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Probabilistic system identification and health monitoring of structures

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ABSTRACT: Global health monitoring of a structure is approached by detecting any significant changes in its stiffness distribution through continual updating of a structural model using vibration measurements. A Bayesian probabilistic formulation is used to treat uncertainties which arise from measurement noise, modeling errors, and an inherent nonuniqueness common in this inverse problem.

INTRODUCTION

Increasing interest has been shown in using system identification approaches to develop a global means of structural health monitoring using vibration measurements from a structure (Natke and Yao, 1988; Chen, 1988; Das, 1990). The basic idea is to use dynamic test data to continually update the stiffness distribution of a model of the structure, and to use any observed local decrease in the stiffness to indicate the location and severity of possible damage. Even if the mass distribution is assumed to be modeled accurately, the inverse problem of using vibration data to determine the stiffness distribution based on a prescribed class of structural models, commonly leads to nonunique solutions. Additional difficulties are caused by measurement noise in the data, and because any mathematical model is only an approximation of the dynamics of a real structure. For these reasons, a reliable methodology for global health monitoring of real structures has not yet been demonstrated.

Beck and Katafygiotis (1991) have presented a general Bayesian probabilistic methodology for system identification applicable to both linear and nonlinear models. The interest here is in using this approach with linear models to detect changes in stiffness in a structure which may be due to damage. This would be useful, for example, if applied to small-amplitude ambient or forced vibration data to continually monitor an off-shore platform for possible fatigue damage caused by cyclic wave loading, or, following a severe loading event such as an earthquake, to use "before" and "after" vibration data from a structure to check for possible event-induced damage. If actual recorded strong earthquake motions are used from a building, care must be taken in interpreting the results since, in addition to the expected nonlinear behavior, past studies have shown that substantial changes in stiffness occur during an earthquake which are not related to structural damage. These changes are possibly due to such effects as temporary loss of stiffness from loosened nonstructural components, micro-cracking in concrete components, and softening in the foundations (see, for example, Beck, 1978; Nisar, Werner and Beck, 1992).

FORMULATION

A class of linear structural models, M_{N_d}, with N_d degrees of freedom, is chosen to model a structure. Thus, the usual multi-degree-of-freedom classically-damped linear model is used except that we allow for the fact that the contributions of the higher modes are usually negligible, so that the model response at the i^{th} degree of freedom corresponding to the measured response $y_i(n)$ at discrete time n, can be expressed as a superposition of N_m modal contributions,

$$q_i(n) \cong \sum_{r=1}^{N_m} q_i^{(r)}(n) \quad ; \quad N_m \leq N_d \tag{1}$$

It is assumed that the mass matrix of the model can be calculated sufficiently accurately from the structural drawings. The uncertain damping of the structure is modeled by uncertain damping ratios for each contributing mode, $\zeta_r, r = 1, \ldots, N_m$. Finally, the uncertain stiffness distribution is modeled by the stiffness matrix :

$$K = K_0 + \sum_{i=1}^{N_\theta} \theta_i K_i \tag{2}$$

where the uncertain nondimensional positive parameters $\theta_i, i = 1, \ldots, N_\theta$ scale the stiffness contributions K_i of each "substructure" and K_0 accounts for the stiffness contributions of those "substructures" with accurately known stiffnesses. Typically, the K_i would come from a finite-element model of the structure.

To resolve the stiffness distribution to high spatial resolution, one might think of choosing a θ_i for each of the bending, shear, axial and torsional stiffness contributions of each element, or at least one θ_i for each structural member. In a typical case, however, the substructures must be chosen on a coarser scale, otherwise the parameters become "unidentifiable" from the available vibration data. Local changes in stiffness in the structure are therefore "smeared" over the whole substructure in the model, reducing the sensitivity to damage and implying that localization of damage can only be done on the broader substructure scale. Despite these shortcomings, a workable methodology can serve as an important global tool to detect the presence of damage and to direct attention to the parts of the structure where the damage is located. A simple visual inspection of the "flagged" substructures, if any, could then be made, or some local non-destructive evaluation technique could be used to pinpoint the damage within the substructure.

