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A method for prediction of hysteretic response to earthquake excitations

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ABSTRACT: A new method for prediction of the stochastic response of hysteretic systems to earthquake excitations is presented. The mathematical model used for ground acceleration can express the nonstationarity of amplitude, strong motion duration and frequency content. The nonlinear model used for hysteretic systems can show the nonlinear behavior from smooth to nearly elastic-perfectly plastic and can consider the stiffness and strength deterioration easily. The response evaluation method is a combination of frequency domain and time domain analyses based on dividing the history of stochastic response into two sets of motions including oscillations and drifts. Comparison of numerical results obtained by the proposed method with those of time history analysis show the high precision and reliability of the proposed method.

1 INTRODUCTION

In earthquake engineering one of the main goals is the optimum aseismic design of structural systems. This means that the design should be both reliable and economical. Stochastic response analysis is an approach which improves reliability and economy of the design. To increase the precision of this approach the nonstationarity of earthquake excitations and the hysteretic behavior of structural systems have been taken into consideration in recent decades. Kaul (1972) has investigated inelastic response of offshore towers by using the Kolmogorov-Fokker-Planck differential equation. Some other methods based on Markov vectors and Galerkin technique and also the stochastic linearization technique have been presented of which the latter can consider the non-Gaussian properties of the response. All of these methods use the Kryolov-Bogoliubov approximation. A method, which does not need this approximation has been also presented by Baber and Wen (1979). In addition to linearization techniques some other methods have been also used for hysteretic response analysis. Goto and Iemura (1973) have been used the energy balance criteria. Irschik and Ziegler (1985) have presented a method based on dividing the displacement of the system into two parts including drift and linear vibration. Ito differential equation and closure techniques have been also used, and Paparizos (1986) has used the district Markov process. The main disadvantage of linearization techniques is their doubtful precision as expressed by some researchers a varying error between 0 and 20% for them. The basic reason for this varying error is the hysteretic behavior of structures as described by Hosseini (1991), and will be discussed briefly in this paper.

On the other hand the models used by researchers for hysteretic behavior of structural systems include bilinear and elastic-perfectly plastic models, Wen model and a few other models which all have some disadvantages like insufficient or imprecise matching with real behavior, difficulty in system identification experiments and showing nonphysical behavior in some special cases, as shown by Jayakumar (1987).

Regarding the shortcomings of existing models and methods, in this paper by using a powerful nonstationary model for ground acceleration and a recently introduced model for hysteretic systems (Hosseini and Ghafory Ashtiany, 1991) a new approach is presented for stochastic response evaluation based on a special combination of frequency domain and time domain analyses and dividing the response history into two separate sets of motions each consisting of either oscillations or drifts. Numerical calculations show the high efficiency of the presented nonlinear hysteretic model and the proposed stochastic method specially in the case of nonstationary excitations. Comparison of the results of proposed method with those of time history analysis show the good precision and reliability of the presented stochastic formulation.

2 THE NONSTATIONARY GROUND ACCELERATION

Among different models presented and used for nonstationary ground acceleration the model of production of a stationary process $\ddot{x}_o(t)$ and a deterministic envelope function $e(t)$ is one of the most advantageous models, which can be expressed by:

$$\ddot{x}_g(t) = \ddot{x}_o(t) \cdot e(t) \tag{1}$$

In this model the nonstationarity of duration and amplitude of the process $\ddot{x}_g(t)$ can be satisfactorily expressed by $e(t)$ and if the power spectral density function (PSDF) of process $\ddot{x}_o(t)$ is considered to be:

$$S_{\ddot{x}o}(w,t) = \sum_{i=1}^{3} \frac{S_i[s_o + (2\beta_{si} w / w_{si})^2]}{(1 - w^2/w_{si}^2)^2 + (2\beta_{si} w / w_{si})^2} \tag{2}$$

as suggested by Hosseini (1990), the nonstationarity of frequency content of process $\ddot{x}_g(t)$ can be easily taken into account. In equation (2) S_i is the spectral intensity of the i^{th} mode of the soil structural model, s_o is a constant between 0 and 1, and $w_{si} = w_{si}(t)$ and $\beta_{si} = \beta_{si}(t)$ are time-dependent modal frequencies and damping ratios given by:

