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Dynamic characteristics of large multiple degree of freedom shaking tables

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ABSTRACT: Large servohydraulic shaking tables are critical tools in the earthquake engineering research and qualification laboratories throughout the world. They are one of the most effective ways to subject specimens to dynamic events similar to the in situ real earthquake events. However, the accurate control of such shaking tables is one of the most difficult technical challenges for today's engineers. In addition to the variable dynamic characteristics of the specimens tested, the components of the total shaking table test system all have inherent dynamic characteristics that are in the testing range of interest. This paper examines the relationships between the achievable system dynamic performance and these inherent characteristics of the system and specimen .

1 INTRODUCTION

A shaking table is a platform excited with servohydraulic actuators to generate artificial earthquakes and other dynamic testing signals of interest in the laboratory. Shaking tables have been used for many years in earthquake engineering research. These tables are used to evaluate the dynamic characteristics of the specimens fastened to them. The number of active degrees of freedom has increased from single axis horizontal testing to systems with six active degrees of freedom. The size has increased from small 1 meter x 1 meter tables for small scale tests up to the giant 15 m x 15 m table with 1000 ton specimen capacity located on Tadotsu Island in Japan.

The ideal shaking table would be a rigid testing instrument that exactly followed the commanded motions. However, as the physical size, maximum frequency of operation and the number of degrees of freedom increases, the significance of the shaking table being a dynamic system itself with its own response characteristics to the command signal, becomes more important. There exist physical limitations due to the size and scale of the shaking table and specimen-table interactions which cause errors with respect to the desired motion waveform. Even the smaller tables will show this when run at higher frequencies for similitude studies.

Based on accumulated field observations and research on a large number of installed shaking tables, various observed dynamic effects can be modeled in a simplified lumped mass way to demonstrate their effect on the system's dynamic response characteristics. When combined with a conventional practical control system using position, acceleration, and force feedback transducers, an analytical system of linearized differential equations can be generated. This system demonstrates the significant dynamic characteristics of the specimen-shaking table system. In control theory language, the analytical 'plant' model of what is to be controlled has been compared with field data characteristics, and reasonable qualitative correlation has been obtained. The dynamic motion signal tracking performance can then be calculated utilizing a practical real time control system model.

2 PHYSICAL SYSTEM DESCRIPTION

A typical large scale shaking table system is shown in Figure 1 in plan and elevation view below. The table is moved by servohydraulic actuators and the excitation forces are reacted by a large reaction mass which is often isolated from the surrounding laboratory building. The table or platform structure is most commonly a welded up steel structure with torsionally stiff shell and internal stiffening honeycomb, although reinforced concrete and aluminum structures also exist. The specimen to be tested is fixed to the table surface with preloaded bolts and tie rods.

The servohydraulic actuators are designed for low friction with full angular swiveling spherical bearings at the head and base. Because of the large hydraulic fluid flows and frequency requirements of the desired testing motions, high performance servovalves are required. The table shown in the figure is actuated in all six rigid body degrees of freedom: longitudinal, lateral, vertical, roll, pitch, and yaw by four longitudinal, four lateral, and four vertical actuators with cylinders, pistons, and rods.

The reaction mass is constructed of reinforced concrete and has a typical mass ratio of 30 to 50 times the specimen-table mass. By Newton's law of action-reaction this generates a reaction mass motion of between 2 and 3 percent of the table's resultant motion for frequencies above the supporting stiffness resonance and within the rigid body frequency range of the reaction mass structure. Some site conditions allow the reaction mass to rest directly on the supporting soil, but for environmental and neighborhood considerations, the reaction mass is often isolated by air suspension springs and the resulting low frequency resonance damped by heavy duty automotive type shock absorbers.

For the purposes of this paper a hypothetical shaking table can be defined by the physical parameters in the following table:

Table 1. Hypothetical Shaking Table Parameters

Size of platform		6.0x6.0 m
Specimen mass		50,000. kg
Table mass		40,000. kg
Actuator moving mass		5,000. kg
Specimen height		6.0 m
Specimen overturning moment		1000. kN-m
Table motion limits:		
Displacement, m	-longitudinal, lateral	± 0.2
	-vertical	±0.1
Velocity, m/s	-longitudinal, lateral	±1.0
	-vertical	±0.5
Acceleration, g	-longitudinal, lateral	±1.0
	-vertical	±0.5
Frequency of operation		±.4-40. hz
Actuator force (12, each unit)		±250. kN
Foundation mass		4,000,000. kg
Foundation dimensions		16.x16.x7. m

Although this represents a relatively large shaking table system, it is a reasonable set of parameters when compared to the larger facilities planned and existing today.

