# Seismic interaction between adjacent buildings under second-order geometric effects

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ABSTRACT: The paper deals with a numerical approach for the problem of earth-quake interaction among neighboring buildings when unilateral elastoplastic/elastic contact under second-order geometric and other instabilizing effects can take place. The method is based on formulating the problem by the finite element method as an inequality one and on solving this by the average-acceleration method of time-discretization and nonlinear mathematical programming. Some results concerning a two-building system under P-Delta effects are given in a numerical example.

#### 1 INTRODUCTION

Earthquake induced pounding between adjacent buildings has been recognized -see e.g. Newmark and Rosenblueth (1971) - as one of the main usual causes of significant damages in seismically active regions. This holds especially for market-areas of cities, where the building codes, due to various socioeconomic reasons, allow partial or full contact between neighboring buildings (Bertero (1987)).

From mathematical point of view, the governing conditions of the relevant problem are equalities as well as inequalities. The latter ones concern on the one hand the possibility to be appeared compressive stresses only (no tension) on the interface, and on the other hand the appearence of retire—ment relative displacements (no penetration) for the same interface points where unilateral contact can take place. So, the problem belongs to so-called inequality problems of mechanics, for which a mathematical study can be done by the variational inequality concept—see Panagiotopoulos (1985).

As regards numerical results, some interesting studies concerning simplified models of single-degree-of-freedom systems have been reported by Wolf and Skrikerud (1980), Anagostopoulos (1988), Penelis and Athanassiadou (1989). These investigations are based on a parametric trial-and-error approach. A more realistic numerical treatment of such inequality problems in earth-

quake engineering for multidegree-offreedom structural systems has been already presented by the first author (A.A.L.) in a series of papers, see e.g. Liolios (1984,1988,1989,1990,1991).

The purpose of this paper is to deal with a numerical approach for the above outlined dynamic inequality problem when some instabilizing effects are taken into account. These effects concern here elastorlastic-softening/fracturing behaviour for unilateral contact P-Delta effects. The method is based on a double discretization, in space by the finite element method and in time by the average acceleration method, and on solving a non-convex linear complementarity problem in each time-step. The proposed method is applied, finally, to a civil engineering example and some conclusions are discussed.

## 2 METHOD OF ANALYSIS

A system of two only adjacent buildings (A) and (B) is considered here for simplicity. Certainly, the extension to systems with more than two buildings is straightforward.

First, following Liolios (1984), the system is discretized in space by the finite element method. Any two associated nodes  $i_A$  and  $i_B$  on the interface are considered as connected by a unilateral constraint, normal to the interface. The stress  $r_i$ , positive when it is compressive, and the corresponding shortening relative displacement

v<sub>i</sub> of the i-th unilateral constraint satisfy the following, in general nonconvex, constitutive relation:

$$r_i \in \partial R_i(v_i, g_i).$$
 (1)

Here  $g_i(t)$  is the existing gap at time t between nodes  $i_A$  and  $i_B$ ,  $\vartheta$  is Clarke's generalized gradient and R is the symbol of non-convex superpotential - see Panagiotopoulos (1985). Relation (1) expresses in a general mathematical way the unilateral frictionless elastoplastic contact taken into account hardening/softening, unloading/reloading, fracturing etc. behaviour. For simplicity, the case of frictionless contact is studied here. The frictional case, which is more complicated, can be investigated in a way similar to that of Liolios (1989, 1991). As known, softening/fracturing behaviour corresponds to descending branches in the diagram of (1), and usually has instabilizing effects the numerical procedures and the structural response. Moreover, the elastoplastic behaviour of unilateral constraint permits us to assume that local impact phaenomena have no significant influence to the global building response.

Now, for the numerical treatment of the problem, the rel. (1) is piece-wise linearized in a way similar to that used by Maier (1971,1973) in elastoplasticity. So, introducing the nonnegative multipliers  $\mathbf{w}_{1}$ , rel. (1) is equivalent to the following linear complementarity conditions:

$$r_i = p_i (v_i - g_i + w_i) + c_i \dot{w}_i,$$
 (2a)

$$w_{i} \geq 0$$
,  $r_{i} \geq 0$ , (2b,c)

$$r_i w_i = 0. (2d)$$

Here c is the damping coefficient and p the stress function for the i-th unilateral constraint. Dots over sym - bols denote, as usually, time-derivatives.