$$w_{si}(t) = w_{oi} - a_i \cdot t \qquad \beta_{si}(t) = \beta_{oi} + b_i \cdot t \tag{3}$$

where w_{oi} and β_{oi} are respectively initial frequency and damping ratio of the i^{th} mode of the soil structural model and a_i and b_i are constants. Eqs. (3) show decreasing predominant frequency and increasing damping ratio of the soil model which have been suggested by some researchers (Nozawa et. al. (1988)). Figures 1 and 2 show PSDF of processes $\ddot{x}_o(t)$ and $\ddot{x}_g(t)$ respectively. As it is seen the predominant frequency of processes has been shifted gradually to lower frequencies.

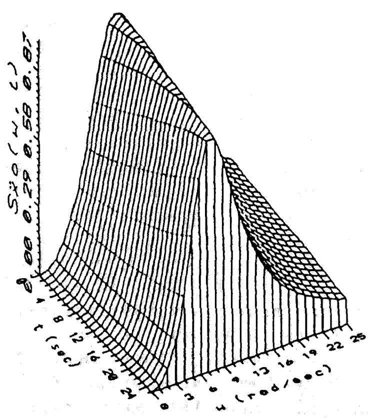


Figure 1. PSDF of process $\ddot{x}_o(t)$

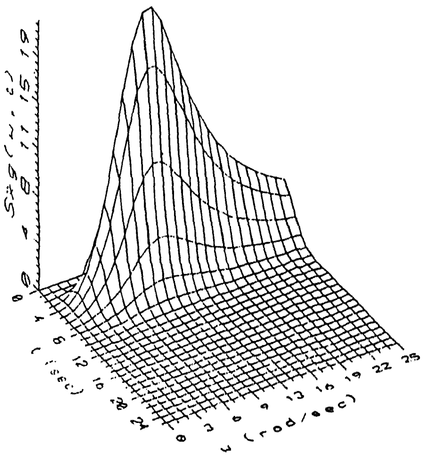


Figure 2. PSDF of process $\ddot{x}_g(t)$

This model of the nonstationary ground acceleration has been used for stochastic response analysis of linear systems by Hosseini and Ghafory-Ashtiany (1989) and for nonlinear systems by Ghafory-Ashtiany and Hosseini (1991) to derive the closed form formula for the response. Application of the model to hysteretic systems has been also shown by Hosseini and Ghafory-Ashtiany (1991) and is developed in this paper.

3 THE NONLINEAR HYSTERETIC SYSTEM

To analyze the stochastic response of hysteretic systems it is necessary to have a mathematical model which can show the real behavior of hysteretic systems as precise as possible. For this purpose Hosseini (1991) has presented a simple mathematical function for expressing the hysteretic resistant force $r(x)$ versus displacement x of the system which for the virgin curve is:

$$r_v(x) = r_u \frac{\text{sign}(x)\{\exp[-A/(|x|/x_y + B)^P] - C\}}{1 - C} \tag{4}$$

where r_u is the ultimate resistance, x_y is the yielding displacement defined as $x_y = k/r_u$ in which k is the initial stiffness of the system, and A, B and C are positive quantities depending on P by:

$$A = \frac{B^P(P+1)}{P} \qquad B = \frac{(P+1)}{(1/C-1)} \qquad C = \frac{1}{\exp(1+1/P)} \tag{5}$$

P is called the model order and controls the rate of change of the system stiffness. Figure 3 shows different curves of the model for different values of P. It is seen that as P increases the model approaches the elastic-perfectly plastic state.