Now suppose that dynamic tests on a structure have produced sampled inputs $Z_{1,N} = \{\underline{z}(n) \epsilon R^{N_I}: n = 1, 2, \ldots, N\}$ and outputs $Y_{1,N} = \{\underline{y}(n) \epsilon R^{N_o}: n = 1, 2, \ldots, N\}$, denoted collectively as the test data D_N. The measured input may consist of time histories of applied forces, or of earthquake-induced accelerations at the structural supports. The measured output is commonly the acceleration time histories at a set of "measured DOF" which is a subset of the DOF for the structural model (so $N_o < N_d$). It follows from the above that the vector of uncertain model parameters which must be determined from these dynamic data D_N is $\underline{a} = [\theta_1, \theta_2, \ldots, \theta_{N_\theta}, \varsigma_1, \varsigma_2, \ldots, \varsigma_{N_M}]^T$.

Beck and Katafygiotis (1991), following the approach originally formulated by Beck (1990), present useful results valid for large N, corresponding to a large number of data points in D_N, which is the typical case for dynamic test data. The posterior PDF (probability density function) which describes the relative plausibility of the uncertain parameters, based on data D_N, is then very peaked at some optimal parameters, indicating that locally they are the most probable values for the class of models M_{N_d}. In this case, if M_{N_d} is *system identifiable*, then the posterior PDF is given by the asymptotic approximation:

$$p(\underline{a}|D_N, M_{N_d}, P_N) \simeq \sum_{k=1}^{K} w_k G(\underline{a}; \hat{\underline{a}}_k, C_N(\hat{\underline{a}}_k)) \quad (3)$$

where $G(\underline{a}; \hat{\underline{a}}_k, C_N(\hat{\underline{a}}_k))$ is a multi-dimensional Gaussian distribution with mean $\hat{\underline{a}}_k$ and covariance matrix $C_N(\hat{\underline{a}}_k)$. Here,

$$\left[C_N^{-1}(\hat{\underline{a}}_k)\right]_{ij} = \frac{1}{2\hat{\sigma}^2} \left.\frac{\partial^2 J_N(\underline{a})}{\partial a_i \partial a_j}\right|_{\hat{\underline{a}}_k} \quad (4)$$

$$\begin{aligned} J_N(\underline{a}) &= \sum_{n=1}^{N} \|\underline{y}(n) - \underline{q}(n;\underline{a})\|^2 \\ &= \sum_{n=1}^{N} \sum_{i=1}^{N_o} [y_i(n) - q_i(n;\underline{a})]^2 \end{aligned} \quad (5)$$

$$\hat{\sigma}^2 = \frac{1}{NN_o} \min_{\underline{a}} J_N(\underline{a}) \quad (6)$$

and the K optimal parameters $\hat{\underline{a}}_k$ are all the global mimima of J_N, that is,

$$J_N(\hat{\underline{a}}_k) = \min_{\underline{a}} J_N(\underline{a}) \quad (7)$$

The weighting parameters in Eq. (3) are given by:

$$w_k = \frac{v_k}{\sum_{k=1}^{K} v_k}, \quad v_k = \det\left[C_N(\hat{\underline{a}}_k)\right]^{1/2} \pi(\hat{\underline{a}}_k) \quad (8)$$

where $\pi(\underline{a})$ denotes the prior PDF used to describe the engineer's judgement regarding the relative plausibilities, before the data D_N are utilized, of different models in M_{N_d}. Eq. (3) can be used to produce the updated predictive PDF for response to further input (Beck and Katafygiotis, 1991), and to calculate "damage probabilities" for each substructure.

