

# Augmentation of inherent damping in buildings with damped resonant appendages

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**ABSTRACT:** It is demonstrated that the addition of a resonant appendage with a relatively small mass and a high damping ratio can be an effective way to increase the inherent damping characteristics of buildings and reduce, thus, their response to earthquake excitations. The demonstration is based on a theoretical formulation and on numerical and experimental studies. The theoretical formulation shows that, if certain conditions are satisfied, the damping ratios in two of the modes of a system formed by a building and a small appendage in resonance are approximately equal to the average of the damping ratios of the two independent components in the corresponding modes and provides, therefore, an explanation as to why damping in a building increases with a damped appendage in resonance. The numerical and experimental studies verify the theory and confirm that the suggested appendages may indeed reduce the seismic response of building structures.

## 1 INTRODUCTION

Civil engineers have now begun to recognize that added damping devices may be a cost-effective way to reduce the vibrations of high-rise buildings, bridges, tall chimneys, and other slender structures that are affected by dynamic forces, and have turned their attention to explore fully the potential benefits of such a technique. This recognition has been brought up by the successful application of vibration absorbers to control wind-induced oscillations in a 34-m steel footbridge in South Africa (Chasteau 1973), a 200-m high chimney (Allaway & Grootenhuis 1965), bridge road deck hangers in Canada (Wardlaw 1973), a 700-ft high tower supported on top of a 150-ft high building in Sydney, Australia (Eng. News Record 1971), 60-story buildings in Boston and New York City (Eng. News Record 1975, 1977), and a 1815-ft high antenna tower in Toronto, Canada (Eng. News Record 1976). It has also stimulated research and implementation of a variety of energy dissipating devices such as viscoelastic dampers, brake lining pad dampers, hysteretic dampers, wire rope dampers, hydrodynamic dampers, and lead extrusion dampers.

In addition to supplemental dampers and vibration absorbers, several other techniques have been suggested to control the response of structures to dynamic loadings. Among the ones that are presently being actively investigated, and in some cases implemented in actual or pilot buildings, are base isolation, tuned mass dampers, active control, or active control in combination with any of the other techniques. All of these new techniques offer a great promise and, without doubt, they will offer viable and cost-effective solutions in the near future. Notwithstanding, independently of how effective a control system might be, it is likely that the design profession at large will only accept those innovations that do not represent large departures

from current, accepted practice. In this sense, the most easily accepted control system will be, and this seems to agree with some of the recommendations emanated from the 1986 ATC workshop on base isolation and passive energy dissipation, those that simply add to a conventional structure, rather than those which require radical changes to the way structures are designed and constructed.

Two forms of structural control which conform to this requirement of simplicity are the addition of hysteretic or friction dampers and the use of tuned mass dampers. Hysteretic and friction dampers seem to be an effective way to reduce seismic response, but, cost aside, there seems to be two problems with them. One is the problem of defining the appropriate yield level and the other is that they encumber the design procedure. Since these devices start working only after their yield or friction level is exceeded, if that level system is set too high, there is the danger of some structural and nonstructural damage before they actually start working. On the other hand, if their yield level is set too low, any mild disturbance might unnecessarily will make these elements yield with the consequent replacement or adjustment cost. Most importantly, however, is the fact that the design of buildings with these energy dissipation elements requires a nonlinear analysis and that, as a result, these energy dissipation elements complicate the design process. In contrast, ever since they were first suggested in 1909 (Frahm 1909) tuned mass dampers have continuously attracted the attention of the engineering profession. They are relatively easy to implement, do not encumber the design process, can be considered in the design of new, conventionally designed buildings as well as in old ones in need of retrofit; do not require radically different design procedures; do not depend on an external power source; do not interfere with the principal vertical and horizontal load paths; can be

designed to respond to small levels of excitation; have been proven useful in reducing the vibrations of a variety of dynamic systems; and, seemingly, can be cost effective. Presently, it is generally accepted that they can be effective in reducing the response of structural systems subjected to harmonic excitations (Den Hartog 1956), a summation of sinusoidal ground accelerations (Warburton 1990), and to wind forces (McNamara 1977). They have been implemented effectively to reduce wind-induced vibrations in high-rise buildings (Eng. News Record 1971, 1975, 1977), and even to reduce floor vibrations induced by occupant activity (Thornton et al. 1990; Webster & Levy 1992). To date, however, the use of tuned mass dampers to reduce seismic response is still in the controversial stage. Some researchers (e.g., Wirsching & Yao 1973; Wirsching & Campbell 1974; Kitamura et al. 1988; Clark 1988) have found that such damping devices may indeed reduce the response of structures to earthquake loads, and some other (Kaynia et al. 1981; Sladek & Klingner 1983) altogether dismiss their effectiveness.

