in Earthquake Engineering. In the PDTM, the structure is theoretically subjected to the true history of deformations, but in a stepped way, performing in fact a static test at each step. Hence, the required load capacity of the equipment is not reduced, whilst the power needs of the installation are drastically diminished in relation to the real-time shaking table test.

This clear advantage of the PDTM is not the only one but, unfortunately, the method also has some disadvantages (see for example Mahin and Shing, 1985, and Donea et al, 1990), most of which arise from

1. Its stepped nature and excesive time elongation which allow important stress relaxation and the appearance of strain rate effect.
2. The errors introduced by the displacement and force transducers.
3. The stability, accuracy and error propagation characteristics of the stepping algorithm.

In relation to the obtained time elongation, in the PDTM the duration of a test can be, e.g., 100 or 1000 times the real duration of the earthquake which is clearly unnecessary for power saving, but is motivated by the size of the required time increment and the obligatory lapse for "hold" and "ramp" phases at each step (Mahin and Shing, 1985).

In the Scaled-Time Test Method (STTM), whose formulation is made in the next section, the time history of deformations is also theoretically obtained, as in the PDTM, but in a continuous way and the duration of the test can be selected to be λ times the real one, e.g., $\lambda=10$. Doing so, the speed is reduced, e.g., 10 times, the acceleration 100 times and the required power is diminished also 10 times with respect to the values of a real-time test.

In relation to the errors introduced by the transducers or the stepping algorithm, the STTM is free of these because it is based on similarity relations and does not need such devices.

2. FORMULATION

Although, the general formulation of the STTM is very simple, it will be restricted here to the specific type of problem for which the method is hoped to be useful.

Considering a discrete system with lumped mass, viscous damping and rate-independent structural restoring forces, the equation of motion in relative coordinates will be written as (Clough, 1975)

$$\underset{\sim}{M}\ddot{\underset{\sim}{u}}(t) + \underset{\sim}{C}\dot{\underset{\sim}{u}}(t) + \underset{\sim}{r}(\underset{\sim}{u}(t)) = \underset{\sim}{p}(t) - \underset{\sim}{M}\underset{\sim}{J}\,\ddot{u}_b(t) \qquad (1)$$

where $\underset{\sim}{u}(t)$, $\dot{\underset{\sim}{u}}(t)$ and $\ddot{\underset{\sim}{u}}(t)$ are respectively the relative displacement, velocity and acceleration, $\underset{\sim}{M}$ and $\underset{\sim}{C}$ are respectively the mass and damping matrices and r is the vector of restoring forces which is a non-linear function of the deformations. The loads are included in vector $\underset{\sim}{p}(t)$ and the acceleration imposed at the base is $\ddot{u}_b(t)$, $\underset{\sim}{J}$ being the influence vector and t the time (see the example of Fig.1a).

In order to obtain in a model a scaled-time response

$$^{m}t = \lambda t \qquad (2)$$

$$^{m}\underset{\sim}{u}(^{m}t) = \underset{\sim}{u}(^{m}t/\lambda) \qquad (3)$$

λ being the time-scale factor ($\lambda>1$), and the corresponding velocities and accelerations

$$^{m}\dot{\underset{\sim}{u}}(^{m}t) = \frac{d}{d\,^{m}t}\,^{m}\underset{\sim}{u}(^{m}t) = \frac{1}{\lambda}\dot{\underset{\sim}{u}}\left(\frac{^{m}t}{\lambda}\right)$$

$$^{m}\ddot{\underset{\sim}{u}}(^{m}t) = \frac{d^2}{d\,^{m}t^2}\,^{m}\underset{\sim}{u}(^{m}t) = \frac{1}{\lambda^2}\ddot{\underset{\sim}{u}}\left(\frac{^{m}t}{\lambda}\right) \qquad (4)$$

the properties of the structure must be modified as well as the load functions. However, it will be assumed that in the new equation of motion, i.e.

$$^{m}\underset{\sim}{M}\,^{m}\ddot{\underset{\sim}{u}}(^{m}t) + \,^{m}\underset{\sim}{C}\,^{m}\dot{\underset{\sim}{u}}(^{m}t) + \underset{\sim}{r}(^{m}\underset{\sim}{u}(^{m}t)) = \,^{m}\underset{\sim}{p}(^{m}t) - \,^{m}\underset{\sim}{M}\underset{\sim}{J}\,^{m}\ddot{u}_b(t) \qquad (5)$$

equilibrium is still possible with the original members of the structure, so that the restoring forces function $\underset{\sim}{r}$ is not altered.

