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Inelastic spectra for eccentric systems

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ABSTRACT: The torsional inelastic response is a significant research topic since some resisting elements of asymmetric plan-wise buildings experience larger inelasticity than if they were located in symmetric systems. Then, the system total strength should be designed by using inelastic spectra provided by the analysis of eccentric models. In this paper, inelastic spectra are computed by response analysis of a two degree of freedom stiffness eccentric model. The spectral values are determined by imposing a target maximum ductility among the resisting elements. For a given total strength two criteria have been considered for designing element yield forces: the first one is based on assuming equal design levels for all elements while the second one selects the element capacity by using location dependent design levels. Overstrength factors of the eccentric system with respect to the correspondent symmetric system are evaluated in order to provide a concise measure of effects due to torsional coupling on dynamic inelastic response.

1 INTRODUCTION

It is well known that asymmetric plan-wise buildings undergo translational as well as torsional motions during seismic excitation. In particular, rotation of decks results in non uniform plan distribution of actions such that some resisting elements experience much larger deformations than if they were located in a symmetric system. An updated approach for evaluating torsional effects requires the analysis of inelastic dynamic response, since resisting elements are expected to deform significantly beyond the yield limit under strong ground motions.

Reduction of peak ductility demand and structural damage appears to be the main design goal. Two ways seem convenient for this purpose: the first one reduces the maximum plastic action by giving the asymmetric systems a larger total strength compared to that of the equivalent symmetric systems while defining the element yield forces with equal design levels.

The second criterion consists in properly distributing a fixed total strength. In fact, it has been demonstrated (De Stefano et al. 1991) that, given the elastic response and the total capacity, an uniform plan distribution of plastic actions, with a subsequent reduction of maximum ductility demand, can be achieved by using location dependent design levels in defining the element yield forces. This is seen to be equivalent to selecting proper values for parameters which globally characterize the strength plan-wise distribution, such as the strength eccentricity and the strength radius of gyration. Adopting such a design procedure leads to a significant improvement of the inelastic response compared to that of systems designed with the same total strength, but equal design levels.

Obviously, the above procedure, which takes into account the actual inelastic behaviour, can be somehow difficult to be applied. In fact, even considering simplified models, the values of inelastic parameters which optimize dynamic response vary with the elastic system parameters and the seismic input, even though a rather narrow range where values are included can be defined (De Stefano et al. 1991). Moreover, giving the plan-wise strength distribution selected values of inelastic parameters can be complex as the number of resisting elements increases.

Therefore, assigning the eccentric system a larger total strength with respect to an equivalent symmetric system still appears a suitable design criterion. In light of this remark, it is of great interest to determine the overstrength to be supplied in order to achieve a target value of maximum ductility demand. This evaluation allows to identify cases for which this procedure can be used without being prohibitively costly and it can be performed by computing the inelastic spectra with imposed ductility for eccentric systems. In the past, evaluation of inelastic spectra has been carried out for SDOF systems and the extension to coupled systems requires that such a procedure is developed with reference to the element characterized by the maximum damage or ductility demand, since plastic action is not uniform in plan.

As regards the damage index to be considered in the analysis, it is widely recognized that ability of structures to resist severe earthquakes manly relies on ductility and energy dissipation. Thus, a reliable damage index should account for both aspects in order to provide an effective measure of structural deterioration. However, recent studies (Cosenza et al. 1989) have shown that earthquake type loading histories are usually characterized by a single large plastic excursion and many plastic cycles with low ductility demands. In such cases, the hysteretic energy dissipation is a parameter less suitable than ductility demand.

In this paper, inelastic dynamic analysis of a simple two-degree-of-freedom system allows to evaluate spectra with imposed ductility, being the resisting element capacities determined by affecting the peak elastic forces with equal reduction factors. Then, the above spectral values are compared to values obtained for symmetric systems having period equal to the first period of the coupled system (Mahin and Bruneau 1990). In this way, an overstrength factor to be assumed is determined as the elastic system parameters vary and the cases are identified where effects of torsional response are more significant. Moreover, range of values are defined in which the overstrength factor is to be included in order to achieve a target ductility demand. Finally, the spectral values have been compared to that obtained from systems designed with selected values of inelastic parameters.

