

# Nonlinear earthquake response of R/C space frame with triaxial interaction

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**ABSTRACT :** This paper represents the investigation on nonlinear earthquake response of reinforced concrete space frame structure by taking into account the interaction among varying axial force and bi-directional bending moments. A simple multi-spring model was proposed used to simulate the interaction. The analysis was carried out to a 12-story reinforced concrete space frame to investigate the effect of the interaction on the response of frame structure under bi-directional lateral earthquake motion. The analysis result shows that the axial force-bending moment interaction affected significantly the member response, but did not appreciably affect the overall response of the frame structure developed a beam yielding mechanism.

## 1 INTRODUCTION

In a frame structure under real earthquake motion, the columns are usually subjected to not only bi-directional bending and shear but also varying axial load. The interaction among the axial load and bi-directional bending moments (N-M interaction) is known to be complex behavior, but it is little known about its effect on the earthquake response of the frame structure.

A model, called as multi-spring model has been proposed by Lai and Otani (1984) and developed by Li (1988,1989) to simulate the N-M interaction for reinforced concrete (R/C) columns. The original model requires the spring's parameters to be determined carefully from the member N-M interaction curve, and may not fit the interaction curve thoroughly, because of its simple constitution with 5-concrete spring and 4-steel spring. The original model is difficult for the practical use of analysis of a real frame structure to prepare the model parameters for a large number of members. Therefore, modification was made to the model to improve its performance and to simplify the calculation of model's parameters.

A general purpose computer program was written for the response analysis of R/C space frame structures. The modified multi-spring model was used for the column member to take into account the N-M interaction. The option of considering the interaction has been included in the program to investigate the effect of the interaction on the earthquake response of the structure.

## 2 MULTI-SPRING MODEL

The column member was idealized to be a linear element with its length equal to the column clear height and two multi-spring elements with zero-length at the base and the top ends (Fig.1). The multi-spring element consists of several uniaxial steel and concrete springs to represent the inelastic flexural rotation and the N-M interaction of the column. The plane sections

assumption was used for the multi-spring element. The model parameters were simply determined by the following method instead of using column's N-M interaction curve.

For rectangular section column with reinforcing bar evenly distributed on all faces, the division of steel and concrete was shown in Fig.2. The spring was simply put in the center point of every divided area, and the strength for the spring was calculated as the product of the area and the material strength, and the spring initial stiffness  $K_{si}$  from assumed plastic zone by the equation

$$K_{si} = \frac{E \cdot A_i}{\eta \cdot L_0} \quad (1)$$

where, E = the material young's modulus,  $A_i$  = the area of the spring, and  $\eta \cdot L_0$  = assumed plastic zone length.

The linear element with elastic flexural deformation may also represent inelastic axial and shear deformation by inelastic axial and shear spring. The bending flexibility,  $\delta_f$ , and axial flexibility,  $\delta_0$ , of the linear element were expressed as

$$\delta_f = \frac{\gamma \cdot L_0}{3EI}, \quad \delta_0 = \frac{\gamma_0 \cdot L_0}{EA}, \quad (2)$$

where,  $\gamma$ ,  $\gamma_0$  = the flexibility reduction factors used to balance the total flexibility of the column, and approximately calculated by the equations

$$\gamma = 1.0 - \eta/0.3, \quad (3)$$

$$\gamma_0 = 1.0 - 2\eta. \quad (4)$$

The force-deformation relation for both concrete and steel springs is assumed to be tri-linear curve as shown in Fig.3 to consider the facts of the inelastic behavior of concrete under larger compressive stress, of the cracking developed over a larger zone than assumed,  $\eta L_0$ , and of the rotation caused by bond slip of tension reinforcing bar along its embedded length in

