Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

A new method for seismic analysis of light equipment: The forced structure modes method

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ABSTRACT : When performing the seismic spectral analysis of structurally supported light equipment using the floor response spectra (FRS) method, some approximation are made relative to the differential displacements, rotations and displacements perpendicular to the earthquake direction of supports and to modal summation. Such approximations are necessary because it is either very difficult or impossible to take account correctly of all these parameters with the FRS method.

This paper describes a new method, the "forced structure modes" (FSM) method, developed for seismic analysis of light equipment. This method can take account of all the above mentionned parameters without any approximation or extra hypothesis with respect to the conventional spectral analysis and is even easier to apply than the FRS method.

The FSM method also save engineering time as the calculation of floor response spectra is no more required. The same data as for this calculation are used directly by the FSM method.

1. INTRODUCTION

The dynamic analysis of light equipment supported by structure subjected to earthquake is at the present time most frequently performed using the floor response spectra (FRS) method. This means that beside the computation of the structure itself, the results of which are used as data, two computation steps are necessary:

- the floor response spectra computation,
- the equipment spectral analysis, on the basis of these spectra.

Generally, as it leads to highly complicated computation to solve completely the problem, the following parameters are neglected:

- correlation between the different imposed movements at the different support points/degrees of freedom. This means that the support driving effects for the perpendicular to the earthquake direction translational degree of freedom (d.o.f.) and for all rotational d.o.f. are not taken into account.
- the summation rule for modal quantities is not proved as being valid for such floor spectra: the available rules have been developed for wide band signals, a characteristic of the "free field" movements but probably not true for signals filtered by a structure.

The present paper describes the "forced structure modes" (FSM) method which solves completely these two difficulties and moreover eliminates the first computational step (floor response spectra).

From the same data as the FRS method, the FSM method is able to compute the modal data of a reference model, including both the structure and the equipment, in its equipment part. This makes it possible then to perform either spectral (with the same spectra as for the structure) or time history analysis giving displacements (relative to the ground), pseudo-acceleration (absolute) and internal forces for the equipment.

2. BASIC HYPOTHESIS AND DEFINITIONS

The basic hypothesis of the FSM theory are the same as for the FRS method, i.e.:

- very light equipment with respect to the structure mass,
- linear behaviour and low damping of the both the structure and the equipment,
- the earthquake solicitation is assumed to be a given time history acceleration at the basis of the structure in a given direction. This time history acceleration may be used either directly or by the way of its acceleration, velocity or displacement spectra.

The correct result for the equipment analysis are assumed to be those, in its equipment part, of the reference model treated by the conventional spectral analysis. It is noticeable that this type of model can be difficult to treat with computers because of the bad conditioning of stiffness and mass matrices. However, as the literal solution exists, it can be assumed to be the correct one.

The conventional spectral analysis vocabulary is supposed to be known by the reader. However, because of its importance in this paper and as this notion is seldom precised, we will recall the definition of the applied acceleration field (Δ): it is the vector composed for each d.o.f. of the analysed model of the cosine of the angle between the earthquake direction and the d.o.f. direction. For instance, in an orthogonal basis, for an earthquake along the X axis, the components of the applied acceleration field are 1 for X d.o.f. and 0 for all other d.o.f. including rotation ones. This vector is found in the second member of the dynamic equilibrium equation of the model:

$$M\ddot{X} + C\dot{X} + KX = -M\Delta\ddot{y}_0(t)$$

where $\ddot{y}_0(t)$ is the time history acceleration of the earthquake.

We will also recall that the basic definition of the participation factors is:
The set of coefficients of the unique decomposition of the applied acceleration field on the modal (and then orthogonal with respect to both mass and stiffness matrices) basis. In mathematical terms, it is expressed by:

$$\sum_{i=1}^{n} \Gamma_i \Phi_i \equiv \Delta$$

where Γ_i is the participation factor and Φ_i is the modal shape of the i-th mode.

The classical expression:

$$\Gamma_i = \frac{\Phi_i^T M \Delta}{\Phi_i^T M \Phi_i}$$

is an easy way to compute them, but the previous identity is their real definition.

Four models are refered to further in this paper:

- A model of the structure, the modal data (shapes, frequencies, damping ratios and participation factors) of which are supposed to be already known. This model will be refered to as the "structure model".
- A model of the equipment supposed to be fixed at its support points. It will be refered to as the "equipment model".
- A model of the equipment supposed to have free d.o.f. at each support point. This model has no support and will be refered to as the "free equipment model".
- A model including both the structure and the equipment with connection between them at the support points. It will be refered to as the "reference model".