In Eq. (3), P_N denotes a class of probability models giving the PDF of the output-error sequence $E_{1,N}$ which is the difference between the measured response $\underline{y}(n)$ and the corresponding model response $\underline{q}(n;\underline{a})$ over the discrete times $n = 1, 2, \ldots, N$. The particular choice of the class P_N used to derive the preceding results asserts that $E_{1,N}$ is a zero-mean stationary Gaussian white-noise sequence with covariance matrix $\sigma^2 I_{N_o}$, implying both temporal and spatial statistical independence. This means that the engineer's uncertainty concerning the value of the output error at a specified time and location is not influenced by knowing the output errors at other times, or other locations within the structure. The output error is a combined effect of measurement noise and modeling error, but for modern instrumentation, the former is often negligible compared to that of modeling error. Also, the output-error method used here is applicable even when the measured structural response does not correspond to the complete state vector of the model. This is important, since in practice this restriction usually applies.

SYSTEM AND MODEL IDENTIFIABILITY

The important problem of finding the set of all optimal parameters is a difficult one, since it involves finding all global minima of a non-convex function. First, we define *system identifiability* of the class of models M_{N_d} to mean that either there is a unique optimal structural model (*global system identifiability*) or there is a finite number $K > 1$ of optimal structural models (*local system identifiability*). If J_N in Eq. (5) does not have a finite number of global minima in the region of parameter space of interest, the class is *system unidentifiable.* In this case, the finite sums in Eq. (3) and Eq. (8) must be replaced by infinite sums or integrals, depending on whether the infinite set of optimal parameters is countable or uncountable.

Now define two models in class M_{N_d} as *output-equivalent* if they give the same output under the specified input $Z_{1,N}$. Let $S_{opt}(\hat{\underline{a}}; Z_{1,N})$ denote the set of all parameters $\underline{a}^*$ which give output-equivalent models to an optimal model given by $\hat{\underline{a}}$, then it is clear from Eq. (5) that each $\underline{a}^*$ in this set is also an optimal parameter. If $S_{opt}(\hat{\underline{a}}; Z_{1,N})$ has a finite number $K(\hat{\underline{a}})$ of optimal parameters in the region of parameter space of interest, then $\hat{\underline{a}}$ is defined to be *model identifiable* (*globally model identifiable* if $K(\hat{\underline{a}}) = 1$ and *locally model identifiable* if $K(\hat{\underline{a}}) > 1$). On the other hand, if $S_{opt}(\hat{\underline{a}}; Z_{1,N})$ is an infinite set, then $\hat{\underline{a}}$ is *model unidentifiable.* In analysing model identifiability for models in M_{N_d}, we can use the result of Beck (1978) that the modal damping factors of the modes contributing to the output are globally model identifiable, and the modal frequencies and modeshape components at the "observed" degrees of freedom are also uniquely specified by the model input and output. Therefore, any nonuniqueness in the optimal parameters is due to the stiffness parameters $\underline{\theta}$ because of missing modes and missing modeshape components of the contributing modes.

Beck and Katafygiotis (1991) present a new algorithm to determine $S_{opt}(\hat{\underline{a}}; Z_{1,N})$ for the class of linear models M_{N_d}. The algorithm methodically and efficiently searches the high-dimensional parameter space by following a finite set of one-dimensional curves corresponding to fixing $N_\theta - 1$ of the frequencies given by the "target" optimal model. This solves the model identifiability problem in many cases for an optimal model parameter obtained by applying a minimization algorithm to J_N defined in Eq. (5). This may not give the full set of optimal parameters required in Eq. (3), however, since there may be other optimal models which do not have exactly the same response at the "observed" degrees of freedom, but still give an equally good fit to the data, as measured by J_N. Of course, in the theoretical case of no measurement noise and no modeling error, system and model identifiability are equivalent. At present, the problem of fully resolving system identifiability is unsolved for real data.