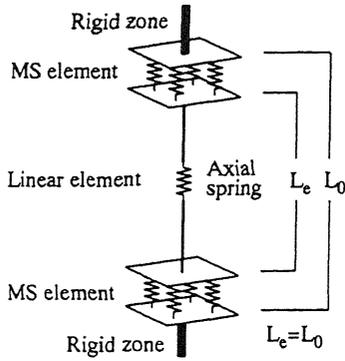
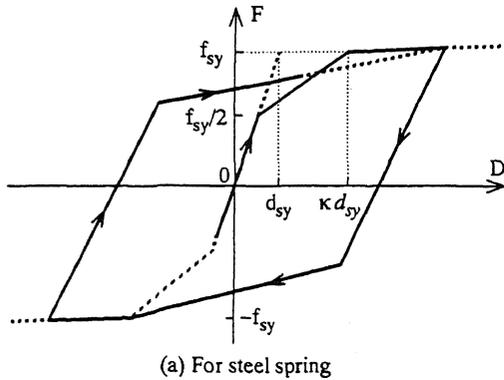
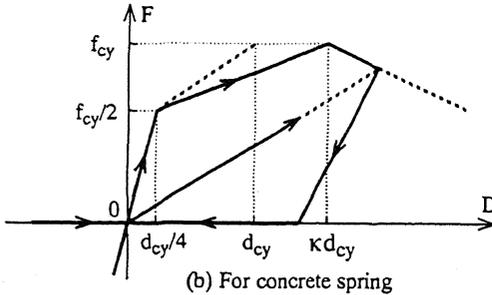


Fig.1 Multi-spring model for column member.



(a) For steel spring



(b) For concrete spring

Fig.3 Spring's force-deformation relations.

joint. The stiffness degradation factor before spring maximum strength was approximated by the equation

$$\kappa = 1.0 + \frac{h_v/D - 1}{h_v/D} \quad (5)$$

where,  $h_v/D$  = the shear span ratio ( $\geq 1.0$ ) of the column.

The performance of the modified multi-spring model has been improved as shown in Fig.4, in which the column N-M interaction curve for the comparison was calculated in diagonal direction ( $45^\circ$ ) by mathematical integration method assuming elasto-plastic stress-strain curve for steel and parabola for concrete. The model also simulated well the observed result of a high strength R/C column specimen under high varying axial load and bi-directional lateral load reversals (Kabeyasawa, 1991) as shown in Fig.5.

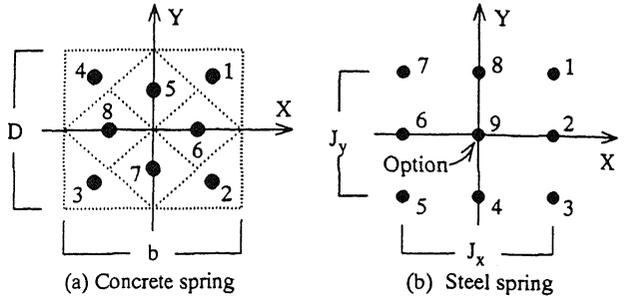


Fig.2 Division of rectangular symmetrical section.

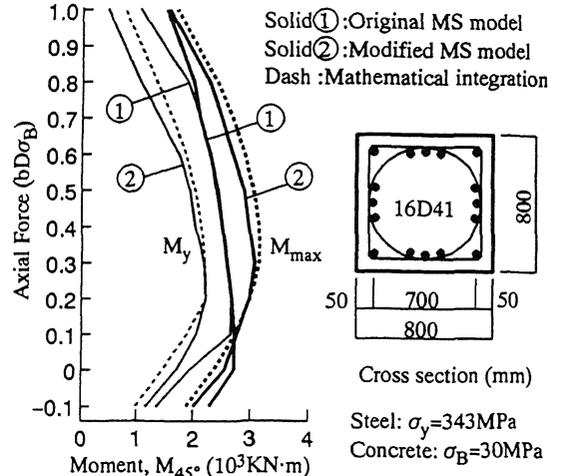


Fig.4 Comparison of N-M interaction curve.