Further, the incremental global equations of dynamic equilibrium for the two buildings (A) and (B) due to a seismic ground displacement history  $\mathbf{x}_{\mathbf{G}}(t)$  are written in matrix notation:

$$\underline{\mathbf{M}}_{\mathbf{A}} \Delta \underline{\mathbf{u}}_{\mathbf{A}} + \underline{\mathbf{C}}_{\mathbf{A}} \Delta \underline{\mathbf{u}}_{\mathbf{A}} + (\underline{\mathbf{K}}_{\mathbf{A}} + \underline{\mathbf{G}}_{\mathbf{A}}) \Delta \underline{\mathbf{u}}_{\mathbf{A}} = \\
= -\underline{\mathbf{M}}_{\mathbf{A}} \ \underline{\mathbf{x}}_{\mathbf{G}} + \Delta \underline{\mathbf{r}} , \qquad (3a)$$

$$\underline{\mathbf{M}}_{\mathbf{B}} \Delta \underline{\ddot{\mathbf{u}}}_{\mathbf{B}} + \underline{\mathbf{C}}_{\mathbf{B}} \Delta \underline{\dot{\mathbf{u}}}_{\mathbf{B}} + (\underline{\mathbf{K}}_{\mathbf{B}} + \underline{\mathbf{G}}_{\mathbf{B}}) \Delta \underline{\mathbf{u}}_{\mathbf{B}} =$$

$$= -\underline{\mathbf{M}}_{\mathbf{B}} \Delta \underline{\ddot{\mathbf{x}}}_{\mathbf{G}} - \Delta \underline{\mathbf{r}} . \tag{3b}$$

Here, as usually, M, C and K denote the mass, damping and current (tangent) first-order (linear elastic) stiffness matrices, respectively. G is the symmetric constant geometric stiffness matrix, depending linearly on preexisting constant stresses. Thus, via the term Gu alone the geometry changes affect the equilibrium (second-order geometric effects) -see e.g.Maier (1971), Corradi and De Donato (1975), Chen and Lui (1987). u(t) is the node-displacement vector (relative to ground); A denotes increment; and finally, r is the vector of interaction forces between (A) and (B) with elements satisfying rels. (1)-(2).

Thus the problem consists in computing the time-dependent set  $\underline{u}_A$ ,  $\underline{u}_B$ ,  $\underline{r}$ ,  $\underline{w}$  and  $\underline{g}$  satisfying (1)-(3) for given initial conditions and  $\underline{x}_G$ (t). Due to inequality conditions, the

Due to inequality conditions? the problem is a nonlinear one, even in the case of linear structures. To discretize this problem in time, use is made of the average-acceleration method, which belongs to Newmark's family of step-by-step direct time integration methods -see e.g. Weaver and Johnston (1987). So we substitute in (3) for every time-step

$$\Delta \ddot{\underline{u}} = c_1 \Delta \underline{u} + \underline{a}, \qquad (4a)$$

$$\Delta \underline{\dot{u}} = c_2 \Delta \underline{u} + \underline{b}, \qquad (4b)$$

where  $\underline{a}$ ,  $\underline{b}$  known quantities from previous time-steps and

$$c_1 = 4/(\Delta t^2), \quad c_2 = 2/\Delta t$$
 (5)

are method parameters. After the above manipulation we arrive eventually to a linear complementarity problem -see also Liolios (1988,1989) - of the form

$$\underline{z} \geq \underline{0}, \qquad \underline{D}\underline{z} + \underline{d} \leq \underline{0}, \qquad (6a,b)$$

$$\underline{z}^{\mathrm{T}}.(\underline{p}\underline{z} + d) = 0. \tag{6c}$$

This problem is solved by known algorithmes of nonlinear optimization - see e.g. Panagiotopoulos (1985) or Maier (1973). Thus, in each time-step  $\Delta t$  is computed which unilateral constraints are active and which are not.

The so-obtained results concern the response of the coupled system, where

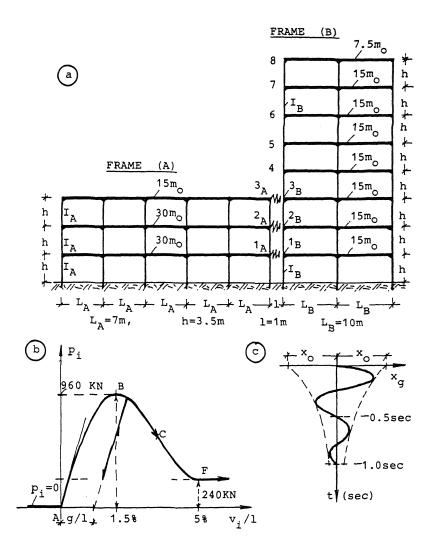


Fig. 1. Numerical example

the unilateral contact and the secondorder geometric effects are taken into account. To compare these results with corresponding ones for the uncoupled system, where the interaction effects are not taken into account and the structures are designed as beeing entirely independent (as was usual until recently in most aseismic computations), the following influence coefficients are introduced:

$$\lambda_{i} = (Q_{i}^{c} - Q_{i}^{u}).100/Q_{i}^{u}.$$
 (7)