2 MODEL SPECIFICATION

Inelastic dynamic response of a two-degree-of-freedom system is examined. The model represents behaviour of a rigid deck of mass M and mass radius of gyration P, supported by lateral load resisting elements, located along x direction as well as y direction. The elements are considered massless and able to resist forces only in their plane. The system, which is subjected to translational ground motion along x direction, is assumed to be symmetric about the y axis (Figure 1a).

The stiffness centre C_S is located at a distance E_S from the mass centre C_M - where the origin of reference system is placed - which is given by:

$$E_S = \Sigma_i \, y_i k_{xi} / K_x \tag{1}$$

while the system torsional stiffness, computed with respect to C_S, is expressed by:

$$K_\theta = \Sigma_i \, k_{xi}(y_i - E_S)^2 + \Sigma_j \, k_{yj} x_j^2 = K_x D_S^2 \tag{2}$$

where D_S indicates the stiffness radius of gyration.

In equations (1) and (2), k_{xi} and k_{yi} denote the lateral stiffness of resisting elements oriented along x and y axis respectively, while K_x is the total stiffness along x direction.

The torsional stiffness arising from elements along y axis is represented by introducing a rotational spring of stiffness k_Φ, which is related to the global torsional stiffness K_θ by parameter γ (Figure 1b):

$$k_\Phi = \Sigma_j \, k_{yj} x_j^2 = \gamma \cdot K_\theta \tag{3}$$

Let e_S represent the dimensionless stiffness eccentricity E_S/L, d_S the dimensionless stiffness radius D_S/L and ρ the dimensionless mass radius P/L. Then, the system elastic response is seen to depend on the uncoupled translational period T, the normalized stiffness eccentricity e_S/ρ, the ratio d_S/ρ and the damping ratio ν.

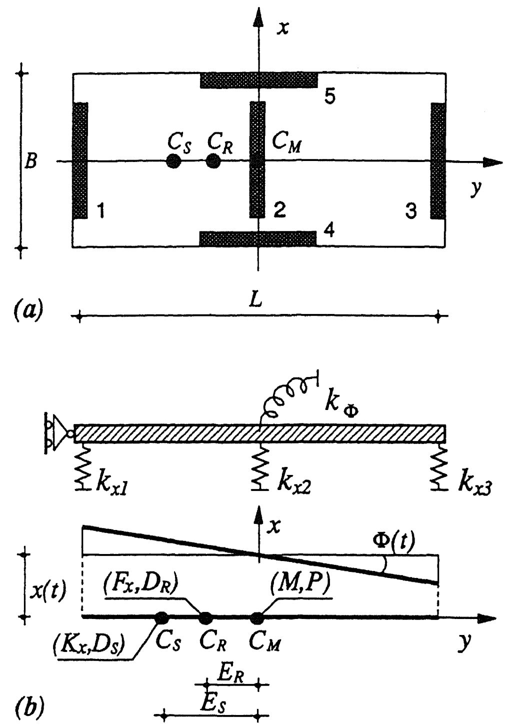

Figure 1. Idealized one-storey system

In addition to the above parameters, the inelastic response also depends on force-displacement relationship and plan-wise distribution of resisting elements. An elastic-perfectly plastic behaviour is assumed for resisting elements along the seismic excitation direction, whereas the rotational spring is supposed to elastically behave. The latter hypothesis is justified by the fact that the y direction elements undergo small deformations because of the symmetry about y axis.

Denoting with F_x the system total capacity and with F_{xi} the yield force of the ith element, the strength distribution in plan can be characterized by the strength eccentricity E_R (Sadek and Tso 1989), which defines the point C_R where the resultant of the forces F_{xi} is applied, and the strength radius D_R, computed with respect to C_R. The correspondent dimensionless parameters are given by:

$$e_R = \frac{E_R}{L} = \frac{\Sigma_i F_{xi} y_i}{L F_x} \tag{4}$$

$$d_R = \frac{D_R}{L} = \sqrt{\frac{\Sigma_i F_{xi}(y_i/L - e_R)^2}{F_x}} \tag{5}$$

3 SYSTEM DESIGN CRITERIA

The total system capacity F_x can be determined by reducing with a factor α the maximum elastic force F_x^e sustained by the system subjected to an assigned ground motion. The force F_x^e can be derived either from elastic dynamic analysis of an equivalent SDOF system - characterized by a period equal to the first period T_1 of the coupled system - or from elastic response of the actual coupled system. Clearly, the above procedures lead to different values of F_x, even because in the second case the force F_x^e is provided by the sum of peak elastic forces F_{xi}^e acting on resisting elements, which are attained at different times. According to the second procedure, the total system strength is provided by:

$$F_x = \sum_i F_{xi} = \frac{F_x^e}{\alpha} = \sum_i \frac{F_{xi}^e}{\alpha_i} \qquad (6)$$

while further specifications are needed in order to univocally define element strengths F_{xi}, whose distribution in plan strongly influences element ductility demands and structural damage.