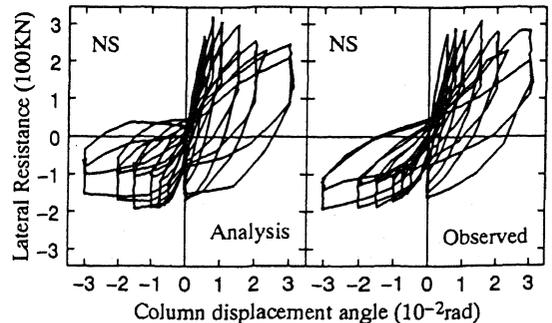
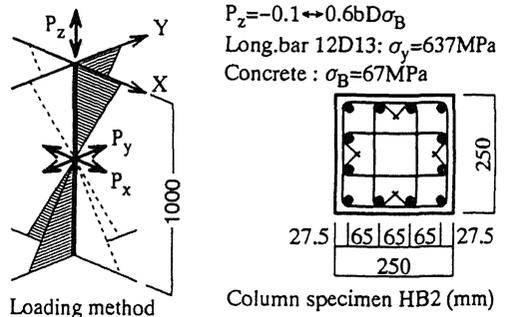


Fig.5 Comparison of calculated and observed results.

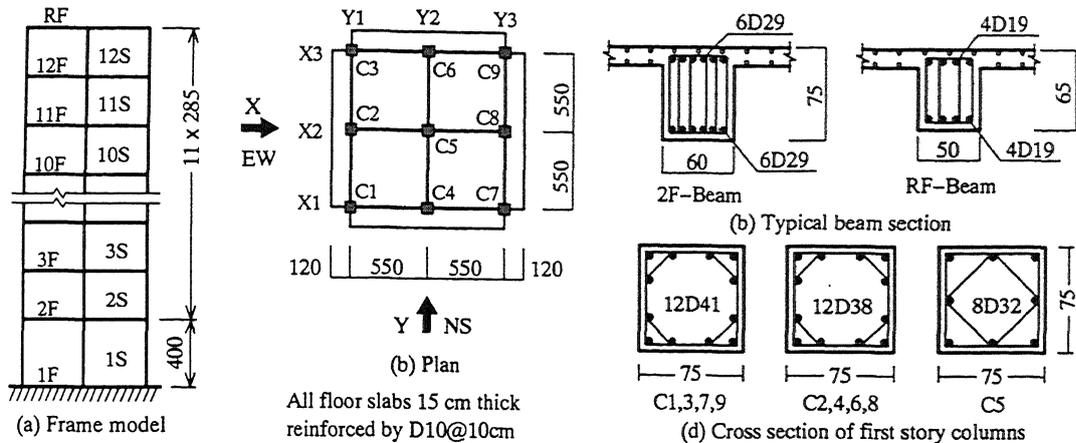


Fig.6 R/C space frame model (unit in cm).

### 3 FRAME MODEL AND ANALYSIS METHOD

#### 3.1 Frame model

A 12-story, 2x2-span R/C space frame (Fig.6) was used in the analysis to investigate the effect of the N-M interaction. The frame was designed symmetrically in either directions using concrete strength from 26.5 MPa at upper stories to 32.4 MPa at lower stories, and reinforcing bar yielding strength 343 MPa for beam and column longitudinal bar and 294 MPa for hoop and floor slab bar, respectively. The base shear coefficient to design beams was 0.24 against the weight per unit floor area of 12 KN/m<sup>2</sup> at roof level and of about 10 KN/m<sup>2</sup> at general floor level. The columns were reinforced against the actions at the beam yielding mechanism without magnification factors.

#### 3.2 Analysis assumptions

the general assumptions for the analysis were (1) rigid floor slab in its own plane to incorporate the structure's torsional oscillation, (2) rigid beam-column joint and consideration of some rigid zone at the member end, (3) three independent degrees of freedom for every joint, i.e. vertical translation and bi-directional rotations, as well as two lateral translations defined by the rigid floor movements, and no torsional rotation at joint due to rigid slab assumption, (4) the mass was concentrated at the center-of-gravity point of every floor level, and all the members were treated as mass-less linear elements, (5) the frame was fixed on rigid base at the base of the first story columns, and earthquake acceleration was input at the base level. (6) the initial gravity load was considered as the beam's initial moment and column's initial axial force, (7) elastic shear deformation was considered for all members.

The beam was assumed to bear only uni-axial flexure in the frame plane and idealized by one-component model (Giberson 1969). Takeda trilinear hysteresis model (1970) was used to simulate the beam flexural behavior.