The purpose of this paper is to demonstrate that the addition of a tuned mass-spring-dashpot system with a relatively small mass and a high damping ratio can be an effective way to increase the inherent damping characteristics of buildings and reduce, thus, their response to earthquake excitations. For such a purpose, a theoretical formulation is presented first to show that the damping ratios in two of the modes of a system consisting of a building and a small appendage in resonance are equal to the average of the corresponding damping ratios of the two components. Then, on the basis of this formulation, it is shown that an appendage with a high damping ratio and tuned to the fundamental frequency of a building may be used to increase the damping ratio in the first mode of the building to a value close to half the damping ratio of the appendage and, hence, to reduce its response to seismic disturbances. The paper also presents the results of numerical and experimental studies conducted to verify the theoretical findings and assess the effectiveness of such a technique in reducing the seismic response of buildings.

## 2 THEORETICAL BACKGROUND

### 2.1 Damping ratios and natural frequencies of building-appendage systems

A complication that arises when a building is considered with a high damping resonant appendage is that the combined system formed by the building and the appendage cannot be considered as a system that possesses classical modes of vibration (Villaverde & Newmark 1980). Nevertheless, if it is assumed that each of the two components has by itself a damping matrix proportional to its own stiffness matrix and, hence, classical modes of vibration, it is possible to derive an approximate expression for the damping ratios and natural frequencies of the building-appendage system in terms of their independent dynamic properties. In particular for the modes of the combined system which result from tuning one of the natural frequency of the absorber to one of the natural frequencies of the building, such an expression can be

obtained as follows:

Consider the free vibration equation of a nonclassically damped building-appendage system:

$$[A] \{\dot{q}\} + [B] \{q\} = \{O\} \quad (1)$$

where

$$[A] = \begin{bmatrix} [O] & [M] \\ [M] & [C] \end{bmatrix}; [B] = \begin{bmatrix} -[M] & [O] \\ [O] & [K] \end{bmatrix}; \{q\} = \begin{Bmatrix} \dot{x} \\ x \end{Bmatrix} \quad (2)$$

in which the matrices  $[M]$ ,  $[C]$ , and  $[K]$  respectively represent the mass, damping, and stiffness matrices of such a building-appendage system, and  $\{x\}$  denotes its vector of relative displacements. If, however,  $[M]$ ,  $[C]$ , and  $[K]$  are written in terms of  $[M]_b$ ,  $[C]_b$ , and  $[K]_b$ , the mass, damping and stiffness matrices of the building without the appendage, and  $[M]_a$ ,  $[C]_a$ ,  $[K]_a$ , the corresponding matrices of the appendage by itself when the end connected to the building is considered fixed, Eq. 1 can be written alternatively as

$$\begin{bmatrix} [A]_b & [O] \\ [O] & [a]_a \end{bmatrix} \begin{Bmatrix} \{q\}_b \\ \{q\}_a \end{Bmatrix} + \begin{bmatrix} [B]_b & [O] \\ [O] & [b]_a \end{bmatrix} \begin{Bmatrix} \{q\}_b \\ \{q\}_a \end{Bmatrix} + \quad (3)$$

$$\begin{bmatrix} [O] & [P] \\ [P]^T & [O] \end{bmatrix} \begin{Bmatrix} \{q\}_b \\ \{q\}_a \end{Bmatrix} + \begin{bmatrix} [V] & [T] \\ [T]^T & [O] \end{bmatrix} \begin{Bmatrix} \{q\}_b \\ \{q\}_a \end{Bmatrix} = \begin{Bmatrix} \{O\} \\ \{O\} \end{Bmatrix}$$

where  $[A]_b$ ,  $[B]_b$ , and  $\{q\}_b$ , and  $[a]_a$ ,  $[b]_a$ , and  $\{q\}_a$  are defined as in Eq. 2 but with the displacements and the mass, damping, and stiffness matrices of the building and the appendage, respectively, and where the matrices in the last two terms of the left-hand side of the equation are simply matrices that account for the coupling between the two subsystems.

