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Characterization and identification of the non-linear behaviour of buildings

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ABSTRACT : This paper deals with the characterization and the identification of non-linearities in the behaviour of buildings under seismic motion. A non parametric representation of the system by Wiener series is used to quantify the effect of non-linearities in the whole response. The non-linearities are characterized by using a modal analysis technique within a moving time window. The time variation of the identified parameters allows to detect a possible stiffness degradation. An attempt of identification of non linear models is then presented. Among the studied visco-plasticity and damage models, the two models studied reproduce accurately the stiffness degradation phenomenon.

1 INTRODUCTION

During strong earthquakes, a building often exhibits a dynamic behaviour whose model can no longer be linear. After the San Fernando earthquake in 1971, many buildings have been instrumented with accelerometers to record their actual seismic motion. Many building response data of good quality were recovered during recent californian earthquakes by C.S.M.I.P. (Californian Strong Motion Instrumentation Program) to study their true structural dynamic behaviour. The object of this study concerns the characterization and the identification of the non-linear behaviour of two reinforced concrete buildings instrumented by C.S.M.I.P. :

- The Pacific Manor building (P.M.B) in Burbank which is ten stories high and shear wall lateral resistance system. It recorded the "Whittier Narrows" earthquake of 10/1/1987 ($a_{max} = 0.22g$ - ground- and $a_{max} = 0.54g$ - structure-)
- The Imperial County Services building (I.C.S.B) which is six stories high with both shear walls and moment resisting frame. It suffered severe structural damage during the "Imperial Valley" earthquake of 10/15/1979 ($a_{max} = 0.34g$ - ground- and $a_{max} = 0.45g$ - structure-).

A previous research work in Afra, Argoul & Bard (1990), Afra & Argoul (1990) and Bard, Afra & Argoul (1992) has indicated that the response of these buildings is markedly non-linear. So, several techniques have been elaborated for the characterization and the identification of these non-linearities. The first one uses a non-parametric identification method based on Wiener series decomposition. It allows to separate the linear part from the total roof response of the buildings. The second technique uses a linear system identification method presented in Afra, Argoul & Bard (1990) applied to the building responses, but with a moving time window. An equivalent linear model made of a succession of linear models on each time window is then obtained.

The other techniques presented consider the response of the building roof as that of an oscillator, by low-pass filtering the signal to eliminate the contribution of higher frequencies, and to retain only the fundamental mode. Two characteristics of great interest, the restoring force and the effective stiffness, are then computed to characterize the encountered non-linearities. A parametric identification method is finally applied to try to identify the non-linear response of these buildings. Three non-linear models have been tested to check their ability to model the observed responses : the perfectly elastoplastic model, the hardening elastoplastic model and the damage model with deteriorating elastoplastic elements.

2 QUANTIFICATION OF NON LINEARITIES

The technique presented here permits to quantify the non linear part in the whole response of the structure. It is based on the decomposition of the input-output relation into Wiener series - see Schetzen (1980).

Typically, the data used are the basement record $\ddot{x}_g(t)$, and the roof record $x_r(t)$. The structure is then treated as a "black box" : a S.I.S.O. - single input single output - model defined by its output located at the roof level to a given input located at the base level. Udwadia and Marmarelis (1976) showed that the ouput $x_r(t)$ can be largely influenced by the feedback due to reflections and modifications in the propagation of the seismic wave in the building whose effect shall not be considered in detail in this paper.

Wiener showed that the system response $x_r(t)$ to a stationary Gaussian white noise input $\ddot{x}_g(t)$ can be expanded as an infinite sum of integral terms as :

$$x_r(t) = \sum_{n=0}^{\infty} G_n [h_n , \ddot{x}_g(t)] \qquad (1)$$

where the G_n represent a set of functionals depending on the excitation and on symmetric terms h_n called Wiener kernels which characterize entirely the studied system.