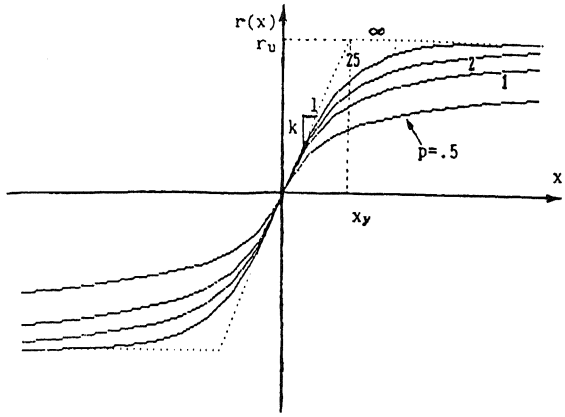


Figure 3. Virgin curves of nonlinear model

For descending and ascending curves based on Masing rules the following relation is used:

$$r_i(x) = r_{r,i} + 2r_u \cdot \mathrm{sign}(x - x_{r,i}) \frac{\exp[-A/(|x - x_{r,i}|/2/x_y + B)^P] - C}{1 - C} \quad (6)$$

where $r_i(x)$ is resistance value on the i^{th} branch of the hysteretic curves, and $x_{r,i}$ and $r_{r,i}$ are respectively the displacement and resistance of the system at i^{th} reversing. Regarding that the rate of system deterioration is proportional to the hysteretic dissipated energy as expressed by Baber and Wen (1979), the model is capable to show deterioration easily by supposing a linear variation for the ultimate resistance and the model order with respect to hysteretic dissipated energy, namely:

$$r_u(E_h) = r_{uo} - a_r \cdot E_h \qquad P(E_h) = P_o - a_k \cdot E_h \quad (7)$$

where E_h is hysteretic dissipated energy, r_{uo} and P_o are respectively the initial values of ultimate resistance and model order, and a_r and a_k are the reduction coefficients of resistance and stiffness respectively. Figure 4 shows the hysteretic curves of the model in different states. It is seen that the presented model has three advantages: 1) a simple and explicit mathematical form, which is useful in the stochastic analytical response evaluation, 2) capability of showing the behavior of a great variety of nonlinear softening system, which make possible to use the model for different structural systems, and c) having only three main parameters, which make the system identification very easy. Considering the advantages of presented model it can be introduced as an effective tool in stochastic response calculation of nonlinear hysteretic systems.