Consider now that when the building-appendage system is vibrating in free vibration in its  $r$ th mode, the building and the absorber by themselves can be considered as vibrating under the action of an external force whose magnitude is equal to the interaction force between the two components and whose variation with time is given by  $e^{\lambda_r t}$ , where  $\lambda_r$  denotes the  $r$ th complex natural frequency of the building-appendage system. In doing so, and since the response of a system without classical damping is also given by the sum of the response in each of its modes (Hurty & Rubinstein 1964), the response of the building and the absorber to such an interaction force in the  $r$ th mode of the combined system may be then written as

$$\{q\}_b = \sum_{i=1}^{2N_b} \{S\}_b^{(i)} Z_i e^{i\lambda_r t}; \{q\}_a = \sum_{j=1}^{2N_a} \{s\}_a^{(j)} z_j e^{i\lambda_r t} \quad (4)$$

which, by considering only the dominant mode in each case, may in turn be approximated as

$$\{q\}_b = \{S\}_b^{(I)} Z_I e^{i\lambda_r t}; \{q\}_a = \{s\}_a^{(J)} z_J e^{i\lambda_r t} \quad (5)$$

In these equations, respectively for the building and the appendage,  $Z_i$  and  $z_j$  represent generalized coordinates,  $N_b$  and  $N_a$  their total number of degrees of freedom,  $I$  and  $J$  the number that corresponds to their dominant modes, and  $\{S\}_b^{(I)}$  and  $\{s\}_a^{(J)}$  complex eigenvectors of the form

$$\{s\}_r = \{\lambda_r \{w\}_r, \{w\}_r\}^T \quad (6)$$

where  $\{w\}_r$  denotes a complex mode shape and  $\lambda_r$  the corresponding complex natural frequency.

By substitution of Eq. 5 into Eq. 3, and if the upper and

lower component equations of the latter are respectively premultiplied by the transpose of  $\{S\}_b^{(1)}$  and  $\{s\}_b^{(2)}$ , the free vibration equation of motion for the building-appendage system may then be reduced to

$$\begin{bmatrix} \lambda_r A_{br}^* + B_{br}^* + \lambda_r Q_I^* + V_I^* & \lambda_r P_{Ur}^* + T_{Ur}^* \\ \lambda_r P_{Ur}^* + T_{Ur}^* & \lambda_r a_{ar}^* + b_{ar}^* \end{bmatrix} \begin{Bmatrix} Z_I \\ z_J \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (7)$$

where

$$A_{br}^* = \{S\}_b^{(1)T} [A]_b \{S\}_b^{(1)}; B_{br}^* = \{S\}_b^{(1)T} [B]_b \{S\}_b^{(1)} \quad (8)$$

$$a_{ar}^* = \{s\}_a^{(2)T} [a]_a \{s\}_a^{(2)}; b_{ar}^* = \{s\}_a^{(2)T} [b]_a \{s\}_a^{(2)} \quad (9)$$

and where  $Q_I^*$ ,  $V_I^*$ ,  $P_{Ur}^*$ , and  $T_{Ur}^*$  are similarly defined in terms of the coupling matrices  $[Q]$ ,  $[P]$ ,  $[V]$  and  $[T]$ . Thus, after taking into account that  $B_{bI}^* = -\lambda_{bI} A_{bI}^*$  and  $b_{aJ}^* = -\lambda_{aJ} a_{aJ}^*$  (see Hurty & Rubinstein 1964), where  $\lambda_{bI}$  and  $\lambda_{aJ}$  respectively represent the  $I$ th and the  $J$ th complex natural frequencies of the building and the absorber, Eq. 7 leads to the following eigenvalue problem:

$$\begin{bmatrix} A_{br}^*(\lambda_r - \lambda_{bI}) + (\lambda_r Q_I^* + V_I^*) & \lambda_r P_{Ur}^* + T_{Ur}^* \\ \lambda_r P_{Ur}^* + T_{Ur}^* & a_{ar}^*(\lambda_r - \lambda_{aJ}) \end{bmatrix} = 0 \quad (10)$$