Wiener shows that because of the orthogonality proper

-ty according to a statistical average E{.} resulting from the gaussian property of the excitation signal, the Wiener functionals G_n can be computed by means of cross-correlation techniques.
Our aim is only to isolate the linear part from the whole seismic response of the building. So, we limited our work to the computation of the two first Wiener functionals which represent the linear part of the response.
G_0 is a constant equal to the average value of $x_r(t)$

$$G_0[h_0, \ddot{x}_g(t)] = h_0 = E\{x_r(t)\} \quad (2)$$

G_1 may be seen as the response of a linear time invariant system with the unit impulse response $h_1(t)$

$$G_1[h_1, \ddot{x}_g(t)] = \int_{-\infty}^{\infty} h_1(\tau)\, \ddot{x}_g(t-\tau)\, d\tau \quad (3)$$

G_1 is computed, as previously mentionned, thanks to cross-correlation between the input signal and the output signal

$$h_1(\tau) = \frac{1}{P}\Phi_{\ddot{x}_g x_r}(t) = \frac{1}{P} E\{x_r(t)\, \ddot{x}_g(t-\tau)\} \quad (4)$$

where P is the constant power spectral density function of the input signal $\ddot{x}_g(t)$.
Two main difficulties are encountered for the application of this technique to buildings. The first is the computation of power spectral density function (p.s.d.f.) P of the seismic excitation which is not a constant for the real input signal. P is obviously not constant over all frequencies but it is reasonable to say that this signal is an approximation to broad-band white process. To estimate the value of P, we propose to minimize, in a least squares sense, the difference between the measured response $x_r(t)$ and the one given by the sum of the two first terms G_0 et G_1.
The optimum value of P is then obtained

$$P = \int_0^T f^2(t)dt \Big/ \int_0^T x_r(t)\, f(t)\, dt - h_0 \int_0^T f(t)\, dt \quad (5)$$

where $f(t) = \int_0^T \hat{\Phi}_{\ddot{x}_g x_r}(\tau)\, \ddot{x}_g(t-\tau)\, d\tau$ and T is the length of the time interval of measurements.
The second difficulty is the numerical computation of the inter-correlation function appearing in relation (4).
We used the unbiased estimate of the sample cross-correlation function at lag numbers $r \in [1,m]$, $m<N$

$$\hat{\Phi}_{\ddot{x}_g x_r}(r\Delta t) = \frac{1}{N-r} \sum_{i=1}^{N-r} x_r((i+r)\Delta t)\, \ddot{x}_g(i\Delta t) \quad (6)$$

where N is the number of points in the discretization of the time intervals of the two signals.
Finally, this technique has been applied to the I.C.S.B data. We find $h_0 = G_0 = 2.25\ 10^{-2}$. The p.s.d.f. of the seismic signal estimated by relation (5) is equal to P = 859,4 . The "linear" response in displacement $G_0 + G_1$ of the Wiener model is estimated from data measured on the time interval [0 , 20sec]. It is drawn in figure 1 in dotted line and compared to the global response drawn in continuous line.
The normalized difference in a mean square sense between the measured response and the "linear" one obtained by Wiener approach is about 34%. That permits to characterize the importance of non linearities with regard to the global response.

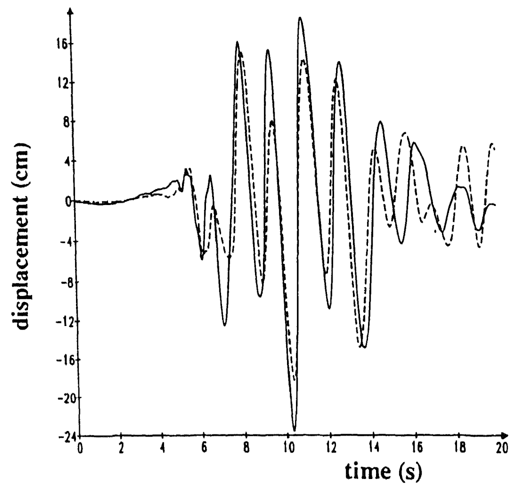


Figure 1. Comparison between the linear part of the response (---) and measured one (—).

Table 1. Identified parameters for I.C.S.B.