The problem of dynamic response of the shaking table itself is caused by the physical size, common practical economical construction materials, and the desired frequency range of operation. The basic tradeoff of control theory between accuracy and stability arises here. In the attempt to raise the gain of the control loops, the lowly damped structural resonances of the physical system cause the loops to go unstable. Compensators can be designed, and modern control theories can be applied, but the multitude of resonances in the frequency range of interest are too numerous to make this a practical task. For example, an examination of the lowest 19 natural frequencies of a completely free square plate shows 5 groups of 2 to 4 natural frequencies within 5 percent of each other with very different mode shapes (Leissa, 1969).

In addition, there is the variation in motion across the table surface itself. To transfer the motion across the table from the actuators to the specimen requires some finite deflection which causes larger and larger acceleration deviation as the frequency increases. For example, the basic torsional wave speed defines the

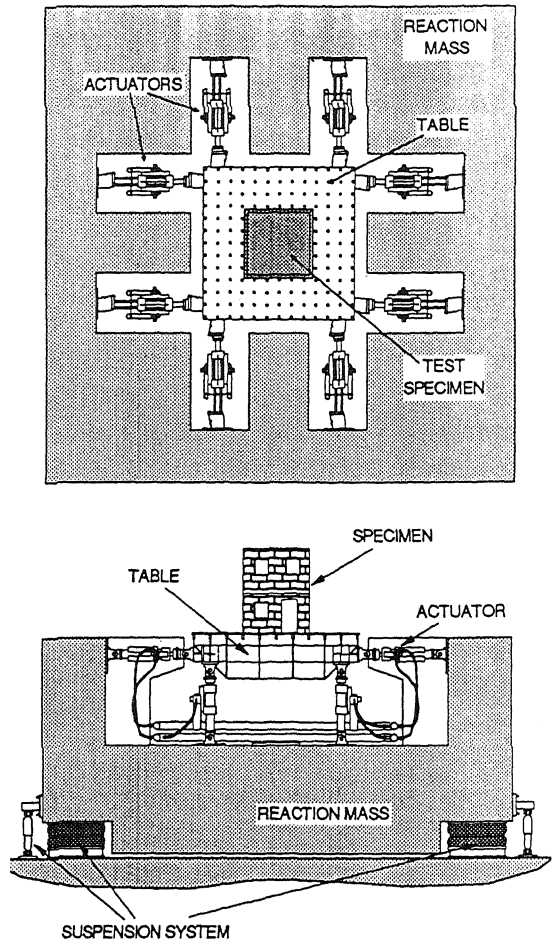

Figure 1. Typical shaking table mechanical configuration - plan and elevation view.

ideal possible upper limit resonant frequency in the free-free boundary condition for the table and foundation structures (Clark and Burton, 1978).

The following resonances were determined to be significant in effecting the dynamic performance of the shaking table system: 1) the reaction mass on the airspring/shock absorber isolation system, 2) the reaction mass internal flexibility, 3) the hydraulic actuator load train compliance (swivel joints and local stiffness of the reaction mass/actuator and the local stiffness of the table/actuator), 4) the hydraulic oil column bulk modulus stiffness, and 5) the lateral bending ('bow string') of the actuators (Kleine-Tebbe and Hirsch, 1983), and 5) the internal table structural flexibility resonances.

3 MATHEMATICAL MODEL

3.1 Mechanical-hydraulic

A linearized, lumped parameter model can be developed to study the complex dynamic response of the shaking table system. Previously performed studies, (Clark and Cross, 1984), focused on the sensitivity of specimen dynamics to the use of specimen force feedback compensation. Calculations were performed at the time assuming a rigid table, rigid reaction mass, and rigid actuator load train. The mathematical model presented in this section and used for this paper's analysis includes a first mode approximation for the significant resonances listed above.