Furthermore, if it is considered that: (a) under the assumption of components with proportional damping,  $A_{bI}^* = 2i\omega'_{bI} M_{bI}^*$  and  $a_{aJ}^* = 2i\omega'_{aJ} m_{aJ}^*$ , where  $\omega'_{bI}$  and  $\omega'_{aJ}$ , and  $M_{bI}^*$  and  $m_{aJ}^*$ , respectively represent the damped natural frequencies and generalized masses of the building and the appendage in their  $I$ th and  $J$ th modes; (b) when the  $J$ th natural frequency of the absorber is tuned to the  $I$ th natural frequency of the building, the undamped natural frequencies  $\omega_{bI}$  and  $\omega_{aJ}$  are the same and equal to  $\omega_o$ ; (c) by assumption the mass of the absorber is much smaller than the total mass of the building; and (d) the complex natural frequencies can be written in terms of natural frequencies and damping ratios as

$$\lambda_r = -\xi_r \omega_r + i\omega_r' \quad (11)$$

where  $\omega_r'$  and  $\xi_r$  respectively denote the  $r$ th damped natural frequency and damping ratio of the system; such an eigenvalue problem leads to the following approximate solution for the building-appendage complex frequency  $\lambda_r$ :

$$\lambda_r = -(\xi_{bI} + \xi_{aJ})\omega_o/2 + i\omega_o \pm (\omega_o/2)[(\xi_{bI} + \xi_{aJ})^2 - \Phi_k^2 \gamma_{IJ}]^{1/2} \quad (12)$$

where  $\xi_{bI}$  and  $\xi_{aJ}$  respectively denote the damping ratios of the building and the appendage in their  $I$ th and  $J$ th modes;  $\omega_o$  is a frequency that is common to the building and the appendage;  $\gamma_{IJ} = m_{aJ}^*/M_{bI}^*$ , where  $m_{aJ}^*$  and  $M_{bI}^*$  are the  $J$ th and  $I$ th generalized masses of the building and the appendage; and  $\Phi_k$  is the amplitude of the point of the building to which the appendage is attached in the  $I$ th mode shape of the building, after it is multiplied by the building's  $I$ th participation factor.

In accordance with Eq. 11 and for relatively small mass and damping ratios, it may be seen thus that two of the damping ratios and two of the natural frequencies of the building-appendage system are given approximately by

$$\omega_r = \omega_o; \xi_r = (\xi_{bI} + \xi_{aJ})/2 \pm (1/2)[(\xi_{bI} - \xi_{aJ})^2 - \Phi_k^2 \gamma_{IJ}]^{1/2} \quad (13)$$

if  $|\xi_{bI} - \xi_{aJ}| \geq |\Phi_k \sqrt{\gamma_{IJ}}|$ , and by

$$\omega_r = \omega_o \pm (\omega_o/2)[\Phi_k^2 \gamma_{IJ} - (\xi_{bI} - \xi_{aJ})^2]^{1/2}; \xi_r = (\xi_{bI} + \xi_{aJ})/2 \quad (14)$$

if  $|\xi_{bI} - \xi_{aJ}| \leq |\Phi_k \sqrt{\gamma_{IJ}}|$ .

## 2.2 Selection of appendage parameters

Equation 14 indicates that if  $|\xi_{bI} - \xi_{aJ}| \leq |\Phi_k \sqrt{\gamma_{IJ}}|$ , the damping ratios in two of the modes of a system formed by a building and an appendage in resonance are approximately equal to the average of the damping ratios in the modes in resonance of the independent building and appendage. This in turn suggests that, at least in principle, the attachment to a building of an appendage that has a high damping ratio, is in resonance with the fundamental mode of the building, and satisfies the above inequality may be a means to augment its inherent damping and hence a convenient method to reduce its response to earthquake ground motions and other dynamic loads. Furthermore, one may note that according to these conditions the mass, stiffness constant, and damping ratio of the appendages that can be utilized to increase the damping of a building may be selected on the basis of the known dynamic properties of the building and the following recommendations: (a) select for the appendage the highest possible damping ratio without exceeding the critical; (b) calculate its mass from the equality  $|\xi_{bI} - \xi_{aJ}| = |\Phi_k \sqrt{\gamma_{IJ}}|$ ; and (c) determine its stiffness constant from the condition  $\omega_a = \omega_b$ , where  $\omega_a$  and  $\omega_b$  are respectively the fundamental natural frequencies of the appendage and the building.