The obvious solution is to adopt greater damping

$$^{m}\underset{\sim}{C} = \lambda\underset{\sim}{C} \qquad (6)$$

and mass

$$^{m}\underset{\sim}{M} = \lambda^2\underset{\sim}{M} \qquad (7)$$

and apply the load and shaking at a reduced speed

$$^{m}\underset{\sim}{p}(^{m}t) = \underset{\sim}{p}\left(\frac{^{m}t}{\lambda}\right) \qquad (8)$$

$$^{m}u_b(^{m}t) = u_b\left(\frac{^{m}t}{\lambda}\right) \qquad (9)$$

$$^{m}\ddot{u}_b(^{m}t) = \frac{d^2}{d\,^{m}t^2}\,^{m}u_b(^{m}t) = \frac{1}{\lambda^2}\ddot{u}_b\left(\frac{^{m}t}{\lambda}\right) \qquad (10)$$

which can be verified if (2-4) and (6-10) are introduced in (5), i.e.

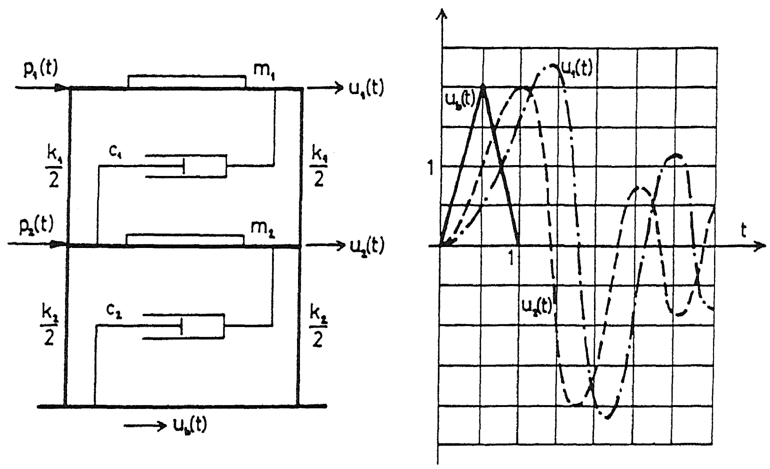
$$\lambda^2\underset{\sim}{M}\frac{1}{\lambda^2}\ddot{\underset{\sim}{u}}\left(\frac{^{m}t}{\lambda}\right) + \lambda\underset{\sim}{C}\frac{1}{\lambda}\dot{\underset{\sim}{u}}\left(\frac{^{m}t}{\lambda}\right) + \underset{\sim}{r}\left(\underset{\sim}{u}\left(\frac{^{m}t}{\lambda}\right)\right) = \underset{\sim}{p}\left(\frac{^{m}t}{\lambda}\right) - \lambda^2\underset{\sim}{M}\underset{\sim}{J}\frac{1}{\lambda^2}\ddot{u}_b\left(\frac{^{m}t}{\lambda}\right) \qquad (11)$$

which is in fact true because it represents the original equation (1) particularized for the instant