Although many shaking tables in operation today can be programmed in the full rigid body six degrees-of-freedom, the most important characteristics can be studied by considering the longitudinal and pitch table degrees of freedom. This is one of the most important test requirements for the shaking table system: program as pure and as high a fidelity longitudinal motion as possible, while maintaining pitch as close to zero as possible.

By restricting the analysis to longitudinal and pitch, the computations can be kept to a reasonable amount of time, including design iterations. Table 2 gives the resonant frequencies and damping ratios calculated for the system shown in Table 1. A conceptual diagram showing the stiffness, damping, and mass modeling elements is shown in Figure 2. The values shown in the table and the numerical values of the masses, springs, and dampers used in the calculations were obtained parametrically from field observations of component acceleration transfer function measurements.

Table 2. Model natural frequencies and damping ratios

	Natural Frequency, hz		Damping, %	
	Long	Pitch	Long	Pitch
Air spring suspension	.80	1.0	16.	20.
Reaction mass flexibility	60.	70.	1.	1.5
Load train compliance	40.	60.	5.	5.
Servovalve response	100.	100.	70.	70.
Oil column compliance	20.	40.	10.	15.
Actuator lateral bending	45.	50.	5.	5.
Table internal flexibility	110.	90.	1.	1.
Specimen flexibility	10.	80.	1.	1.

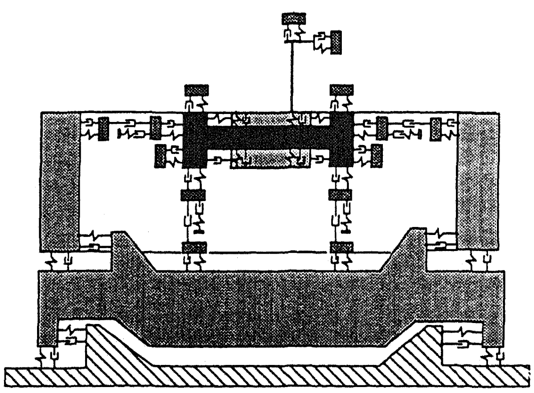

Figure 2. Longitudinal-pitch mechanical mathematical diagram.

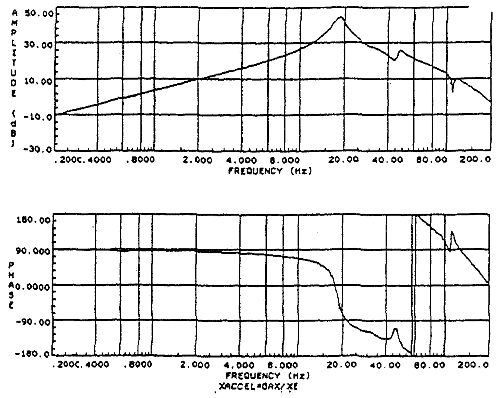

Figure 3. Longitudinal acceleration to longitudinal servovalve command transfer function frequency response - bare table.

In addition, to the mechanical and hydraulic modeling shown in Figure 2, the servovalve flow response to command signal was modeled as a second order subsystem transfer function. The hydraulic oil was modeled in 2 parts, an incompressible part, and a compressible part. The output of the servovalve and the pumping was the incompressible part. This was used as the base input to the oil column compliance spring (the compressible part).

From the linearized dynamics of these elements, a set of 20 coupled, ordinary differential equations were derived. These equations were evaluated as a function of frequency, and the unknown motions solved with logarithmic spacing for a frequency range from .2-200 hz with 800 lines of resolution. Although the shaking table system has a specified operating frequency range from .4-40 hz, the dynamic tracking performance of the system is effected by coupled dynamics 5-10 times the highest frequency of operation, especially in determining the tradeoff decisions between system stability and accuracy.