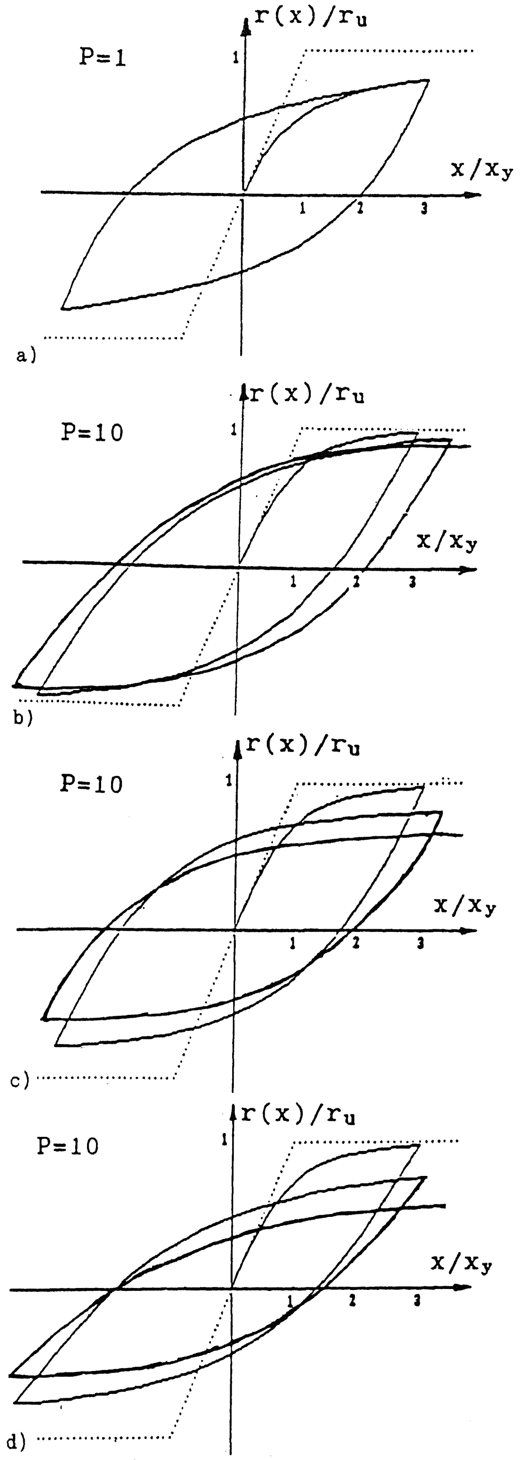


Figure 4. Hysteretic loops of the model in different states, a) without deterioration, b) with stiffness deterioration, c) with resistance deterioration, d) with combined deterioration

4 THE RESPONSE PREDICTION METHOD

There is two great differences between the behavior of hysteretic and elastic systems. The first is that in elastic system after oscillations even with very large amplitude the system returns to its initial static equilibrium state, whereas in hysteretic state if the system sustains a large displacement due to strong excitations, it does not return to its initial equilibrium situation and stops in a different position. The second difference is in the quality of change of system stiffness, which for elastic systems is gradual during the history of motion, but for hsyteretic systems is sudden at reversing instants of the motion. In fact in the hysteretic case at reversing instants stiffness of the system suddenly jumps to its initial value. This sudden change in the stiffness of the systems gives it a dual behavior, which can not be stated mathematically by a single equation. In the case of elastic-perfectly plastic behavior the motion of the system can be divided exactly into two separate sets of motions, namely oscillations and drifts. This can be expressed by:

$$m\cdot\ddot{x}_e(t)+ c\cdot\dot{x}_e(t)+ k\cdot x_e(t)=-m\cdot\ddot{x}_g(t) \qquad t_{e,i-1}\le t\le t_{b,i} \qquad (8)$$

$$m\cdot\ddot{x}_d(t)+ c\cdot\dot{x}_d(t)+ r_u=-m\cdot\ddot{x}_g(t) \qquad t_{b,i}\le t\le t_{e,i} \qquad (9)$$

where x_e and x_d are system displacement in elastic and plastic states respectively, and $t_{b,i}$ and $t_{e,i}$ are the beginning and ending instants of the i^{th} drift motion. In deterministic analysis these instants can be computed exactly. In stochastic analysis, finding $t_{b,i}$ instants is in fact a level crossing time problem, which has been solved only for stationary state. To solve this problem for nonstationary excitations paying attention to the conditions in which the amplitude of the system motion increases and exceeds a specified level is helpful. This exceeding can occur in two different situations. One is resonance and the other is intense shock. In the case of earthquake excitations the first situation is more reasonable. In fact it can be said that before going to plastic phase the system accomplishes at least one oscillation with a period near to its natural period, so it can be supposed that the motion is similar to an sinusoidal oscillation with the frequency w_d of the system, namely:

$$x_e(t)= x_m\cdot\sin(w_d t) \qquad (10)$$

where x_m is the maximum amplitude. On this basis the passage condition of the system from the level x_y can be written as:

$$E[x_e^2(t)]\ge x_y^2/2 \qquad (11)$$

Considering the response evaluation method of linear system presented by Hosseini and Ghafory-Ashtiany (1989), the mean square response (MSR) to nonstationary excitations can be calculated as a function of time by using a suitable time step. Then using the criterion given by relation (11) the passage time can be computed by interpolation in that time step within which the passage condition is satisfied. After passage the governing equation of motion is equation (9). In deterministic analysis the mathematical form of ground acceleration is known and equation (9) can be solved easily, but the solution form is such that the instant at which the system velocity becomes zero, namely the reversing instant can not be derived analytically. To have a solution with the desired mathematical form a simplification is used considering the little effect of damping in dissipating energy in comparison with the effect of plastic deformation of the system. Then by omitting the damping term of the left hand side of equation (9) it reduces to:

$$m\cdot\ddot{x}_d(t)+ r_u= -m\cdot\ddot{x}_g(t) \qquad (12)$$

By solving equation (12) one can obtain:

$$\dot{x}_d(t)= \dot{x}_e(t_b)+\dot{x}_g(t_b)-\dot{x}_g(t)-r_u(t-t_b)/m \qquad (13)$$

$$x_d(t)= x_e(t_b)+x_g(t_b)-x_g(t)+[\dot{x}_e(t_b)+\dot{x}_g(t_b)](t-t_b)-r_u(t-t_b)^2/(2m) \qquad (14)$$