Identification on	ω	ξ	ρ	d_0	v_0	E%
The global response	3.94	0.159	1.31	0.30	-1.78	27
The linear part	4.06	0.116	0.943	0.092	-0.24	10

We have then applied the linear identification method presented in Afra, Argoul & Bard (1990) to the global displacement and to the "linear" part, over the time interval [0, 15sec]. The results for the fundamental mode (frequency ω, damping ratio ξ, participation factor ρ and the modal initial conditions in displacement and velocity d_0 and v_0) are given in Table 1. We observe that the minimization error E is lower when identification is made with the "linear" part -10%- than with the non linear global response -27%-.

3 CHARACTERIZATION OF NON LINEARITIES

The usual non-linearities occuring in the accelerometric responses of buildings under strong motion earthquakes are of two types : the geometric non linearities when the terms of first order in the relationship between strains and displacements are insufficient to well reproduce the actual response of the structure and the non-linearities of behaviour when the constitutive law between stress and strain is no longer linear. Only the last case will be studied here and especially the phenomenon of the degradation of stiffness which is very frequently encountered in the dynamic behaviour of concrete work constructions. For example, Iemura and Jennings (1974) showed that for the Milikan Library building on the campus of the

California Institute of Technology during the San Fernando earthquake of 9 February 1971, its fundamental frequency decreased from 1 to 1.5 Hz previously observed in pre-earthquake vibrations. That means that during the earthquake, the building has lost 34% of its initial rigidity.

3.1 Modal analysis on moving time window

The stiffness degradation is a non stationnary phenomenon. This property of non stationnarity can be emphasized by using the technique presented below and which permits to characterize the time variation of the structural model parameters. It is based on the application of the linear system identification method within a moving time window.

First, we briefly remember the main features of this method. The dynamic behaviour is described by the modal equations of a discretized system with N degrees of freedom and with proportional viscous damping. Thus we obtained a set of NxN_m uncoupled equations (N_m is the number lower than N of the vibration modes taken in the model) which can be written as

$$\ddot{x}_i^r + 2\,\xi_r\,\omega_r\,\dot{x}_i^r + \omega_r^2\,x_i^r = \rho_i^r\,\ddot{x}_g \qquad (7)$$

in which ω_r and ξ_r are the r^{th} vibration frequency and modal damping ratio, respectively.

ρ_i^r is the participation factor of the r^{th} mode at response point i, defined as:

$$\rho_i^r = -\phi_{ir}\frac{\{\phi_r\}^t[M]\{1\}}{\{\phi_r\}^t[M]\{\phi_r\}}$$

where $\{\phi_r\}$ is the r^{th} eigen mode and ϕ_{ir} its value at the data point i. [M] represents the mass matrix and $\{1\}$ the column vector whose components are equal to 1. $\ddot{x}_g$ is the ground acceleration.

The modal initial conditions in displacement and velocity are: $d_i^r = x_i^r(t_0)$ and $v_i^r = \dot{x}_i^r(t_0)$ which are usually zero because the structure is initially at rest, but are necessary for a moving time window analysis. The displacement at point i is given by combining all the modal contributions : $x_i = \sum_{r=1}^{N_m} x_i^r$. If we have N_p measurement points, the model with viscous damping is described by $N_m(2+3N_p)$ parameters written in vector $\{\gamma\}$: $\{\omega_r, \xi_r, \rho_i^r, d_i^r, v_i^r\}$ with $r = 1, N_m$ and $i = 1, N_p$.

This method is based on the minimisation in a least-squares sense of the difference, measured in frequency or time domain, between a MDOF linear, planar and time invariant model and the actual responses.

$$E(\{\gamma\}, f_1, f_2) = K\sum_{i=1}^{N_p}\int_{f_1}^{f_2}\left|H_{ei}(f) - H_{mi}(\{\gamma\}, f)\right|^2 df \qquad (8)$$

where $K = \left(\sum_{i=1}^{N_p}\int_{f_1}^{f_2}\left|H_{ei}(f)\right|^2 df\right)^{-1}$. H_{ei} and H_{mi} are respectively the measured response and the one given by the model at point i and at frequency or time f.

This method has been applied to the two buildings previously mentioned and the results are given in table 2. The minimisation error, between the displacement at the roof of the model and that, recorded, is very important (30% for I.C.S.B and 44% for P.M.B). After the study of torsional motion for the two studied buildings in Bard, Afra & Argoul (1992) that shows the smallness of torsion (the spectral ratio of the roof torsional motion upon the average roof relative transverse motion less than 20%), we conclude that they have exhibited a non linear behaviour.