A design criterion adopting equal design levels - α_i=α - for all elements results in an automatic definition of inelastic parameters e_R and d_R provided by equations (4) and (5).

Alternatively, for a given global reduction factor α, the element capacities F_{xi} can be determined by using different local design levels α_i, depending on element location, which corresponds to varying parameters e_R and d_R. In particular, if the considered model presents only three elements along seismic excitation direction, equations (4), (5) and (6) univocally provide the yield forces. In this case, design is performed with reference to inelastic system properties and one can search for the strength plan-wise distribution, defined in terms of e_R and d_R, that makes the plastic action uniform in plan.

4 INFLUENCE OF STRENGTH DISTRIBUTION PARAMETERS

In a recent study (De Stefano et al. 1991), the response of systems with the same elastic properties and total strength, but designed with several values of the inelastic parameters e_R and d_R, has been evaluated through the most widely used damage indices. In fact, evaluation of plastic action on a structural element can be carried out by introduction of indices that measure both maximum plastic deformation and energy dissipation (Mahin and Bertero 1981). Since analyses have shown similar variations of all damage indices with strength eccentricity and strength radius, comparison can be carried out with reference to any of them.

As an example, in Figure 2 the Park-Ang damage index D_i of resisting elements (Park and Ang 1985), multiplied by the available *monotonic* ductility μ_m, is presented for systems with T_1=0.40 *sec*, ρ=0.35, e_S=-0.10 and d_S=0.40 and subjected to an accelerogram from Friuli earthquake (Tolmezzo EW - 6/5/1976).

For the stiff side element 1, the damage index, shows an increasing behaviour as the strength centre C_R moves towards the flexible side element 3. An opposite trend characterizes the curves D_i for the element 3, even if variation is included into a closer range, while values of D_2 are almost independent of e_R. The increase of parameter d_R results in reduction of damage index for the element 1 and 3 whereas the values for element 2 grow. This behaviour is easily explained by the mechanical meaning of d_R.

An inelastic SDOF with period equal to 0.4 *sec* and design level equal to 4, excited by the same ground motion, shows $D_i \cdot \mu_m$ equal to 8. Then, the lateral torsional coupling results in increment of damage index, as well as ductility demand, on the stiff-side element and reduction on the flexible-side element, while D_2 on the central element is slightly larger.

Therefore, in systems sized so that the strength centre C_R coincides with the mass centre C_M (e_R=0), the stiff side element is subjected to larger plastic excursions whereas the flexible side element experiences larger plasticity as C_R moves towards C_S (Goel and Chopra 1991). Examination of these results demonstrates that an uniform plan-wise distribution of ductility demands, along with a significant reduction of maximum damage among resisting elements, can be achieved provided that proper values of the inelastic parameters are selected.

Moreover, attainment of such a condition is mainly influenced by the strength eccentricity e_R, whereas the strength radius d_R plays a less significant role. Obviously, as the elastic system characteristics vary, the values of e_R which correspond to uniformly distributed ductility demands are different.

In a previous paper (De Stefano et al. 1991), systems with periods T_1 varying from 0.3 to 0.8 *sec* have been considered and the best inelastic response has been seen to occur as the strength eccentricity is included between 0 and e_S, being sometimes close to $e_S/2$, while a value of d_R slightly larger or equal to that of a system with constant design levels appears to be suitable. The improvement of torsional response deriving from the above design criterion is remarkable, especially for torsionally stiff systems whose strength eccentricity is very different from $e_S/2$ as they are sized with equal design levels.