## 3 NUMERICAL STUDY

Numerical studies have been conducted previously (Villa-verde 1985) whereby it is verified that the use of the resonant appendages suggested above indeed reduce the response of buildings to earthquake excitations. The buildings involved in these studies were a single-story building and a ten-story one, both structured with three-dimensional frames and under three different earthquake excitations. The reduction in their response is achieved by means of appendages with damping ratios of 40 and 90 per cent. Such a reduction, however, is accomplished at the cost of adding at their roofs a mass that represents a relatively large percentage of the building's total mass. It is of interest, therefore, to assess the extent to which these resonant appendages can be effective when they are designed with relatively small damping ratios and, as a result, relatively small masses. With this objective in mind, the 10-story shear building shown in Figure 1 is analyzed under three different earthquake records, for the cases in which (a) the building has no appendage; the building has a 20 per cent damping attached to its roof; and (c) the building also has an appendage attached to its roof, but the damping of the appendage is 30 per cent. The building is considered to behave elastically at all times and with a damping ratio of 2 per cent in its fundamental mode. Its damping matrix is assumed proportional to its own stiffness matrix. The appendage consists of a mass, a spring, and a damper, and it is also modeled as a shear beam. The records used are the first ten seconds of the following accelerograms from California earthquakes: (a) S00E 1940 El Centro; (b) N21E 1952 Taft earthquake; and (c) S16E 1971 Pacoima Dam. In each case, the natural frequency of the appendage is tuned to the fundamental frequency of the building, which in this particular case is

equal to 1.0 Hz, and its parameters selected according to the design recommendations established above. For the 20 per cent damping appendage, these recommendations lead to a mass of 19 Mg and a stiffness constant of 0.769 MN/m. Similarly, for the 30 per cent damping appendage, its mass and stiffness constant result as 47 Mg and 1.861 MN/m, respectively. Note that mass of the appendage with 20 per cent damping represents 1.4 per cent of the total mass of the building and 19.4 per cent of the roof mass, whereas for the appendage with 30 per cent damping those percentages are 3.4 and 48.0, respectively.

Level	Stiffness	Mass
10	137.24 MN/m	98 Mg
9	149.71	107
8	162.19	116
7	174.66	125
6	187.14	134
5	199.62	143
4	212.09	152
3	224.57	161
2	237.05	170
1	249.88	179
0		

Figure 1. 10-story shear building in numerical study

Table 1. Maximum displacements of 10-story building under the 1940 El Centro ground acceleration record

Level	Maximum displacements (m)		
	with no appendage	with 20% damping appendage	with 30% damping appendage
1	0.0320	0.0249	0.0195
2	0.0643	0.0503	0.0399
3	0.0957	0.0750	0.0592
4	0.1251	0.0980	0.0777
5	0.1514	0.1185	0.0944
6	0.1739	0.1359	0.1088
7	0.1921	0.1502	0.1212
8	0.2059	0.1613	0.1316
9	0.2201	0.1694	0.1419
10	0.2280	0.1743	0.1486

Table 2. Maximum displacements of 10-story building under the 1952 Taft ground acceleration record

Level	Displacement (m)		
	with no appendage	with 20% damping appendage	with 30% damping appendage
1	0.0093	0.0085	0.0078
2	0.0187	0.0173	0.0159
3	0.0281	0.0260	0.0240
4	0.0371	0.0343	0.0318
5	0.0456	0.0423	0.0393
6	0.0532	0.0495	0.0463
7	0.0597	0.0558	0.0525
8	0.0648	0.0608	0.0577
9	0.0681	0.0644	0.0617
10	0.0698	0.0664	0.0643

The results of the study are summarized in Tables 1 through 3, where it is shown the maximum floor displacements for the building without the appendage and with the 20 and 30 per cent damping appendages. It is observed from these tables that for the two damping values and each of the three ground accelerations considered the maximum

displacements are reduced with the addition of the resonant appendage. As expected, however, this reduction depends on the ground excitation and increases as the damping of the appendage is increased.

Table 3. Maximum displacements of 10-story building under the 1971 Pacoima Dam ground acceleration record

Level	Maximum displacement (m)		
	with no appendage	with 20% damping appendage	with 30% damping appendage
1	0.0658	0.0596	0.0549
2	0.1332	0.1197	0.1105
3	0.1992	0.1784	0.1656
4	0.2601	0.2349	0.2188
5	0.3142	0.2876	0.2690
6	0.3627	0.3353	0.3153
7	0.4093	0.3771	0.3565
8	0.4478	0.4115	0.3918
9	0.4759	0.4371	0.4194
10	0.4911	0.4527	0.4378

#### 4 EXPERIMENTAL STUDY

To verify experimentally the effectiveness of the proposed resonant appendages, a simple wooden structural model and a damper were built and tested in a shaking table under random and sinusoidal excitations.