EXAMPLE APPLICATIONS

By integrating the damping out of Eq. (3) and replacing the Gaussian distributions by delta functions for large N, the posterior PDF of the stiffness parameters $\underline{\theta}$ can be approximated by:

$$p(\underline{\theta}|D_N, M_{N_d}, P_N) \simeq \sum_{k=1}^{K} w_k \delta(\underline{\theta} - \hat{\underline{\theta}}_k) \qquad (9)$$

The weighting coefficients w_k can be calculated from Eq. (8) considering the particular case $\underline{a} \equiv \underline{\theta}$, but Katafygiotis (1991) showed that Eq. (8) can be replaced by the following:

$$w_k = \frac{v_k}{\sum_{k=1}^{K} v_k} \quad , \quad v_k = \pi_{\underline{\theta}}(\hat{\underline{\theta}}_k) J^{-1}(\hat{\underline{\theta}}_k) \qquad (10)$$

where $J(\hat{\underline{\theta}}_k) = |\nabla \underline{\omega}(\hat{\underline{\theta}}_k)|$ is the Jacobian of the transformation $\underline{\theta} \rightarrow \underline{\omega}(\underline{\theta})$ calculated at $\underline{\theta} = \hat{\underline{\theta}}_k$. The term $\pi_{\underline{\theta}}(\hat{\underline{\theta}}_k)$ reflects the contribution of the prior PDF to the weighting of the k^{th} optimal model. Before using the data D_N, each θ_i is assumed to be independently distributed with a prior PDF $\pi_{\theta_i}(\theta_i)$. This distribution is chosen subjectively and is selected to be of a convenient mathematical form roughly consistent with the engineer's judgement regarding the relative plausibilities of the different values of θ_i.

The following distribution is suggested for damage detection studies:

$$\begin{aligned} &\pi_{\theta_i}(\theta_i; \lambda_i) \\ &= c_1(\lambda_i)\frac{1}{\theta_i}\exp[-\frac{(\ln\theta_i - \lambda_i^2)^2}{2\lambda_i^2}] \;;\; 0 < \theta_i \leq 1 \\ &= c_2(\lambda_i)\exp[-\frac{(\theta_i - 1)^2}{2\mu(\lambda_i)^2}] \qquad ;\; \theta_i > 1 \end{aligned} \qquad (11)$$

where the functions $\mu(\lambda), c_1(\lambda), c_2(\lambda)$ are given by:

$$\begin{aligned} \mu(\lambda) &= \lambda[1 - \mathrm{erf}(\frac{\lambda}{\sqrt{2}})]\exp(\frac{\lambda^2}{2}) \\ c_1(\lambda) &= \frac{1}{\sqrt{2\pi}\mu(\lambda)}\exp(\frac{\lambda^2}{2}) \\ c_2(\lambda) &= \frac{1}{\sqrt{2\pi}\mu(\lambda)} \end{aligned} \qquad (12)$$

The proposed prior PDF $\pi_{\theta_i}(\theta_i)$ implies that the most probable value is $\theta_i = 1$ and there is an equal probability of one half for θ_i being below or above unity. Note that the K_i in Eq. (2) can always be chosen so that the most probable value for each θ_i is unity, by simply absorbing a factor into each K_i. The parameter λ_i is a *confidence* parameter describing how peaked the distribution is about $\theta_i = 1$; the more confident the engineer is in this nominal value of θ_i, the smaller the chosen value for λ should be. This prior PDF is plotted in Figure 1 for three choices of the confidence parameter $\lambda = 0.5, 1.0, 8.0$; all curves are plotted with a maximum of unity, although in applications they are correctly normalized to have unit area.

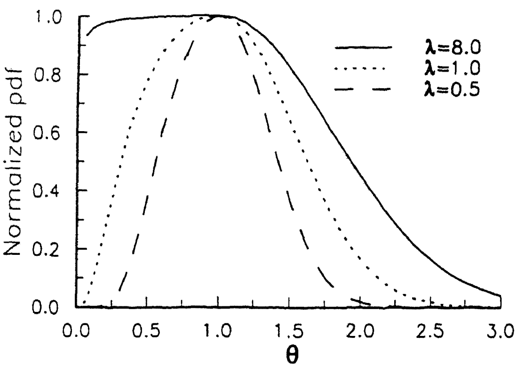

Figure 1. Scaled prior PDF for a stiffness parameter θ for three different values of the confidence parameter λ.