Figure 3 shows the 'open loop' or 'plant to be controlled' characteristics for the bare table (without specimen mass or dynamics), and Figure 4. shows the same frequency response function for a 50,000. kg. specimen with effective first mode line of moment at 2. meters vertically above the table surface, and a lateral offset of 0.5 meters. The plots show the frequency response function with the horizontal axis logarithmic in frequency and the amplitude expressed in decibels (20 times the decimal logarithm of the magnitude) as

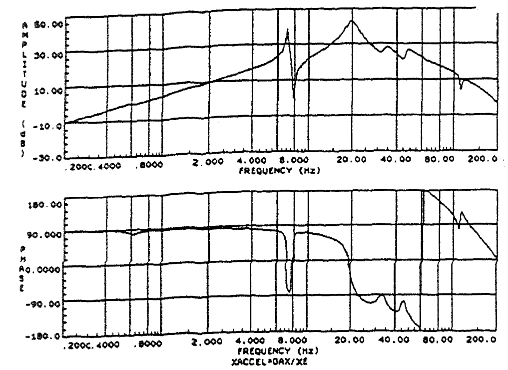

Figure 4. Longitudinal acceleration to longitudinal servovalve command transfer function frequency response - 50,000. kg specimen.

typical in control system signal analysis. The phase angle is shown with horizontal range between -180. to +180. degrees.

The system output to be controlled is the table platform acceleration average as sensed by distributed filtered accelerometers with the servovalve command as input. The interaction of subsystem dynamic characteristics (as given in Tables 1 and 2) can be seen in Figures 3 and 4. At low frequency the response behaves like a differentiator on the displacement of the oil compliance (amplitude increasing with frequency). Then the longitudinal oil column resonance occurs at about 20 hz, and after the resonance the system behaves like a mass integrator (decreasing amplitude with frequency).

Small anomalies on these dominant dynamic characteristics can be observed: the small phase bump at .6 hz from the reaction mass and the 45 hz resonance-antiresonance from the actuator bending dynamics, and finally the sharp anti-resonance observed at 110 hz from the high amplification of the welded steel table structure. The interaction of the servovalve dynamic response and the load train force impedance is not directly observable on the acceleration signals.

Figure 4 shows the same signal, longitudinal acceleration output to the servovalve command input with the mounting to the table of the 50,000. kg, 6 meter high specimen with 10 hz natural frequency longitudinally and 80 hz vertically. The large specimen to table mass ratio results in shifting the 10 hz longitudinal natural frequency down to a 7 hz resonance, 8 hz anti-resonance, pair. The interaction of the coupling of the specimen also causes some additional dynamics response variation at from the reaction mass .6 hz, and from 30 to 70 hz from the pitch oil column and the actuator bending resonances.

The other significant variable to be controlled is the pitch acceleration of the table. The pitch degree of freedom is excited from the overturning moment coupling of *the tall specimen. Figures 5 and 6 show* the pitch acceleration response to longitudinal axis servovalve excitation. Without specimen, Figure 5,

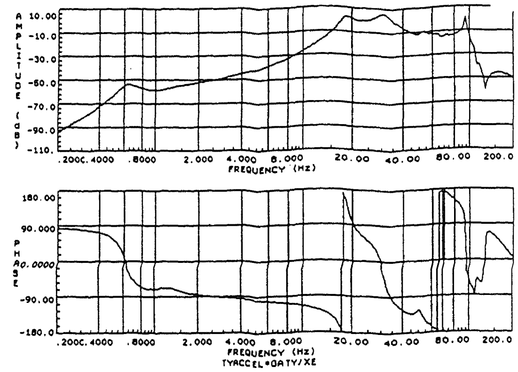

Figure 5. Pitch acceleration to pitch servovalve command transfer function frequency response - bare table.

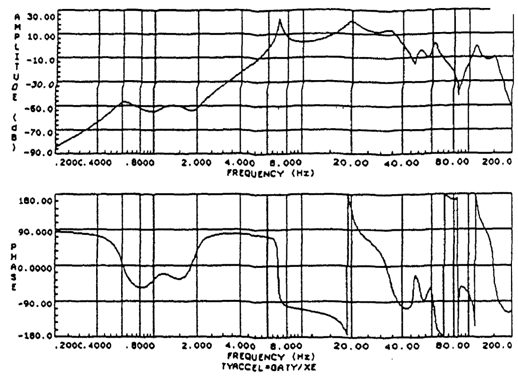

Figure 6. Pitch acceleration to pitch servovalve command transfer function frequency response - 50,000. kg specimen.

the response is much lower than the longitudinal acceleration (80 dB at .2 hz) but at 20 to 30 hz the difference is less 40 dB. Additionally, if a longitudinal monitor point is chosen 6 meter above the table surface, 16. dB of amplitude must be added to the value shown. The pitch diagram shows more variation in dynamic response from 90 to 130 hz. When compared with the longitudinal diagram, from the effects of actuator bending, table flexibility, and reaction mass flexibility.