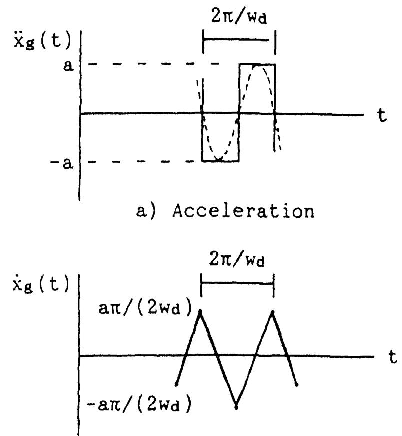
From equation (13) the duration of drift motion is obtained as:

$$t_e-t_b= m/r_u\cdot[\dot{x}_e(t_b)+\dot{x}_g(t_b)-\dot{x}_g(t_e)] \qquad (15)$$

Equation (15) shows that the drift duration depends on velocity of the system at the beginning instant of drift and also ground motion velocity at beginning and ending instants of drift motion of the system, which are all unknown. To overcome this difficulty it is suggested to use the energy balance equation at beginning and ending instants of drift motion and an assumption about the variation of ground acceleration before and during drift motion of the system based on the conditions in which the system state changes from oscillatory to drift. Energy Balance equation, neglecting damping effect, can be written as:

$$\tfrac{1}{2}m[\dot{x}_e(t_b)+\dot{x}_g(t_b)]^2= \tfrac{1}{2}m[\dot{x}_g(t_e)]^2+r_u[x_d(t_e)-x_e(t_b)] \qquad (16)$$

On the other hand, based on the necessary condition for change of system state from elastic to plastic phase, ground acceleration variation just before and during drift motion of the system can be assumed as a sinusoid with frequency w_d. With this assumption, duration can be calculated by equation (13), but not analytically. To have an analytical solution for drift duration the aforementioned sinusoid can be approximated by a rectangular wave as shown in figure 5.



a) Acceleration

b) Velocity

Figure 5. Assumed variation for ground acceleration and velocity for calculating system drift duration

The parameter "a" in figure 5 is the time-dependent average amplitude of ground acceleration and can be determined statistically by using recorded accelerograms. Based on the described assumption equations (13) to (16) reduce to:

$$\dot{x}_d(t)= (a-r_u/m)(t-t_b)+\dot{x}_e(t_b) \quad t_b<t_e<t_b+\pi/w_d \tag{17}$$

$$x_d(t)= \tfrac{1}{2}(a-r_u/m)(t-t_b)^2+\dot{x}_e(t_b)(t-t_b)+x_e(t_b) \tag{18}$$

$$t_e-t_b= -\dot{x}_e(t_b)/(a-r_u/m) \tag{19}$$

$$[1+ r_u/(m\cdot a-r_u)-a^2/(a-r_u/m)^2][\dot{x}_e(t_b)]^2- a^2\pi/[w_d(a-r_u/m]\cdot\dot{x}_e(t_b)= 0 \tag{20}$$

It should be noted that depending on values of "a" and $\dot{x}_e(t)$ the drift duration might be less or more than π/w_d, and if it is more than this value at first system displacement and velocity values at instant $t_e+\pi/w_d$ should be calculated by using equations (17) to (20) and then by using the equations of the second half of the assumed oscillation, the drift value and its duration can be calculated. After each drift, the system goes again to the elastic phase and the governing equation is equation (8), but there is in it an initial displacement of x_y with respect to the last equilibrium situation. Then, the system displacement after drifts is consisted of one free oscillation part x_f due to the initial displacement and one forced part due to the existing excitations, namely:

$$\underline{x}_e(t)= x_f(\theta)+ \int_0^\theta -m\cdot\ddot{x}_g(\tau+t_e)\cdot h(\theta-\tau)\cdot d\tau \tag{21}$$