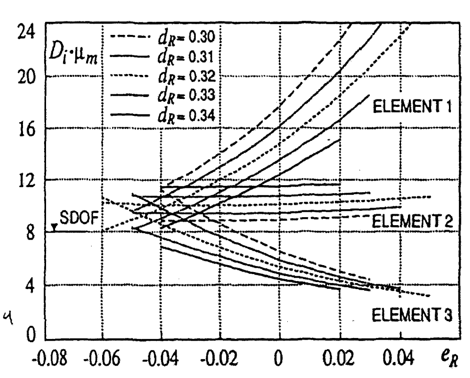

Figure 2. Park-Ang damage index $D_i \cdot \mu_m$ of systems with T_1=0.4 *sec*, e_S=-0.10 and d_S/ρ=1.143

5 CONSTRUCTION OF INELASTIC SPECTRA

Given the elastic system parameters, the linear response is evaluated and the peak element forces F_{xi}^e are determined. Subsequently, both models with equal design levels and models with selected values of inelastic parameters can be designed as already specified.

As the global design level varies with fixed values of the elastic parameters, the ductility demand μ_i for each element has been calculated such that the maximum ductility demand among resisting element $\mu_{i,max}$ can be defined. Therefore, the global design level $\bar{\alpha}$ is computed which corresponds to the assigned value $\bar{\mu}$ for $\mu_{i,max}$. Thus, the spectral inelastic value S_E for the eccentric system is given by:

$$S_E = \frac{\sum_i F_{xi}^e}{\bar{\alpha} M g} \tag{7}$$

The above quantity represents the dimensionless total strength needed by the asymmetric system in order to get the target ductility $\bar{\mu}$.

With a similar procedure, which is simplified by the fact that all resisting elements present equal ductility demands, the spectral values S_S for equivalent symmetric systems are obtained. Obviously, in addition to the imposed ductility $\bar{\mu}$ and the damping ratio ν, parameter S_S depends on the period and the seismic input. Thus, a factor O_F, representing the overstrength needed by the coupled system with respect to the symmetric system with period equal to T_1, can be defined as a function of the elastic parameters:

$$O_F(T_1, e_S, d_S) = \frac{S_E(T_1, e_S, d_S)}{S_S(T_1)} \tag{8}$$

having fixed ρ at constant value.

Such a parameter provides a concise measure of effects due to structural asymmetry on dynamic inelastic response, at least in terms of ductility demands.

6 ANALYZED SYSTEMS

The analyzed systems present three resisting elements oriented along the direction of seismic excitation, since it has been shown that a system with few elements can represent with a good approximation response of systems having a larger number of elements (Goel and Chopra 1990). Moreover, a significant torsional stiffness arises from perpendicular elements, since parameter γ has been fixed at a value of 0.40. All systems are characterized by ρ equal to 0.35, which corresponds to a ratio L/B=1.459. Values of the first period T_1 have been considered varying between 0.1 and 2.0 *sec*, while two values are chosen for the stiffness eccentricity: e_S=-0.10 and e_S=-0.20. The stiffness radius d_S is assigned values of 0.35 and 0.40, such that behaviour of torsionally-flexible and torsionally-stiff systems is analyzed.

Three values are chosen for the imposed ductility - $\bar{\mu}$=3,4,5 - since it should be noticed that an available ductility of 4 is usually believed to be representative of well designed structures against earthquake type loading. The inelastic spectra have been plotted assuming damping ratio ν to be equal to 0.05 for both modes of vibration. As regards seismic input, a record from Friuli earthquake (Tolmezzo EW - 6/5/1976) has been used. The accelerogram presents duration of 36.40 *sec*, maximum acceleration of 0.312 *g* and an acceleration spectrum which amplifies over a short range of periods about 0.6 *sec*.

7 INELASTIC SPECTRA AND OVERSTRENGTH FACTORS

A first analysis has been carried out in order to evaluate effect of torsional coupling on systems designed with equal reduction factors α_i. In Figures 3 and 4 the inelastic spectra for torsionally-stiff systems with e_S=-0.10