For the reference model, we will define the "structure modes" and the "equipment modes".
The "structure modes" (resp. "equipment modes") are those modes for which the displacements of the structure d.o.f. are significant (resp. insignificant) with respect to those of the equipment d.o.f..

3. PHYSICAL DESCRIPTION OF THE SOLUTION

The FSM method formulation gives in modal displacement, acceleration and internal forces terms, the limit of the exact solution of the reference model when the equipment masses and stiffnesses tend proportionnaly to zero. In fact it is an extension to a multi-degree-of-freedom and multi-support equipment of the computation scheme used in ref. (2) to evaluate the response of a single d.o.f. oscillator connected to a point of a structure. In this extension the following physical scalar quantities of the original scheme:

- supporting d.o.f.
- independent d.o.f. of the equipment

have been replaced by vectorial quantities corresponding directly to the structure and equipment characteristics. The scalar mass and stiffness have been replaced by the corresponding matrices.

As the equipment characteristics are supposed to be known, it is no more necessary to treat the problem for several frequencies and damping ratios as it is done in order to compute floor response spectra.

The FSM method formulation can be derived in exactly the same formulation through three different approaches :

- the first one consists in building-up the basic equation of the reference model and to assume for the structure d.o.f. their values as computed in the structure model for each structure mode and zero for each equipment mode. It is then possible to derive the expression of modal displacements for each equipment d.o.f. for each considered mode. Then the applied acceleration field is decomposed on the modal basis, giving the participation factor for all modes.
- the second one is identical to the first one in its first part, say until the derivation of modal displacements for equipment d.o.f.. Then, for equipment modes, a more precise approximation of structure d.o.f. displacements is given using the equipment reactions on the structure. At ast, computed with the classical formula, the participation factor are found.
- the third approach consists in considering the movements at support d.o.f. as imposed to the equipment and to solve directly taking account of the literal expression specificities of these movements, say:
  - the movements is the sum of several modal movements which are, for each support d.o.f. proportional to a single time function.
  - each this time function satisfies a specific equation that gives a relationship between its two first derivative, it self and $\dot{y}_0(t)$

The basic equations for the reference model and the derived complete FSM solution are wholly given in ref. (1). The physical meaning of the resulting formulation for the first approach can be described as follows:

3.1 About the structure modes

- the structure modes are not modified by the presence of the equipment because of its ligthtness.
- if one gives the structure part of the reference model a structure model mode shape and free it instantaneously without initial velocity, at every point d.o.f. of the structure, and thus at the support points of the equipment, the movement is proportional to its initial value with a cosine shape at this mode frequency with respect to time,
- the support points movements for this free vibration of the structure is then an harmonic solicitation for the equipment,
- the response of the equipment to this harmonic solicitation can be easily formulated with the help of the free equipment model. The obtained equipment shape can be considered as the extension, in the equipment part of the reference model, of the structure mode shape,
- the participation factor of this mode is unchanged with respect to its value in the structure model, because in its formulation, the terms relative to the equipment are negligibe with respect to those relative to the structure,
- for a similar reason the damping ratio of this mode is unchanged with respect to its value in the structure model,
- as a conclusion about the structure modes of the reference model, they remain unchanged with respect to the modes of the structure model and are imposed to the equipment with their shapes, frequencies, damping rations and participation factors. That's why this method is called the "forced structure modes" method.

3.2 About the equipment modes.

- the equipment displacement do not induce any displacement of the structure because of its low stiffness. In other terms, the structure can be considered as a fixed support for the independent movement of the equipment (by "independent" we mean the movement that are induced by forces directly applied to the equipment). This means that the equipment model mode shapes, with zero displacement for the structure give the equipment modes of the reference model,
- as they are unchanged, the different equipment mode shapes remain orthogonal two by two with respect to the equipment model mass matrix.

3.3 About the earthquake applied acceleration field modal decomposition

- the structure modes components in the decomposition of the applied acceleration field in the equipment part of the model are directly known as the product of the known modal shapes by the known participation factors,
- the sum of equipment modes components can be computed as the difference between the whole applied acceleration field for all modes and the known sum, as hereabove computed, of all structure modes components .
- the sum of equipment modes components can be decomposed on the orthogonal basis of the equipment mode shapes. This decomposition is made only for the independent equipment d.o.f., all other ones being zero.