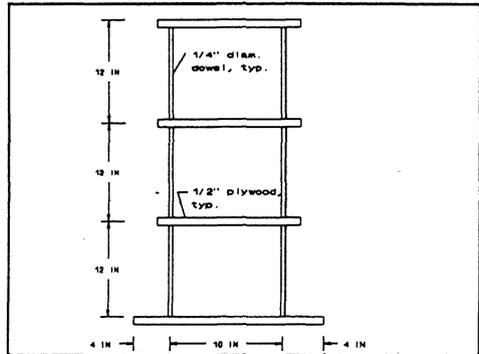


Figure 2. Structural model in experimental study

##### 4.1 Structural model

The structural model was designed as a three-story building structure loaded with three equally spaced lumped masses. One-half inch plywood was used for the floors and 1/4 inch diameter dowels were used for the columns. Figure 2 provides the configuration and dimensions of the model. In order to prevent rocking, wood block strips 1-1/2" by 3-1/2" by 2'-0" were attached to the top of the base of the model. To add mass to the floors, steel weights were attached with hot glue to each story along the center line of the floors in the direction to be tested. The total weights for the model were of 4.76 lb for the top story, 3.84 lb for the middle story, and 3.83 lb for the bottom one. The frequency, damping ratio, and mode shapes of the structure were obtained through an impact hammer test. The values obtained are summarized in Table 4. It was determined by the various means of exciting the structure that the first, third and fifth modes were bending modes, while the

second, fourth and sixth were torsional modes.

Table 4. Natural frequencies and damping ratios of structural model

Mode	Frequency (Hz)	Damping (%)
1	2.183	3.948
2	4.522	2.617
3	6.841	2.053
4	10.885	1.269
5	13.886	1.621
6	21.203	1.673

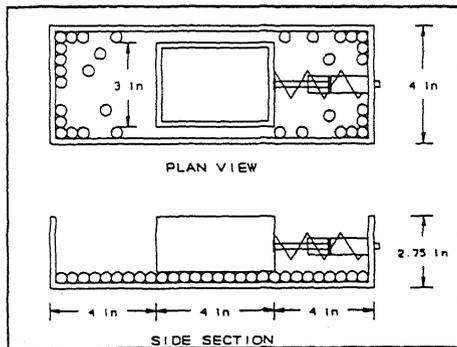


Figure 3. Damper model in experimental study

#### 4.2 Damper model

The damper model was designed as a single-degree-of-freedom system consisting of a weight attached to a spring and a dashpot as shown in Figure 3. The model also incorporates an inner box attached to an outer box via the spring and the dashpot, and ball bearings with a diameter of 1/4 inch used to prevent friction between the inner and the outer boxes. The inner box is used to hold the mass of the damper and the outer one serves to contain the ball bearings. The boxes were constructed with 1/4-inch plywood. The damper was designed to obtain a damping ratio of 53.5 per cent. The dashpot consisted of a Fortuna 5 ml syringe, with the plunger filed down slightly to prevent friction along the sides of the syringe. The dashpot value was measured to be 0.063 lb-sec/in. The constant of the spring was of 0.8341 lb/in. Weights were added to inner box to obtain a damper weight of 1.70 lbs. The weight, spring constant, and damping constant of the damper approximately correspond to the design parameters obtained using the design recommendations given in Section 2.2 for the structural model under consideration and an appendage with a damping ratio of 53.5 per cent. These design parameters turned out to be as follows: weight = 1.735 lbs; spring constant = 0.846 lb/in; and damping constant = 0.656 lb-sec/in.

The damper model was subjected to an impact hammer test by itself to obtain its natural frequency and confirm that its design frequency was close to the fundamental natural frequency of the structure. The results of this test revealed a first peak in its frequency response curve at 2.406 Hz; this corresponds closely to the design frequency of 2.184, with a percent difference of 10.16 percent.

#### 4.3 Shaking table tests with random excitations

In the shaking table tests, the structure was tested first under random excitations generated by a HP3562A dynamic signal analyzer. The structure was tested over a period of time in which a number of averages were taken and the frequency response curves were obtained. The damper model was then placed on the top story, removing beforehand some of the steel weights on the top floor to compensate for the weight of the device beyond the design weight of 1.70 lb. The testing was repeated and new frequency response curves were obtained. The frequency response curves obtained from the tests without and with the damper are shown back to back in Figure 4.