Table 2. Identified parameters from linear identification.

Building	ω	ξ	ρ	E%
P. M.B	11.89	0.048	-1.32	44
I.C.S.B	3.98	0.158	-1.35	30

The length of the time window must be chosen to be very large as possible to estimate reliable parameters of the equivalent linear models. In the other hand, it must be very short as possible to ensure the linearity and the property of stationary process assumed within the time window. So, the time window length is chosen roughly to be six times the vibration period (in seconds) of the structure. We choose a length of 6 sec. After every linear identification method, a sensitivity analysis on the minimization criterion E previously defined in relation 8, is performed to determine the confidence interval for every identified parameters.

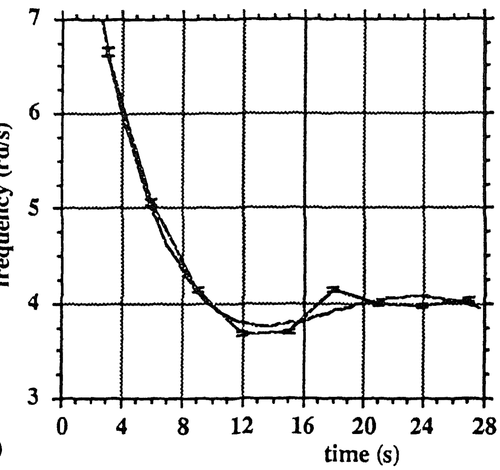
Figures 2-3 show for these buidings, the draw of the successive values with the confidence interval of the frequency and damping ratio identified within every interval [1,6] , [6,12] , [12,18] , [18,24] , [24,30] . An equivalent linear model made from a succession of linear models on each time window is then obtained.

We observe first a strong variation of ω and ξ of the identified equivalent linear model, secondly a difference of the behaviour of the two buildings. For P.M.B, no specific non linear behaviour seems to stand out and the draw of damping ratio is not given here because it is not significant of a particular non linearity; for I.C.S.B, we notice an irreversible decrease in time of the frequency. This decrease which illustrates the phenomenon of degradation of rigidity will be studied in more detail in the next section. The curves of frequency and damping ratio have been then fitted by polynomials. For I.C.S.B, the explicit form of the time variation of the frequency and damping ratio on a polynomial form of fourth degree are

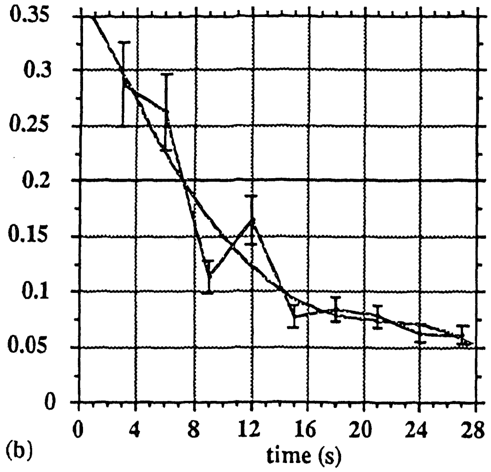
$$\omega(t) = 9.575 - 1.198t + 0.086t^2 - 0.0025t^3 + 2.6.10^{-5}\,t^4$$

$$\xi(t) = 0.369 - 0.023t - 0.0003t^2 + 6.9E\text{-}05t^3 - 1.4.10^{-6}\,t^4$$

The comparison between the displacement given by this model and the one recorded is given in figure 4. The error E obtained on displacement is mainly reduced from 30% with time invariant linear model to 11% with the time variant one for I.C.S.B and from 40% to 16% for P.M.B. This model allows a good fit of the seismic data.



a)



(b)

Figures 2. Time variation of a) the frequency and b) the damping ratio (I.C.S.B - Longitudinal vibrations -)