Here  $Q^U$  and  $Q^C$  are the absolutely maximum values, which a response quantity Q takes during the seismic exci-

tation when the structures are uncoupled and coupled, respectively.

# 3 NUMERICAL EXAMPLE

In the building system of Fig. 1a, the frame (A) is of reinforced concrete with elastic modulus  $\rm E_b=3.4*10^7 KN/m^2$  and column sections 40/60 in cm, and the frame (B) is of steel with  $\rm E_s=21*10^7 KN/m^2$  and columns IPBl 500. Damping ratio is 5% for (A) and 3% for (B). Both frames are considered as having rigid beams with total vertical loads  $\rm a_1m_0$ g, where  $\rm m_0$ g=98.1 KN, g=9.81 m/sec² and  $\rm a_1$ =coefficients as in

Fig. 1a. Unilateral contact can take place at joint-points  $i_{\rm A}$ ,  $i_{\rm B}$ , where i=1,2,3. The corresponding to (2a) function  $p_{\rm i}$  is assumed to be as shown in Fig. 1b, where AB and BCF are parabolas of 2-nd and 3-rd degree, respectively. The above simulation of unilateral contact is certainly a very complicated task and can be estimated on the basis of experimental results. P-Delta effects for the steel frame (B) are taken into account. The plane system is subjected to an horizontal earthquake ground displacement

$$x_g(t) = x_0 e^{-2t} \sin(4\pi t)$$
 (8)

with x = 10mm and diagram as in Fig. 1c. The herein presented numerical approach has been applied to estimate quantitatively the interaction effects on the seismic response of frames (A) and (B).

From the so-obtained results are shown indicatively in Table 1 only those concerning the storey-shear-forces of the frames (A) and (B).

Table 1. Influence coefficients (in %) for the storey shear forces

Storey	Frame (A)	Frame (B)
1	-24.19	+67.42
2	-15.98	+12.54
3	-25.67	+26.41
4		+119.73
5		+46.88
6		+46.05
7		+41.79
8		+44.96

As the table results show, the uncoupled stress-state of the three-storey frame (A) is reduced about 16% -26% due to interaction. On the contrary, the uncoupled stress-state of the eight-storey frame (B) is increased about 13% - 120%. As was expected, the most significant increase is for the 4-th floor of (B). Thus, if the columns of this floor are designed without taken into account the sei smic interaction effects, then these columns are overstressed about 120% more than the designed capacity. This remarkable result shows the significance of computing the interaction influence on the seismic response of adjacent buildings.

#### 4 CONCLUSIONS

The herein presented numerical method

can be used effectively in practical civil engineering applications, where a quantitative estimation of the seismic interaction between adjacent buildings under second-order effects is required. For this purpose, the realization on computer of the method seems to be indispensable. This is obtained by using available computer codes of the finite element method, the direct time integration methods and nonlinear optimization.

As the results of the numerical example show, the interaction influence on the earthquake response of neighboring buildings may be significant. Therefore the usual aseismic design and control without taking into account such a possible interaction under second-order effects may be no realistic. Certainly, the sufficient aseismic joint among adjacent buildings seems to be an effective rule for seismically active regions. If this rule can be applicable, and if the simulation of the unilateral contact behaviour can be done in a realistic way (e.g. by experimental results), then the seismic joint gap can be adjusted suitably by the herein procedure. So, a parametric application of the presented method, having as one parameter the joint gap, can be used effectively to control the seismic interaction effects in a desirable level.

### REFERENCES

Anagnostopoulos, S.A. 1988. Pounding of buildings in series during earthquakes. Earthq. Enging and Struct. Dynamics 16: 443-456.

Anagnostopoulos, S.A. and Spiliopoulos K.V. 1991. Analysis of building pounding due to earthquakes. In: Krätzig W.B. et al (eds.), Structural Dyna mics: 479-484. Rotterdam: Balkema.