Figure 6. shows the pitch acceleration response to longitudinal servovalve command after the specimen is mounted. Because of the large coupling and the relatively small pitch response to longitudinal excitation , compared to longitudinal response, there is much more dynamic response variation in the specimen induced pitch response over a broader frequency range. For example, the reaction mass dynamics at .4-2. hz is more visible, and the frequency range from 40 to 150 hz shows much more variation compared to the longitudinal response, caused by the interaction of the *dynamic elements.*

Not shown is a similar set of longitudinal and pitch acceleration responses to pitch servovalve command.

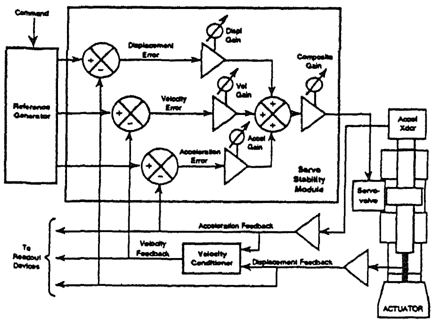

Figure 7. Three variable control block diagram.

These are similar in nature to Figures 3, 4, 5 and 6. shown. In normal horizontal seismic testing, there is not a direct desired pitch command.

In summary, these figures characterize the linearized dynamic model of the object (system) to be controlled.

3.2 *Electronic*

The servocontroller modeled for the shaking table system is shown in Figure 7. A control technique reduced to practice is known as three variable control. It features two benefits demonstrated in laboratory use. Three feedback sensors or transducers are used on an averaged longitudinal and pitch degree of freedom basis: displacement, acceleration, and actuator force.

As shown in Figure 7, three variable control simultaneously achieves closed loops on displacement, velocity, and acceleration. A command signal reference generator filter obtains three kinematically compatible signals for the commands from a single desired command. Three summing junctions subtract the feedback for each of the three variables from the corresponding reference signals, forming the three errors. The three errors are then passed through user set gains and summed to form the composite error. Electronic lead is available by introducing the velocity and acceleration signals into the composite error with additional weightings.

Velocity feedback is obtained with crossover filters combining the displacement and acceleration signals together. The displacement signal is used to obtain the low frequency component of the velocity, and the acceleration transducer is used to compute the high frequency component of the velocity. The result is an accurate, wide bandwidth, and low noise velocity measurement without using additional velocity transducers.

With the dynamics of this technique, more accurate real time feedback control over a wider bandwidth is provided by the displacement loop at low frequencies and by the acceleration loop at high frequencies. In addition, the presence of the three reference signals (displacement, velocity and acceleration) adds a quadratic lead term to the system dynamics. This may be tuned to cancel the lowest frequency lag of the system, which in a large earthquake simulator is a result of the table plus specimen mass acting on the

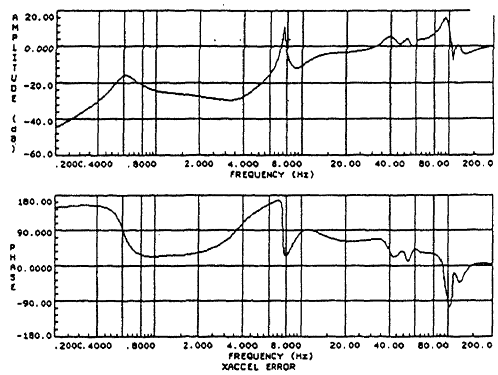

Figure 8. Longitudinal acceleration error to longitudinal command acceleration transfer function frequency response.

actuator oil column compliances. Since the resulting frequency may be quite low relative to the maximum desired operating frequency, this three variable control technique is often helpful.

The differential equations for the control algorithm presented here was added to the 20 plant equations in 20 unknown motions. In addition actuator force feedback is effective in stabilizing the system with heavy specimens. This signal is not shown in Figure 7, but was included in the calculated response.