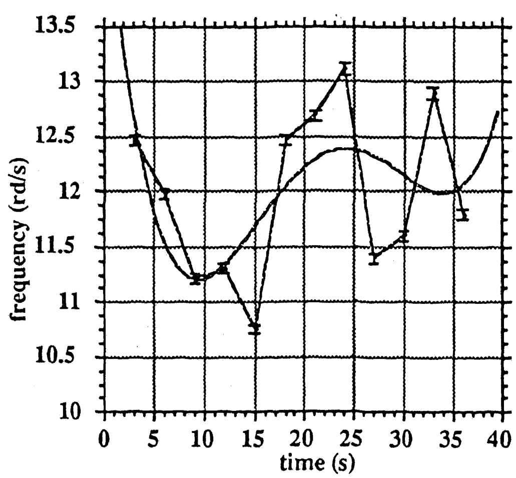


Figure 3. Time variation of the frequency (P.M.B - Longitudinal vibrations -)

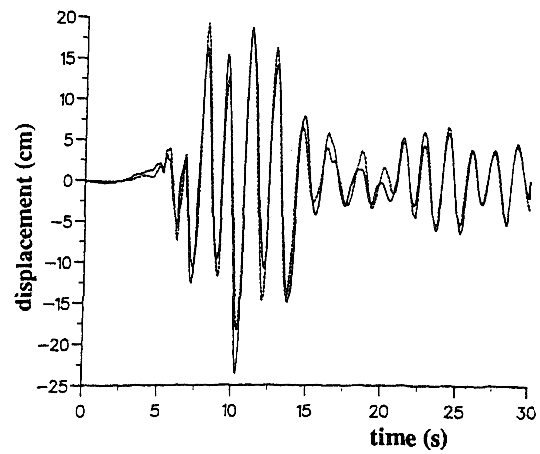


Figure 4. Comparison between response of identified model (---) and measured one (—).

3.2 Notions of restoring force and of effective stiffness

The linear identification technique previously presented has been applied to 13 buildings in Afra & Argoul (1990) and the results have shown that the fundamental mode is strongly predominant in the displacement measured at the top of the building. So, we propose to work only on this roof response and to model it by a non linear oscillator.
The restoring force is defined as the difference between the excitation forces and the inertial forces; it is opposed to the internal forces. For an oscillator, the restoring force per unit of mass is expressed as

$$f(x,\dot{x}) = \rho \ddot{x}_g - \ddot{x} \qquad (9)$$

The presence of higher modes in the response strongly disturbs the interpretation of non linear effects in the study of the fundamental mode. So, to reduce their influence, the response is filtered by a low-pass filter. The filtered restoring force is computed for the I.C.S.B data. The acceleration spectrum at the roof shows that the higher modes appear beyond 1.8 Hz that will be the optimum cutting frequency of the filter.

Figure 5 gives the diagram of the filtered restoring force. We recall that the slope of the "major axis" of the loop defined for every cycle described the evolution of the stiffness of the structure and its area represents the energy dissipated by the structure during the cycle. We notice that the loops of hysteresis rotate around the origin and the slope of the "major axis" of the loops slants towards the right. It means that the value of the restoring force taken at the maximum displacement point decreases, therefore the rigidity of the structure decreases. A way to describe this phenomenon is to draw the time evolution of the effective stiffness.
The effective stiffness during a given vibration cycle is defined by the ratio of the value of the restoring for-ce taken at the absolute maximum displacement, upon the absolute maximum displacement x_{max} for that cycle

$$k_{eff_i} = \frac{f_i}{x_i} = \frac{f(x_{max},0)}{x_{max}} \qquad (10)$$

For a linear oscillator with viscous damping, the loop during a cycle of vibration is an ellipse and k_{eff} is

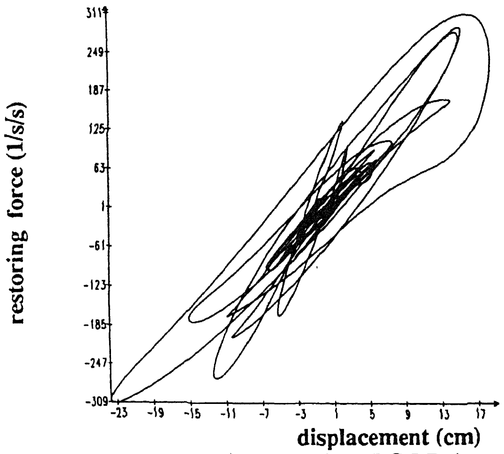


Figure 5. Filtered restoring force from I.C.S.B data.

constant and equal to the square of the pulsation $k_{eff} = \omega_0^2$. In figure 6, k_{eff} versus x_i is drawn for I.C.S.B. longitudinal and transversal vibrations; we see a quick decrease of the effective rigidity until the maxi-mum of x_i is reached, then nearly a stretch when x_i decreasing.