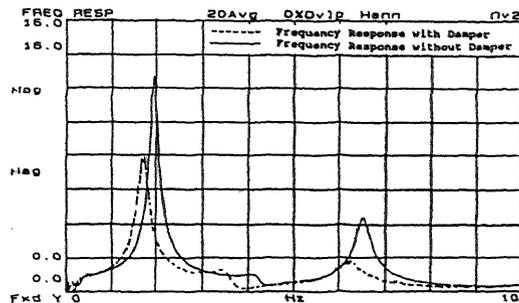


Figure 4. Frequency response diagrams of structural model without and with damper

A comparison between the frequency response curves in Figure 4 indicates that the addition of the damper to the structural model induces a reduction of 38.3 percent in the value of the ordinate of the frequency response function that corresponds to the fundamental frequency of the system. A reduction is observed too in the value for the third natural frequency. In contrast a slight increase is observed for the second mode, which, given that this mode is a torsional one, could have been the result of an increase in the moment of inertia of the system when the damper was added to its top story. Although the mass of the top story is the same in both cases, the damper has its weight distributed over a larger area, thus resulting in a larger polar moment of inertia and an increase in the torsional motion of the structure.

#### 4.4 Shaking table tests with sinusoidal excitations

Since the reduction in structural response by a resonant appendage is caused by the large response of the appendage mass and hence by the large dissipation of energy by the appendage damper, it was considered of interest to evaluate the effectiveness of the suggested damping devices for different levels of the appendage response. For such a purpose the shaking table experiment was extended to include tests under different sinusoidal excitations. Sinusoidal excitations with a frequency of 1.5, 2.0, and 3.0 Hz were considered. The frequency of 2.0 Hz represents a frequency close to the natural frequency of the structural

model and hence a frequency that would induce a large structural and appendage response. In contrast, the frequencies of 1.5 and 3.0 Hz are separated from the fundamental frequency of the system, with the frequency of 3.0 Hz being the one that is the farthest apart, and thus they are frequencies for which one would expect a relatively lower structural and appendage response.

The structure responded as expected when tested under the sinusoidal excitations. As expected, too, at 2.0 Hz the damper performed at its best. In this case, the damper reduced the acceleration response of the structure's top floor by 45.2 percent. When the input frequency was reduced to 1.5 Hz, the damper worked, but not to the extent as at 2.0 Hz. For this frequency, the reduction factor was of 5.9 per cent. At 3.0 Hz the damper still functioned, but not very effectively. The reduction factor obtained for this frequency was of 2.9 per cent.

## 5 CONCLUSIONS

It has been shown that if a small appendage with a relatively small mass and a high damping ratio is attached to a building structure and tuned to one its modes of vibration, then the combined structure-appendage system results in a system for which two of its modes of vibration have a damping ratio that is approximately equal to the average of the damping ratios of the two independent components. On the basis of this finding, it has been observed that the inherent damping in a structure may be significantly augmented by attaching to the structure a relatively small appendage with a high damping ratio, a natural frequency equal to the natural frequency of the dominant mode of the structure, and parameters that satisfy a relationship between the dynamic properties of the building and the appendage. This observation, and the corresponding reduction in structural response by such an appendage, has been verified by means of numerical and experimental tests. It is concluded, thus, that the use of these resonant appendages has the potential to become an effective technique to reduce the response of structures to earthquakes.

The results of the numerical and experimental studies also indicate that there exists a dependence of the reduction in response attained by means of the appendages in question on the characteristics of the ground motion. It is also observed that as one increases the damping ratio of the appendage, the size of the appendage mass also increases. At first, these two factors may appear as a hindrance to the application of the technique since one might think that a large mass is needed to attain a significant reduction in response and that the system might work for some but not all possible excitations. On second thoughts, however, it should be noted that what the application of the technique does to a structure is simply to increase its damping. This implies that a very high damping ratio, and hence a large appendage mass, will not be necessary in most cases since beyond a certain limit additional damping will not reduce a structure's response significantly enough. That implies too that the technique will be effective under those ground motions that in the absence of damping would induce a large structural response and, hence, under the critical ground motions that govern the design of the structure.

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