4 CLOSED LOOP FREQUENCY RESPONSE

With the mathematical description of the object to be controlled and the control algorithm for both the average longitudinal and the average pitch degrees of freedom described above, the tuning of the system for best closed loop performance can proceed. As described above, gains on the feedback and feedforward variables of displacement, velocity, and acceleration can be tuned as well as acceleration rate (jerk) and actuator force feedbacks. These state variables of the total system can be tuned manually for best performance.

The resulting optimum tuning system error responses are shown in Figures 8 and 9. They both are the results of tradeoffs between accuracy and stability. A simple way to think about the control system ability is that there are enough feedback control signals or energy state variables to stabilize a certain number of dynamic modes of response as the loop gain is raised to achieve higher tracking accuracy.

Since all real systems are in reality distributed parameter systems, there will always be more dynamic modes of response than there are stabilizing feedback signals. Therefore, in the process of raising the gain by online feedback control, or adaptive feedback control schemes, the system will always eventually go unstable. Therefore, a compromise between high accuracies at low to mid range operating frequencies and instabilities or ringing oscillations at higher frequencies must be found.

Figure 8 is a plot of the longitudinal sinusoidal

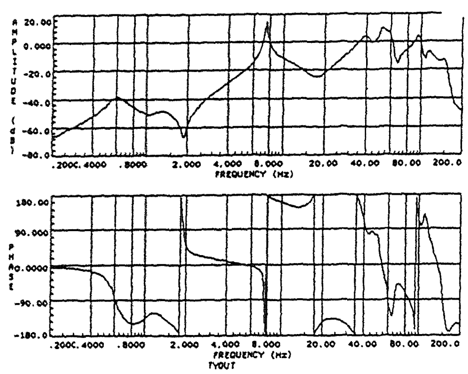

Figure 9. Pitch acceleration to longitudinal command acceleration transfer function frequency response.

amplitude tracking error as a function of frequency after tuning for good response. At the low frequency of .2 hz, we see an error of -44. dB or .63 percent, with a peak of -16. dB or 15.8 percent at the reaction mass resonance of .6 hz, another peak at the effective specimen resonance of 7.5 hz of 316. percent (10 dB), a local maximum at the actuator bending bow string frequency of 40 hz (158. percent) (3.97dB) and finally a largest global maximum in the frequency range (.2-200. hz) of +16. dB or 630. percent of desired program at 95. hz. Within the desired operating range or .4-40. hz, the worst error is +58. percent, although any actual physical system with significant nonlinearities would excite the 95 hz from higher harmonics.

The other significant error comes from the undesired pitch response from longitudinal excitation. Figure 9 is a plot of the pitch response acceleration amplitude and phase when referenced to the longitudinal program unity amplitude and zero phase.

The first maximum pitch acceleration response occurs at .6 hz (reaction mass isolation interaction resonance) at -40. dB. (1. percent) and at the effective specimen natural frequency of 7.5 hz, +14. dB or (501. percent) with a final maximum of +18. dB (251. percent) at 50. hz.

If longitudinal overtest due to pitch is monitored at the top of the specimen location (6. meter), then an additional +2. dB after equal linear scaling must be added to the rotational acceleration results. These are the practical realtime feedback control system performances that can be achieved using the physical model, the control system, and tuning described. A good discussion of a spcific case of the phemomena described was reported by the University of California, Berkeley, Shaking table laboratory (Rinawai, Clough, and Blondet, 1988).

5 CONCLUSIONS

The physical size and current practical construction materials dictate the inherent dynamic characteristics of servohydraulic multiple degree of freedom shaking tables. This paper has modeled a typical large system and demonstrated the practical performance achievable based on the assumptions of the model. As the frequency of excitation increases, the errors also increase. The performance demonstrated correlates qualitatively with actual systems in use.

Future improvements may be possible as new technologies are developed and applied. An example could be the use of multiple distributed actuators and sensors applied to the shaking table platform structure. Also, the use of composite materials for the table structure, with improved stiffness to weight and resulting dynamic properties, could be applied. Finally, as technology advances in real time digital control and computing, more complex control algorithms may be implemented.

A careful cost to benefit analysis for the application of these technologies to the shaking table laboratory must be conducted, however. Although the ideal goal of perfect fidelity of command acceleration reproduction has been defined, it has been observed that the real earthquake and excited structure's boundary conditions are random and flexible, respectively.

6 ACKNOWLEDGMENTS

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