4 IDENTIFICATION OF THE NATURE OF STIFFNESS DEGRADATION

After the study of simple models as the viscoplastic perfect model, the hardening visco-plastic model, we hold the damage model, proposed by Cifuentes & Iwan (1989). This model is composed as shown in figure 7 of a succession in parallel of a linear spring, an elasto-plastic element, a set of N deteriorating elements which consist of 'breaking' elasto-plastic elements and a dash-pot. An elasto-plastic element is characterized by a linear spring with stiffness k_{ep} in series with a slip element that allows a maximum force equal to $k_{ep}\, x_{y_{ep}}$; the i[th] 'breaking' elasto-plastic element no longer contributes to the restoring force when the absolute value of displacement exceeds a value $\beta\, x_{y_i}$ (see figure 8). The coefficient β must be greater than 1; if it is equal to 1, then the deteriorating element acts as a linear element till the breaking and when β tends to infinity, the deteriorating element acts as a elasto-plastic element.

When the number N of deteriorating elements has been chosen, this model has 2N + 6 parameters to identify. N can be reduced by making the following assumptions as Cifuentes and Iwan did

1. $\beta = 2$;
2. each deteriorating element absorbs the same maximum energy D that implies that $k_i = D / x_{y_i}^2$;
3. the deteriorating elements are organized in ascending order of the displacements x_{y_i} which are chosen equally spaced over the displacement range of interest. It ensues that $x_{y_i} = (x_{max} / N)\, i$ where x_{max} is the

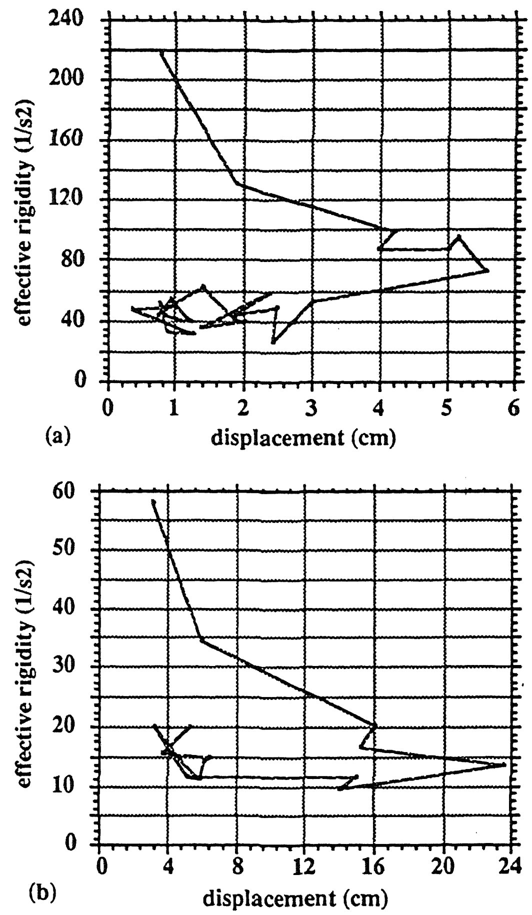


Figure 6. Effective rigidity obtained from I.C.S.B. data (a) transverse vibrations, (b) longitudinal vibrations.

maximum relative displacement experienced by the structure.

Finally, D is estimated directly from the effective stiffness diagram

$$D = (K_0 - K_f) \Big/ \sum_{i=1}^{j} \frac{1}{x_{y_i}^2} \quad \text{with } K_0 = k_e + k_{ep} + \sum_{i=1}^{j} k_i$$

and $K_f = k_e + k_{ep} \sum_{i=j+1}^{N} k_i$ where j represents the number of deteriorating elements having reached the failure threshold at the end of the seismic excitation. K_0 and K_f are respectively the effective rigidity of the first and the last cycle of vibration and are estimated from the diagram of effective stiffness.