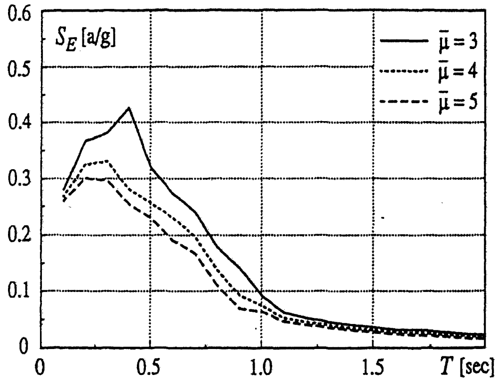

Figure 3. Inelastic spectra for systems with d_S/ρ=1.143 and e_S=-0.10

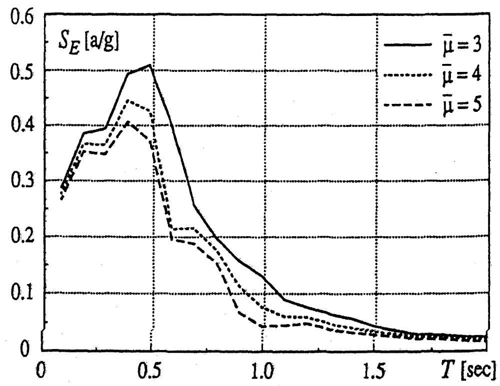

Figure 4. Inelastic spectra for systems with d_S/ρ=1.143 and e_S=-0.20

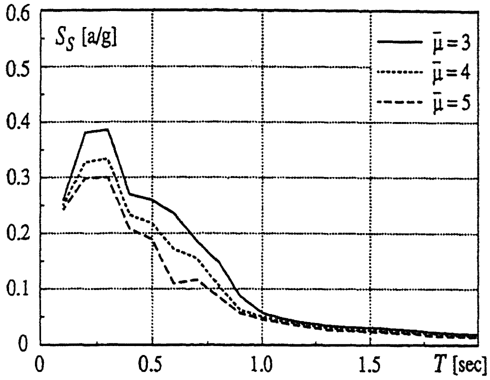

Figure 5. Inelastic spectra for symmetric systems

and e_S=-0.20 are plotted, while Figure 5 contains inelastic spectra for the correspondent symmetric systems.

From comparison of figures 3 and 4 with figure 5 asymmetric systems are seen to require a larger strength - compared to that needed by a symmetric system - in order to get an imposed ductility on the element with the greater inelastic excursion. A concise examination of results can be clearly performed by calculating the above defined overstrength O_F. This factor is represented in Figures 6 through 9 as the elastic system parameters vary and it not only allows to relate asymmetric systems to symmetric systems but also to compare asymmetric systems having different stiffness eccentricities and stiffness radii.

Curves of O_F frequently show an irregular trend with period T_1; however, all laterally stiff models are characterized by factors O_F very close to unity. The required overstrength is nearly independent of target ductility for short period and long period systems whereas $\bar{\mu}$ strongly influences O_F for medium period systems. In the latter cases a clear trend cannot be recognized.