where $\theta=t-t_e$. On this basis the mean square response with respect to the last static equilibrium position of the system can be written as:

$$E[\underline{x}_e^2(t)]= x_f^2(\theta)+ \int_{-\infty}^{\infty} |\underline{H}_n(w,\theta)|^2\cdot S_g(w,t)\cdot dw \tag{22}$$

in which:

$$\underline{H}_n(w,\theta)= \int_0^\theta e(\tau+t_e)\cdot h(\theta-\tau)\cdot \exp[-iw(\theta-\tau)]\cdot d\tau \tag{23}$$

Performing the described calculation in a recursive manner the drift displacements and their durations as well as the mean square response with respect to each new equilibrium situation of the system can be obtained easily.

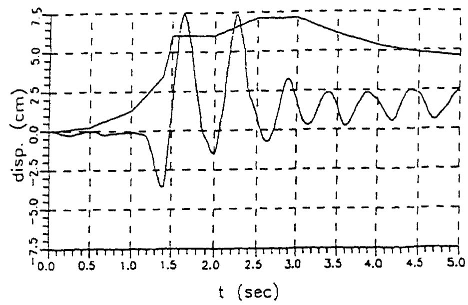
The explained procedure for stochastic response analysis which is exact for the elastic-perfectly plastic hysteretic systems can be used for smooth hysteretic system with a few modifications. In the smooth hysteretic case instead of exact linear vibration and plastic drift motion there is a linear-wise vibration in which the mean square response can be calculated by using the equivalent linearization technique in the nonstationary state presented by Ghafory Ashtiany and Hosseini (1991), and a plastic-wise motion in which the drift values can be estimated by using the explained procedure only by substituting r_u by an average value:

$$r_d= (r_y+ P\cdot r_u)/(P+1) \tag{24}$$

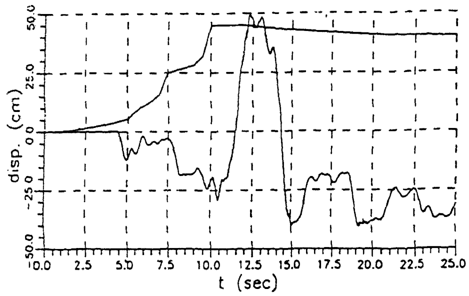
where r_d is the average resistance of the system during drift motions, $r_y=r_v(x_y)$ and P is the order of the hysteretic model. In equation (24) if P approaches infinity then r_d approaches $r_y=r_u$ as is expected according to the description given about the nonlinear hysteretic model.

5 NUMERICAL RESULTS

To show the efficiency of the proposed model and method, and the high precision and reliability of the predicted hysteretic response some numerical calculations have been done with different values of system initial frequency and damping, and also model order, and the result of stochastic method have been compared with those of time history analysis method. A sample of numerical values used for the hysteretic system characteristic is given in table 1 showing a case with medium x_y and another case with small x_y. The response curves of these to systems subjected to two different Iranian earthquakes one short and the other long



a)



b)

Figure 6. Comparison of stochastic and deterministic responses, a) system with medium x_y subjected to Naghan earthquake, duration=5 sec., b) system with small x_y subjected to Tabas earthq., duration=25 sec.

Table 1. Sample values used for the characteristics of the hysteretic systems

nat.freq.(cps)	damp.ratio (%)	P	x_y
2.0	10	5	0.05
2.0	10	5	0.005

duration are presented in figure 6 showing both stochastic and deterministic responses. Regarding that the stochastic procedure does not care the direction of drift motions the closeness of absolute values of these two responses are quite satisfactory.

6 CONCLUSIONS

Based on the numerical results it can be concluded that:

a) The nonlinear hysteretic model is an effective tool for stochastic response analysis of hysteretic systems.

b) The proposed response prediction method is an easy and useful procedure with high precision and reliability, specially in the case if nonstationary excitations.

c) Number of drifts depends on the strong motion duration of excitations. For short earthquakes usually one great drift occurs.

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