The $\{\gamma\}$ vector of the model parameters has only five components : $\{\gamma\} = \{\rho, k_e, k_{ep}, x_{y_{ep}}, c\}$. Cifuentes & Iwan (1989) propose to still reduce the number of parameters by estimating the participation factor ρ from the dynamic analysis of the building during its conception. The three parameters k_e, k_{ep} and $x_{y_{ep}}$ are then identified by a one dimensional minimization of the

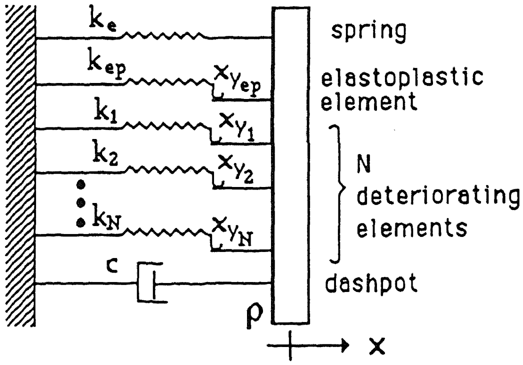


Figure 7. Damage model

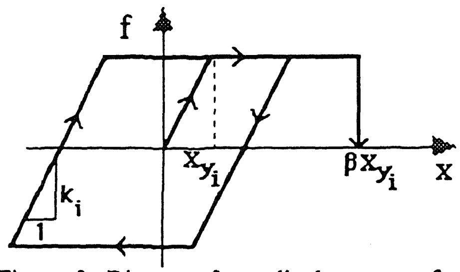


Figure 8. Diagram force-displacement of the ith deteriorating element of the damage model

Table III Parameters identified of the damage model

ρ	k_e	k_{ep}	$x_{y_{ep}}$	c	N	x_{max}	d_{max}	E%
1.20	7.92	6.38	11.3	0.55	10	23.7	24.1	27

difference between the restoring force given by the model and that obtained from the earthquake records - c being zero-. Then, by successive approximations (2 or 3), they try to approach the value of the parameter c that gives the best fit for the low amplitude portion of the response. We have used a one dimensional minimization of the difference between the accelerations given by the model and that measured on structure by taking the five components of {γ} as the unknowns of the problem. This technique has been applied to the I.C.S. B. data, we find : D = 52 cm^2/s^2 and the results are given in Table III. We observe that the maximum displacement d_{max} given by the identified model is similar to x_{max}. The damage model permits a good fit of accelerometric data as shown in figure 9. The error E on displacement of 27% is nevertheless greater than this of 11% obtained with the time variant model.

CONCLUSIONS

The use of Wiener series decomposition allows to detect and quantify the part of non linearities in the seismic response of buildings. The results obtained with modal analysis on moving time window show that there is a consistent time variation of the fundamental frequency and of the modal damping ratio of

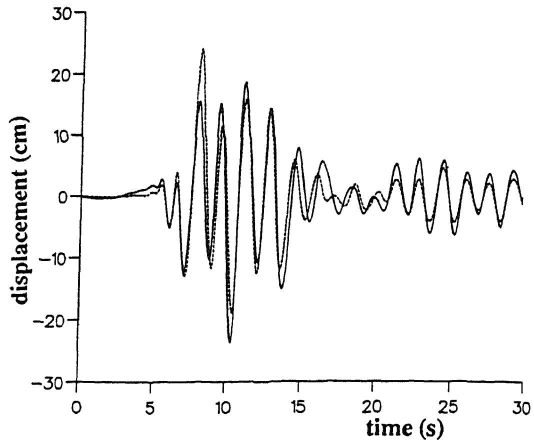


Figure 9 Comparaison between response of damage model (---) and measured one (—).

the equivalent linear model during strong earthquakes. This variation may be often interpreted as a degradation of the rigidity which can be corroborated by the computation of the effective rigidity . It is found that the time variant linear model provides an efficient model for the degradation effect. At last, when we limit our study to the first vibration mode, the damage model gives a good and simple representation of the stiffness degradation.

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