Başar, Y., Eller, C., Krätzig, W.B.and Quante, R. 1991. Finite element analysis of nonlinear dynamic instability phenomena of arbitrary shell structures. In: Krätzig, W.B. et al (eds.), Structural Dynamics: 91-96. Rotterdam: Balkema.

Bertero, V.V. 1987. Observations on structural pounding. Proc. of the Intern. Conf. "The Mexico Earthqua - kes", ASCE, 264-278.

kes", ASCE, 264-278.

Bathe, K.-J. and Wilson, E.L. 1976.Numerical methods in finite element analysis. N.J.: Prentice-Hall, Inc.

Bouwkamp, J.G., Schneider, B. and Kanz

R. 1991. Composite construction in earthquake design. In: Askar, G. (ed.)

- Proc. 4-th Intern. Coll. Structural Stability: 390-409, Istanbul.
- Chen, W.F. and Lui, E.M. 1987. Structural stability-Theory and implementation. New York: Elsevier.
- Clough, R.W. and Penzien, J. 1975. Dynamics of structures. New York: McGraw-Hill.
- Corradi, L. and De Donato, O. 1975.Dynamic shakedown theory allowing for second-order geometric effects. Meccanica 10: 93-98.
- Liolios, A.A. 1984. A finite-element central-difference approach to the dynamic problem of nonconvex unilateral contact between structures. In: Sendov, B. et al (eds.), Numerical methods and applications: 394-401. Sofia: Bulg. Academy Sciences. Liolios, A.A. 1988. Seismic interacti-
- Liolios, A.A. 1988. Seismic interaction between adjacent structures: A linear complementarity approach for the unilateral elastoplastic-softening contact with friction. In: Kounadis, A.N. and Krätzig, W.B. (eds.) Structural dynamics and earthquake engineering: 157-165. Athens: Hellenic Society for Theor. Appl. Mech. Liolios, A.A. 1989. A linear comple -
- Liolios, A.A. 1989. A linear comple mentarity approach to the nonconvex dynamic problem of unilateral con tact with friction between adjacent structures. Journal Appl. Math.Mech. (ZAMM) 69(5): 420-422.
- Liolios, A.A. 1990. A numerical approach to seismic interaction between adjacent buildings under hardening or softening unilateral contact.

  Proc. 9-th Europ. Conf. Earthq.Eng. vol. 7-A: 20-25. Moscow: EAEE.
- Liolios, A.A. 1991. A numerical estimation for the influence of modifications to seismic interaction between adjacent structures. In: Savidis S.A. (ed.), Earthquake resistant construction and design: 461-468. Rotterdam: Balkema.
- Maier, G. 1971. Incremental elastoplastic analysis in the presence of large displacements and physical instabilizing effects. Int. Jnl Solids and Structures 7: 345-372.
- Maier, G. 1973. Mathematical programming methods in structural analysis. In: Brebbia, C. and Tottenham, H. (eds.), Variational methods in engineering:8/1-8/32. Southampton: Southampton University Press.
- Newmark, N.M. and Rosenblueth, E.1971. Fundamentals of earthquake engineering. Englewood Cliffs:Prentice-Hall
- Panagiotopoulos, P.D. 1985. Inequality problems in mechanics and applications-Convex and nonconvex energy functions. Basel: Birkhäuser Verlag.

- Papadrakakis, M., Mouzakis, H. and Bitzarakis, S. 1991. Dynamic contact between adjacent structures using a formulation with Lagrange multipliers. In: Krätzig, W.B. et al (eds.), Structural dynamics: 913-920. Rotterdam: Balkema.
- Penelis, G. and Athanassiadou, Ch. 1989. Discussion on a paper by S.A. Ana gnostopoulos. Earthq. Engng. Str.Dyn. 18: 445-446.
- Weaver, W.Jr. and Johnston, P.R. 1987. Structural dynamics by finite ele ments. Englewood Cliffs, N.J.: Prentice-Hall-Inc.
- Wiegel, R.L. (ed.), 1970.Earthquake
   engineering. Englewood Cliffs, N.J.:
   Prentice-Hall, Inc.
- Wolf, J.P. and Skrikerud, P.E. 1980. Mutual pounding of adjacent structures during earthquakes. Nuclear Enging and Design 57: 253-275.
- Zienkiewicz, O.C. and Taylor, R.L. 1989. The finite element method, 4-th ed. New York, London: McGraw-Hill.