In order to illustrate the procedure for damage assessment for different cases of model identifiability, two different structural models are considered using numerically generated dynamic data: a six-story planar shear building model, and a finite-element model of a two-span bridge.

In both examples, it is assumed that the structure is *undamaged* to start with, and the corresponding structural model is "calibrated" using the measured response at some observed degrees of freedom for a known excitation. This can be done by minimizing the measure-of-fit function $J_N(\underline{a})$ in Eq. (5) to find one optimal solution $\hat{\underline{a}} = [\hat{\underline{\theta}}, \hat{\underline{\varsigma}}]^T$ and then employing the proposed algorithm to resolve the model identifiability of the optimal stiffness parameters $\hat{\underline{\theta}}$, so that the full set of output-equivalent optimal models is obtained. Next, it is assumed that the structure has been *damaged*. Utilizing a new set of measured input and output data corresponding to the damaged structure, and applying the same algorithms as in the undamaged case, all optimal models for the damaged structure are obtained. Each of the optimal solutions found in either the undamaged or damaged case is weighted in accordance to Eq. (10), where the prior PDF $\pi_{\underline{\theta}}(\underline{\theta})$ is assumed to be constructed from the proposed Eq. (11), controlled by a uniform confidence parameter λ for each θ_i. Integrating Eq. (9) leads to the cumulative marginal probability distributions of each stiffness parameter θ_i for both the undamaged and damaged cases. Any relative shift to the left of the cumulative marginal probability distribution of a parameter θ_i between the before and after damage cases, then indicates possible damage within the i^{th} substructure.

For the first example of the six-story shear building, the uniform mass distribution was assumed known and six stiffness parameters $\theta_i, i = 1, \ldots, 6$ were chosen to parameterize the stiffness distribution, with the i^{th} interstory stiffness being $k_i = \theta_i k_0$, where k_0 is a chosen nominal interstory stiffness. The undamaged case was taken to be a uniform stiffness distribution, that is, $\underline{\theta} = [1.0, 1.0, \ldots, 1.0]^T$, while the damaged case had the first interstory stiffness reduced to 70% of its original value and the stiffnesses of the remaining stories left unchanged, that is, $\underline{\theta} = [0.7, 1.0, 1.0, \ldots, 1.0]^T$. Assuming a "minimal" noise-free data case where only excitation at the base and the corresponding response at the roof is measured, eight different optimal solutions were found in the undamaged case (see Table 1a), while twelve optimal solutions were found in the damaged case (see Table 1b). The last three columns of these Tables display the weighting coefficients of each optimal solution for three choices of the confidence parameter $\lambda = 8.0, 1.0, 0.5$. Notice that, as expected, as the chosen value of λ becomes smaller, the weighting of the optimal solutions closest to the "a priori" most probable uniform stiffness distribution becomes larger. Figure 2 displays the cumulative marginal distributions for just two of the parameters θ_1 and θ_2 for both the *undamaged* and *damaged* cases and for the confidence parameter value $\lambda = 0.5$. Notice that in the case of θ_1, the curve corresponding to the damaged case has shifted to the left of the curve corresponding to the undamaged case,

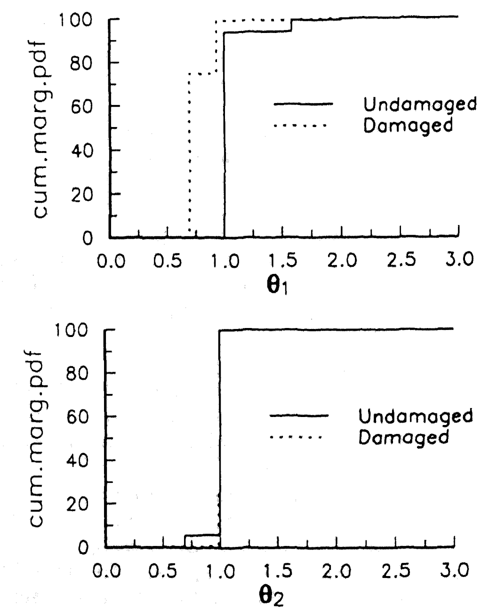

Figure 2. Cumulative marginal probability distributions of the parameters θ_1 and θ_2, scaling the first and second interstory stiffnesses, for the undamaged and damaged six-story shear building.