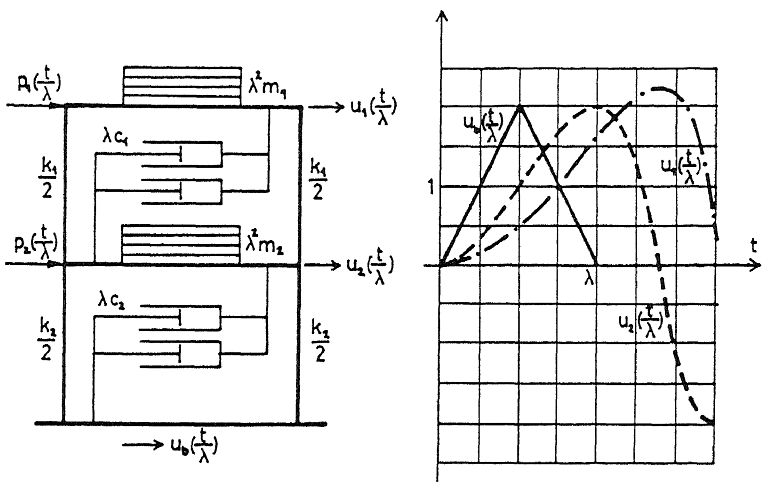
$$t = \frac{^{m}t}{\lambda} \qquad (12)$$

In short, as soon as a time-scale factor λ has been chosen, the STTM consists of physically modifying the damping and the mass of the structure (6-7) and applying the loads at a lower speed (8-10) in order to obtain a scaled-time response (2-4) (Fig.1b).

In the following section these same results are obtained by means of similarity methods.



a) Original system (prototype)



b) Model

Figure 1. Modification of the system in order to obtain a scaled-time response

Table 1. Scale factors for seismic structural models.

CASE	$\lambda_h = \frac{m_h}{p_h}$	$\lambda_{\ddot{u}} = \frac{m_{\ddot{u}}}{p_{\ddot{u}}}$	$\lambda_M = \frac{m_M}{p_M}$	$\lambda_p = \frac{m_p}{p_p}$
1	λ_t	λ_t^{-1}	λ_t^3	λ_t^2
2	λ_t^2	1	λ_t^4	λ_t^4
STTM	1	λ_t^{-2}	λ_t^2	1

3. ALTERNATIVE FORMULATION VIA SIMILARITY METHODS

Similarity methods are widely used for scale model testing in Earthquake Engineering. In fact, small-scale models that are tested on shaking tables constitute an alternative to full-scale pseudo-dynamic tests or, even, medium- scale models have also been used with the PDTM. The main disadvantage of the former models is the size effect affecting the properties of some materials which are widely used in buildings and civil works.

Following Dove and Bennett (1986),

the development of scaling laws for structural dynamics under seismic motions can be based on the following parameters:

$\ddot{u}$ = response acceleration at any point on the structure.
E = stress strain characteristic of the material (not constant)
M = mass of the structure
h = a linear dimension
$\ddot{u}_b$ = input or driving acceleration
t = time
p = any force, except damping

which they gather in dimensionless pi terms within the model law:

$$\frac{\ddot{u}}{\ddot{u}_b} = \phi\left(\frac{Eh^2}{M\ddot{u}_b}, \frac{h}{\ddot{u}_b t^2}, \frac{P}{M\ddot{u}_b}\right) \qquad (13)$$

From expression (13), the following relations can be extracted for the scale factors (ratios between the parameters of the model and the prototype)

$$\lambda_{\ddot{u}} = \frac{\lambda_h}{\lambda_t^2}$$

$$\lambda_M = \frac{\lambda_E \lambda_h^2}{\lambda_{\ddot{u}}}$$

$$\lambda_p = \lambda_M \lambda_{\ddot{u}} \qquad (14)$$

where λ_E is always considered 1 (no change in the material).

Dove and Bennett adopt the scale factor of size λ_h (properly its inverse) as the characteristic factor and propose two main choices among others for development of tests. Both cases are obtained from (14) and are summarized in Table 1, whilst translated to the scale factor of time λ_t as the characteristic one. Note that λ_t is simply called λ outside this section.

The first case has the advantage that the required mass is directly obtained without altering the density of the model. However, its gravity force will be distorted since the scale factors for force and mass are not equal.

In the second case gravity forces are not distorted for the same reasons, but the density of the small-scale model has to be incremented and this requirement is normally fulfilled in an approximated way by use of additional lumped masses.