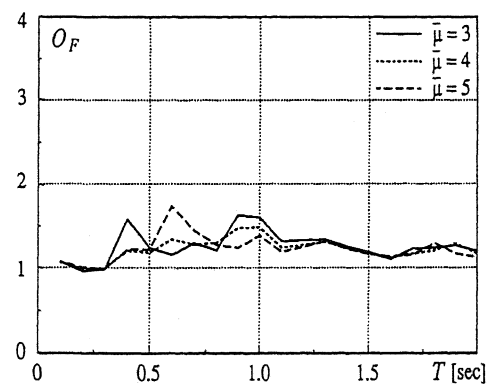

Figure 6. Overstrength factors for systems with d_S/ρ=1.143 and e_S=-0.10

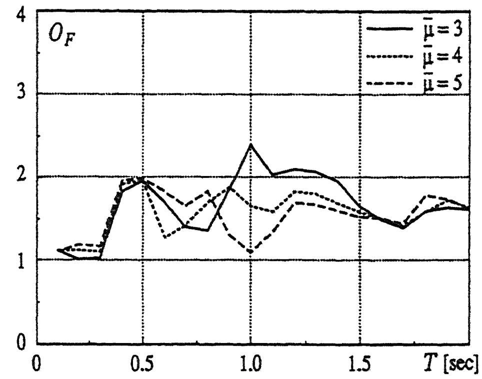

Figure 7. Overstrength factors for systems with d_S/ρ=1.143 and e_S=-0.20

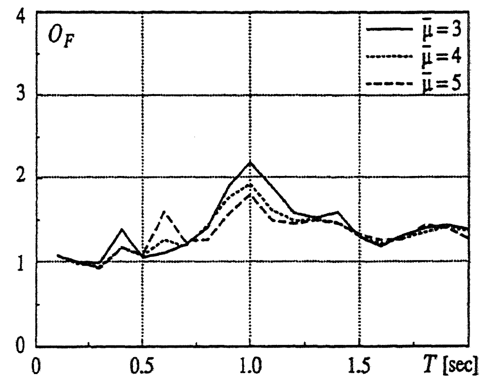

Figure 8. Overstrength factors for systems with d_S/ρ=1.00 and e_S=-0.10

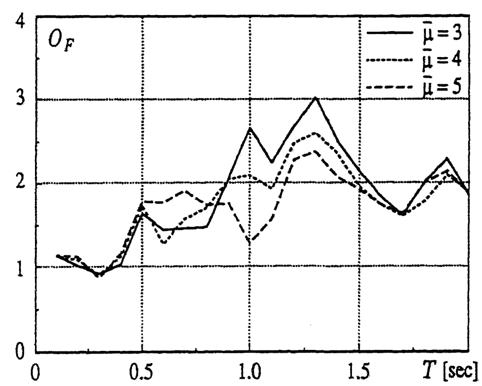

Figure 9. Overstrength factors for systems with d_S/ρ=1.00 and e_S=-0.20

Values of O_F reduce as the torsional stiffness increases, while the overstrength generally grows for systems with larger eccentricity e_S. In particular, curves of models with e_S=-0.10 present a rather flat tendency with variation of T_1 as d_S/ρ=1.143 and values seldom are larger than 1.5. As torsional stiffness decreases (Figure 8) systems generally demand values up to 2.0 for d_S/ρ=1.0, with a larger dependence on period.

Systems having stiffness eccentricity equal to -0.20 are severely influenced by the first period T_1 and they need to be designed with total strength which is usually almost twice as much as symmetric systems require.

The above results evidence that a design criterion based on adopting equal design levels for all resisting elements does not allow an effective use of available total strength. Hence, adoption of location dependent design levels represents a more reliable procedure since ductility demands of eccentric structures can be reduced up to values that characterize response of symmetric systems by adopting the same total strength.

This remark is confirmed by inelastic spectra drawn in Figure 10 where curves for torsionally stiff systems, whose elements have equal α_i, are compared to curves obtained by defining plan-wise strength distribution with selected values of inelastic parameters. In particular, according to observations already outlined in discussing Figure 2, d_R has been assumed equal to the value that characterizes models with α_i=α, while e_R has been fixed at value of $e_S/2$. Design performed by setting inelastic parameters d_R and e_R in such a way is seen to remarkably improve inelastic response, primarily as the period T_1 is shorter than 0.6 *sec*. In fact, in this range of periods a fixed value of ductility demand is achieved by giving the asymmetric systems a total strength virtually equivalent to that needed by symmetric structures. The improvement to be obtained with this design methodology can be extended to longer periods provided that the selected values of d_R and e_R are somehow made dependent on elastic systems parameters. A comparable enhancement of inelastic response is not evidenced for torsionally flexible structures since the criterion adopting constant α_i leads to systems having d_R and e_R very close to selected ones.

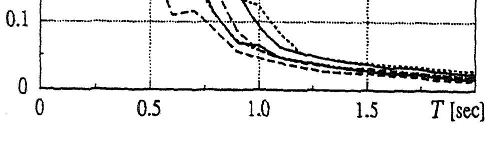

Figure 10. Inelastic spectra for systems with d_S/ρ=1.143, e_S=-0.10 and sized by two design criteria

8 CONCLUSIONS

Analysis of torsional inelastic response of a two degree of freedom stiffness eccentric model has been performed in order to construct inelastic spectra for asymmetric systems. The extension of the usual procedure developed for SDOF systems has been carried out by setting the target value of ductility demand on the element that experiences the largest plastic excursions. The comparison with inelastic spectra evaluated for symmetric systems having period equal to the first period of the coupled systems has allowed to determine the overstrength required by the asymmetric structures. Results obtained using two different design criteria - the first one adopting equal reduction factors for all resisting elements and the second one based on location dependent reduction factors - has confirmed conclusions already drawn by the Authors. In fact, for short and medium periods, the second criterion, which results in defining a proper plan-wise strength distribution, leads to design asymmetric systems demanding a total yield capacity very close to that needed by symmetric systems in order to get a target ductility demand. Further studies are needed for widening this methodology since values of inelastic parameters that achieve a significant improvement of torsional response are seen to depend on elastic system parameters.

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