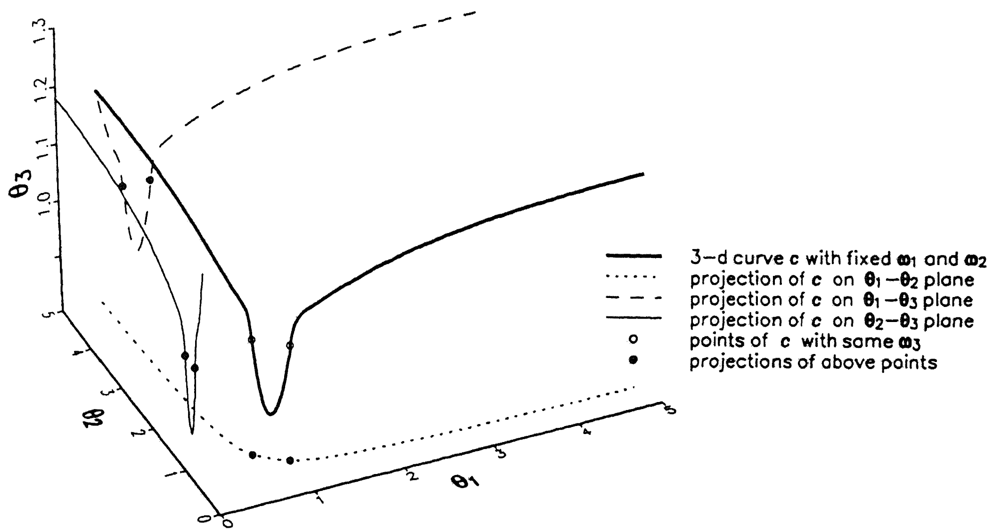

Figure 3. Three-dimensional curve c comprised of all solutions $\underline{\theta}$ corresponding to models having the same first two frequencies ω_1 and ω_2 as the damaged case $\underline{\theta} = [0.7, 1.0, 1.0]^T$ for the bridge model.

Table 1. "Output-equivalent" stiffness distributions for the six-story shear building. The weighting coefficient $w_k(\%)$ corresponding to each optimal solution is shown for three different values of the confidence parameter λ of the prior distribution. (a) Undamaged building. (b) Damaged building.

No.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	$w_k(\%)$ $\lambda = 8.$	$w_k(\%)$ $\lambda = 1.$	$w_k(\%)$ $\lambda = .5$
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	41.44	67.84	93.67
2	1.5848	0.6963	1.2875	0.7574	1.1766	0.7898	18.06	16.32	5.40
3	1.9970	0.7980	0.7095	1.3848	0.7113	0.8980	3.78	1.67	0.11
4	2.0000	1.0000	1.0000	0.5000	1.0000	1.0000	18.38	8.59	0.60
5	2.0932	1.0476	0.7240	0.7374	0.6705	1.2738	11.79	4.44	0.20
6	2.2911	0.6304	0.9321	1.1774	0.9515	0.6631	3.16	0.76	0.01
7	2.4913	0.8777	0.6514	1.1106	0.6672	0.9475	2.34	0.33	0.00
8	2.8252	0.6753	0.8826	0.9021	0.8753	0.7520	1.04	0.05	0.00

(a)