In the last line of table 1, the STTM has been included to show how it can be obtained using these similarity relations (14). As is observed, the STTM uses a full-scale model that will be subjected to lower accelerations (4), because of the time enlargement (2), but with the full forces (8) due to the enlarged masses (7). The gravity forces will be greatly distorted if the additional masses rest on the structure; so, in the next section some posibilities are considered in order to avoid this problem.

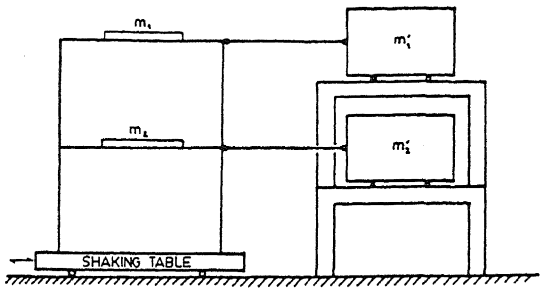
This development can also be used to obtain a scale factor for the power required during the test, which results for the STTM

$$\lambda_w = \frac{\lambda_p \lambda_h}{\lambda_t} = \lambda_t^{-1} \qquad (15)$$

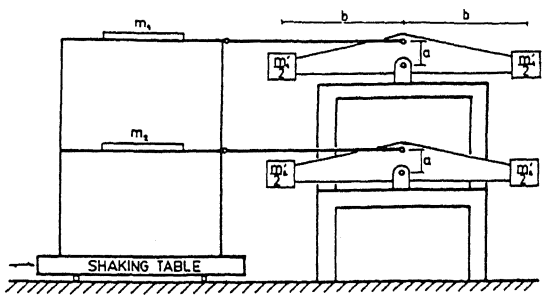
i.e., power is reduced in the same proportion as time is magnified.

Dove and Bennett also extend their study to several types of damping forces. The results for the STMM in this respect could be summarized as follows:

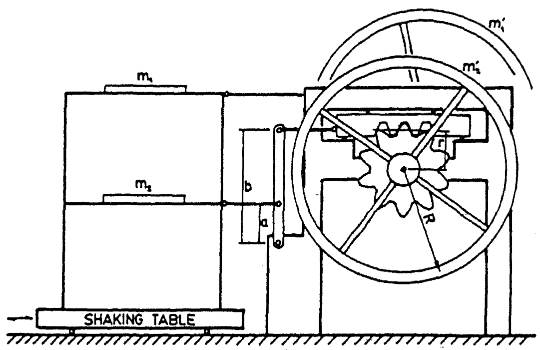
- Viscous damping forces are distorted because they are proportional to the velocities, unless additional viscous damping (6) is present in the model.
- Structural hysteretic damping forces are maintained as long as they are frequency independent.
- For the maintenance of Coulumb damping forces, no distortion of the gravity forces is critical.



a) $m_i' = m_i\ (\lambda^2-1)$



b) $m_i'\ (b/a)^2 = m_i\ (\lambda^2-1)$



c) $m_i'\ (R/r)^2(b/a)^2 = m_i\ (\lambda^2-1)$

Figure 2. Mechanical devices for the STTM

4. SOME PRACTICAL CONSIDERATIONS

In order to make the STTM a useful technique for Earthquake Engineering some patent practical problems have to be solved at this point.

Firstly, as currently is made in the PDTM, the structure must be assimilable to a multistorey frame with lumped mass at the floor levels and one horizontal degree of freedom at each floor. Although the viscous damping forces can be theoretically maintained by instaling additional dampers on the model in order to fulfil (6), neglecting them is probably more reasonable, because, specially if the response is going to be strongly non-linear, most of the dissipation will be produced by other sources (Mahin and Shing, 1985) and these are correctly included in the method.

Secondly, the required enlargement of inertial properties (7) must be made in a way as inexpensive as possible and without affecting the gravity loads on the columns of the structure. Some ideas concerning this problem are now expounded.