With this limitation of d.o.f., it is possible to use the orthogonality property of modes to compute easily the participation factors.

At this stage, we can say that if we know the modal data of the structure and a description of the equipment (d.o.f., masses, stiffnesses, acceleration field), we can simulate the modal analysis of the reference model in its equipment part.

4. MODAL TRUNCATION

As for conventional spectral analysis, the accuracy of the results will depend on the modes used in the modal summation. Using all modes will generally be too much expansive. So it is necessary to give a validity criterion in order to ensure the accuracy of the results with a reduced number of modes. In models of such type as the reference model the effective mass criterion is generally not valid for the calculation of the equipment because the equipment modes may have a significant participation in the internal forces though they have an insignificant participation in the effective mass.
In ref (1) a validity criterion is developped on the basis of the splitting of modes into structure and equipment modes. A required accuracy will be specified for the support displacements and for the effective mass of the equipment relative to the computed sum of equipment modes components of the applied acceleration field.

5. MODAL SUMMATION

An important point about the spectral analysis is the summation rule for modal quantities. It is shown in ref (1) that the choice of this rule should meet the condition that for two modes having the same frequency the result depend only on the algebraic sum of these two modes components.

For this reason the author would recommend the use of the CQC rule developped by Der Kiureghian and al. in ref (3), for those modes that have a frequency lower than the cut- off frequency (f_c) of the spectra. For other modes with a frequency greater than f_c, they have to be algebraically summed because of the simultaneousness of their response. As in conventional spectral analysis, a rigid modes theory, similar to that of the conventional spectral analysis, developed in ref (1) allows to compute directly this sum without computing every mode if all structure modes are known. If this is not the case, an approximation of this sum is given.

6. APPLICATION ON A SIMPLE MODEL

The aim of this paragraph is to show an application of the main part of the FSM method, i.e.:

- the computation of the equipment part of the structure modes,
- the computation of the participation factors for the equipment modes,

and to compare it with the modal results of the reference model,

In order to simplify this visualization, this test is made on a small plane model. The structure model has three horizontal d.o.f. and is represented on figure 1. Its stiffness and mass matrices are (units are kN/m for stiffnesses and tons for masses) given in figure 4.The d.o.f. order in these matrices is x3-x2-x1.

The free equipment model has 6 horizontal d.o.f. and is represented on figure 2. Its stiffness and mass matrices are given in figure 5. The d.o.f. order in these matrices is x7-x6-x5-x4-x3-x2.
The x2 and x3 d.o.f. are the interface d.o.f., i.e. the support points of the equipment.
The reference model including both the structure and the equipment is represented in figure 3.
The equipment is very light with respect to the structure (1/5000 total mass ratio) in order to meet the first basic hypothesis.

Damping ratio have been taken equal to 20% for the structure and 2% for the equipment. Those values do not intend to be representative but have been chosen different enough to make the test convincing.

The tables 1. and 2. give the results of the FSM method compared to those of the reference model.
For the reference model (table 1.) the results are the frequencies, mode shapes multiplied by the participation factors for all (structure and equipment) d.o.f. and modal damping ratios.
For the FSM method (table 2) the results are the modal shappes multiplied by the participation factors, for the equipment d.o.f. . Intermediate results of the structure and equipment models used as data are given in the grey area of the table. Structure d.o.f. for the equipment modes are hypothetically zero.

The different modes of the models are designated by their type ("str" for structure modes and "equi" for equipment modes) and their number in the type in the ascending frequencies order.One can state that the results of the FSM method are in very good accordance with those of the reference model. It is not necessary to compare further results because the same methods for modal summation can be applied in both cases to obtain them. So we can say that the FSM method gives a good approximation of the modal data of the reference model in its equipment area by using almost the same data as the FRS method.

7. CONCLUSION

The "forced structure modes" method for analysis of light equipment supported by a structure subjected to earthquake makes it possible to account for several phenomena that the classical floor response spectra transfer method does not:

- the driving effects in the directions perpendicular to the earthquake direction,
- the driving effects of rotation d.o.f.,
- the effects of differential displacements and rotations between support points,
- the validity domain for modal summation method in a spectral analysis.

Besides, one can state that this method gives results very close to those of the reference model including both the structure and the equipment. The global rigid modes computation has been adapted to the specificity of the FSM method and a validity criterion has been defined which is correct for the equipment though it is virtually integrated in a larger model.