No.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	$w_k(\%)$ $\lambda = 8.$	$w_k(\%)$ $\lambda = 1.$	$w_k(\%)$ $\lambda = .5$
1	0.7000	1.0000	1.0000	1.0000	1.0000	1.0000	16.92	45.50	74.68
2	0.9325	0.9872	0.7847	0.7581	0.8500	1.5039	13.72	27.95	24.33
3	1.6241	0.5339	1.4416	0.6390	1.1820	0.7418	2.32	2.53	0.45
4	1.7669	0.8066	0.6781	0.7085	0.6174	1.6560	2.96	2.30	0.20
5	2.0000	1.0000	1.0000	0.3500	1.0000	1.0000	7.47	4.50	0.18
6	2.2450	0.9373	0.9252	0.3807	0.7915	1.1929	28.28	9.86	0.12
7	2.2703	0.6203	0.6213	1.4750	0.6120	0.8863	0.39	0.12	0.00
8	2.3524	0.9004	0.9018	0.3942	0.7688	1.2087	26.66	7.09	0.05
9	2.4858	0.4789	0.9286	1.2143	0.9161	0.5692	0.43	0.08	0.00
10	2.6908	0.7835	0.6083	0.7301	0.5692	1.3136	0.61	0.06	0.00
11	2.9212	0.6469	0.5690	1.1304	0.6005	0.9157	0.15	0.01	0.00
12	3.1107	0.5047	0.8921	0.8880	0.8398	0.6702	0.10	0.00	0.00

(b)

implying that the first story has most probably been damaged. This is not so in the case of θ_2, however, implying that the second story was not damaged. The probability associated with different degrees of damage can also be established; for example the probability that the stiffness of the first floor is below 60% of its original value is 0%, while the probability it is below 80% of its original value is 75%.

In the second example of the two-span bridge, a simplified finite-element model was employed. The deck was modeled using eight identical three-dimensional beam elements, but with the stiffnesses of the elements of the left and right span scaled by two independent parameters θ_1 and θ_2. The bridge was assumed to be supported at its midlength by a pier, modeled by a single three-dimensional beam element with stiffness scaled by a third independent parameter θ_3. The stiffness of the supporting soil at the abutments and at the base of the pier was modeled using translational and rotational springs with prescribed spring constants. The lumped masses at the translational degrees of freedom of the deck were uniformly distributed and were assumed known. The undamaged case was taken to be a uniform distribution $\underline{\theta} = [1.0, 1.0, 1.0]^T$. A "minimal" data case was assumed, consisting of the measured vertical and transverse excitation of the bridge near the abutments and at the base of the pier, along with the corresponding "measured" vertical and transverse response at the midlength of the left span. In this case, noise was assumed present so that only the first three modes made a detectable contribution to the observed response. Calibration of the undamaged model from the data, including examining model identifiability, led to a globally unique solution $\underline{\theta} = [1.0, 1.0, 1.0]^T$. In the damaged case, the stiffness of the left span was taken as 70% of its original value, while the stiffness of the right span and the pier remained unchanged, that is, $\underline{\theta} = [0.7, 1.0, 1.0]^T$. Calibration of this damaged structure, including examining model identifiability, led to two "symmetric" optimal solutions $\underline{\theta} = [0.7, 1.0, 1.0]^T$ and $\underline{\theta} = [1.0, 0.7, 1.0]^T$. If we assume that the output was also measured at the midlength of the right span, the second solution is eliminated, resulting in a globally identifiable solution.

Suppose now that the response is again measured at only one location, but the noise level is larger so that only two modes (the fundamental vertical and transverse modes) make a detectable contribution to the observed response. The stiffness parameters then become unidentifiable, since there exists an infinite number of optimal solutions represented by the three-dimensional curve c displayed in Figure 3. Additional research is necessary to determine the weighting of each optimal solution along this curve. If measurements at one additional location were available, then global identifiability would occur. Also, as illustrated in Figure 3, when the frequency of the third mode (the second vertical mode) is assumed known, there are only two points on curve c with the correct third mode frequency, leading to the two locally identifiable optimal solutions mentioned earlier.

CONCLUSIONS

This paper presents a probabilistic methodology for structural health monitoring and then focuses on the issue of identifiability. Asymptotic expressions are given for the posterior PDF of the model parameters which can be used to compute "damage probabilities" for each substructure, even when there is nonuniqueness in the optimal model parameters determined from dynamic test data, as shown for the case of local system identifiabilty.

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