Molina and Alarcón (1991) presented the STTM for the first time suggesting the use of an "artificial inertia", i.e., simulated by hydraulic actuators, like those used in the PDTM, whilst controlled with accelerometers in order to produce, at each floor, a horizontal force proportional to its acceleration in the opposite direction. However, we now recognize the existing limitations to the obtention of accuracy and stability in the dynamic range for such a force control system.

Probably the simplest mechanical solution consists of using a complementary structure on which the additional masses can slide, and then connect them to the floors of the structure with the aid of light horizontal truss members (Fig. 2a). This procedure, of course, introduces some friction forces but can be more limitated by the enormous weights that may be needed to manipulate if the time-scale factor λ is in the order of 10, e.g (remember (7)).

Other more interesting mechanical solutions use rotational inertia which allows to materialize large values of λ without the need of said enormous weights. Fig. 2b shows a simple mechanism and Fig. 2c shows a double mechanism, both with rotational inertia. The formulae that these figures include allow the determination of the additional masses and were obtained by imposing a virtual displacement to the corresponding floor and equating the work of the rotational inertial forces to the work of an apparent translational force. Of course, the masses of all the members of the mechanisms could have also been included. These mechanisms or similar ones can be used in an effective way, we think, for the STTM and should be made easily adaptable for different structures.

As shown in Fig. 2, in the STTM a shaking table would be needed, as in a real-time test, and a complementary structure, which is analogous to the reaction wall of the pseudo-dynamic test.

5. CONCLUSIONS

As has been shown, the justification for the proposed STTM is based on the same reasoning that also justifies the PDTM, i.e., mainly the possibility of testing large-scale structures with limited power facilities. Obviously, if the available power for the actuators is large enough, the traditional shaking-table real-time test should be prefered.

Making a theoretical comparison of the STTM in relation to the PDTM, the following advantages can be found for the former:

- The test response is continuous and is theoretically a scaled-time version of the real one.
- The strain rate and stress relaxation effects are less important because of the shorter duration of the test.
- The errors arising from displacement or force measurements are not present.
- The errors arising from the step-by- step integration algorithm are not present.
- The actuators are only needed in the shaking table, whose control is standard.

On the other hand, the STTM needs the lumped mass assumption, as does the PDTM, and presumably has the following drawbacks with respect to it:

- The proposed mechanisms for adding apparent mass may introduce important spurious dissipation as well as jumps at the sign changes of the acceleration.
- There is no possibility of introducing viscous damping or any theoretical change in the model by means of the integration algorithm. Subestructuring techniques are also impossible.

Naturally, some of these advantages and drawbacks need to be assessed for particular cases with the aid of experimental verification.

6. ACKNOWLEDGMENT

This work has been carried out within the framework of the joint research project :"Numerical and Experimental Activities in Support of the Validation of the Pseudodynamic test Method" of the Association of Structural Mechanics Laboratories (ASML), Working Group 4, which is financed by the Commission of the European Communities. However, opinions expressed in this paper do not necessarily represent those of other participants in the project.

The authors are also grateful to the Spanish DGICYT for its support through the research project PB90-0273.

REFERENCES

Clough, R.W., Penzien, J. 1975 Dynamics of Structures. Mc. Graw-Hill.

Donéa, J., Jones, P.M., Magonette, G., Verzeletti, G. 1990. The pseudo-dynamic test method for earthquake engineering: An overview. Commission of the European Communities. EUR 12846 EN.

Dove, R.C. and Bennett, J.G. 1986. "Scale modeling of reinforced concrete category I structures subjected to seismic loading. NUREG/CR-4474, LA 10624-MS, Los Alamos National Laboratory, Los Alamos, NU, January.

Mahin, S.A. and Shing, P.B. 1985. Pseudodynamic method for seismic testing. ASCE Journal of Structural Engineering. Vol. 111, No. 7, July. 1482-1503.

Molina, F.J., Alarcón E. 1991. A scaled time test method. Internal document presented at the meeting of the W.G. 4 of the Association of Structural Mechanics Laboratories held on April 19 at Bergamo.