Last, in engineering terms, this method presents the advantage of the suppression of the floor response spectra computation which delays the equipment calculation and generally brings inaccuracy in the information transmission.

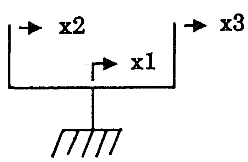

Figure 1.
Structure model

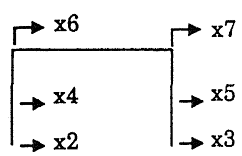

Figure 2.
Equipment model

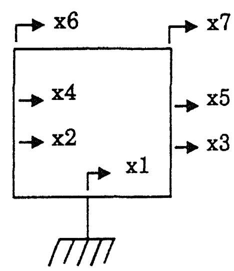

Figure 3.
Reference model

$$M^S = \begin{bmatrix} 500 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \qquad M^E = \begin{bmatrix} .1 & 0 & 0 & 0 & 0 & 0 \\ 0 & .1 & 0 & 0 & 0 & 0 \\ 0 & 0 & .1 & 0 & 0 & 0 \\ 0 & 0 & 0 & .1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K^S = \begin{bmatrix} 200000 & 0 & -200000 \\ 0 & 500000 & -500000 \\ -200000 & -500000 & 1700000 \end{bmatrix} \qquad K^E = \begin{bmatrix} 420 & -20 & -400 & 0 & 0 & 0 \\ -20 & 420 & 0 & -400 & 0 & 0 \\ -400 & 0 & 1400 & 0 & -1000 & 0 \\ 0 & -400 & 0 & 400 & 0 & -400 \\ 0 & 0 & -1000 & 0 & 1000 & 0 \\ 0 & 0 & 0 & -400 & 0 & 400 \end{bmatrix}$$

Figure 4.
Structure mass and stiffness matrices

Figure 5.
Equipment mass and stiffness matrices

Table 1: Reference model results

mode		str	str	equi	str	equi	equi	equi
	type	str	str	equi	str	equi	equi	equi
	number	1	2	1	3	2	3	4
frequency (hz)		2.6965	4.0259	6.4597	7.4144	8.4127	16.323	19.758
damping ratio (%)		19.999	19.998	2.0275	19.978	2.0017	2.0001	2.0000
x1		.41904	.24939	.00089	.33067	.00001	.00000	.00000
x2		.58818	.69428	.00014	-.2826	.00000	-.0000	.00000
x3		1.4893	-.4145	-.0001	-.0746	-.0000	-.0000	-.0000
x4		.74207	.93509	-1.223	.55723	-.0142	.00329	-.0000
x5		1.5655	-.4416	-.1004	-.0799	.05523	.00004	.00111
x6		.84265	1.0262	-1.943	1.0944	-.0185	-.0021	.00001
x7		1.6436	-.4386	-.3097	-.0496	.15480	.00004	-.0004

Table 2: Forced structure modes method results

mode		equi	equi	equi	equi	str	str	str
	type	equi	equi	equi	equi	str	str	str
	number	1	2	3	4	1	2	3
frequency (hz)		6.4594	8.4111	16.323	19.757	2.6970	4.0265	7.4133
damping ratio (%)		2.0000	2.0000	2.0000	2.0000	20.000	20.000	20.000
x1		.00000	.00000	.00000	.00000	.41909	.24956	.33135
x2		.00000	.00000	.00000	.00000	.58816	.69460	-.2827
x3		.00000	.00000	.00000	.00000	1.4895	-.4147	-.0748
x4		-1.227	-.0142	.00328	-.0000	.74196	.93520	.56105
x5		-.1005	.05508	.00004	.00111	1.5656	-.4417	-.0796
x6		-1.949	-.0185	-.0021	.00001	.84250	1.0262	1.1005
x7		-.3104	.15427	.00004	-.0004	1.6436	-.4387	-.0484

This method will thus give as better results than the floor response spectra method as the complexity of the problem increases. This complexity is defined by the occurrence of cross effects, differential displacements, rotations at support d.o.f. and a great number of equipment d.o.f..

The validity limits of this method must therefore be recalled, i.e.:

- linear behaviour of both the structure and the equipment,
- equipment very light with respect to the structure,
- low damping,
- necessity of a modal analysis of the structure.

These limits are in fact the same as those of floor response spectra transfer method.

A particularly interesting case for using this method is the case of an equipment supported by several independant structures.

It is then possible to consider each mode of a support structure as having zero displacements at the interface d.o.f. with the other upport structures.

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