The column was idealized by the multi-spring model and was treated as a substructure with two internal joints between the multi-spring element and the linear

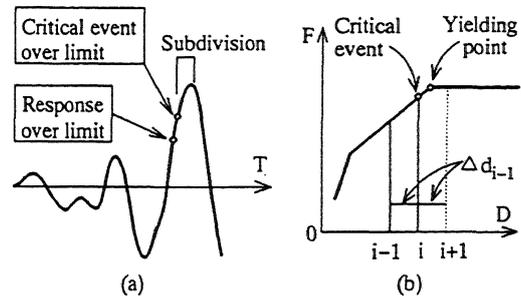


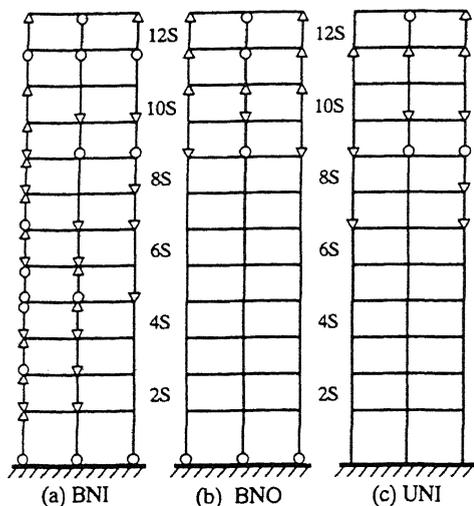
Fig.7 Subdivision of integration time interval.

element (Fig.1). The degrees of freedom of the column model was reduced to consider the column's contribution into overall structural matrix.

#### 3.3 Computer program and analysis method

A general purpose computer program, named CANNY (revision from former version RANMI, 1989), was written for static and dynamic analysis of frame structures and the structures including shear-walls and truss elements. A step by step incremental analysis method was used to integrate the equation of motion. The unbalanced forces caused by stiffness change was corrected in the next time step. To improve the computation precision, subdivision of the integration time interval was considered as response displacement and critical events over some limit as shown in Fig.7a. The critical event was that the yielding may take place in next time step in any spring estimated by present response increment (Fig.7b).

The varying axial load caused by the overturning moment was considered in the analysis. To investigate the effect of the N-M interaction on the earthquake response of structures, the analysis with only bi-directional bending interaction without N-M interaction was calculated for comparison. That is, imagining the frame was taken out from a ideal frame structure with infinite spans in either directions, then the varying axial loads in outside columns of the frame model were



(a) BNI (b) BNO (c) UNI  
 $\Delta \nabla$  Spring yielding,  $\circ$  column yielding,  
 Input : El Centro (NS,EW) $\times 1.8$ ,  
 BNI : Biaxial response with N-M interaction,  
 BNO : Biaxial response without N-M interaction,  
 UNI : Uniaxial response with N-M interaction.  
 Fig.8 Column flexural yielding of frame Y3.

removed by imposing on the columns a fictitious axial load contributed by the imaginary boundary beams.

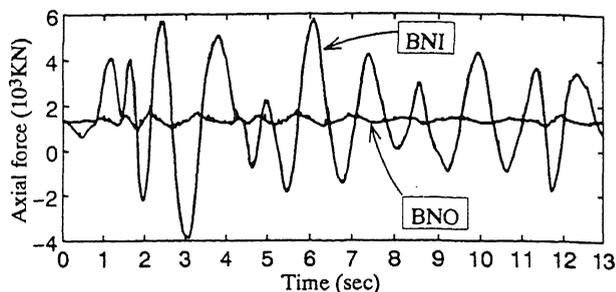
#### 4 COMPUTATION AND ANALYSIS RESULT

The response analysis was computed for 13 seconds of the 1940 El Centro records (NS and EW) amplified by a factor of 1.8, and the 1968 Hachinohe records (NS and EW) by factor of 1.7, inputting in one or two horizontal directions. Newmark's numerical method with Beta value equal 1/4 was used for the response computation at a normal time interval of 1/100 second and subdivision time interval of 1/1000 second. Instantaneous stiffness proportional damping force was considered with an initial damping factor of 0.02.

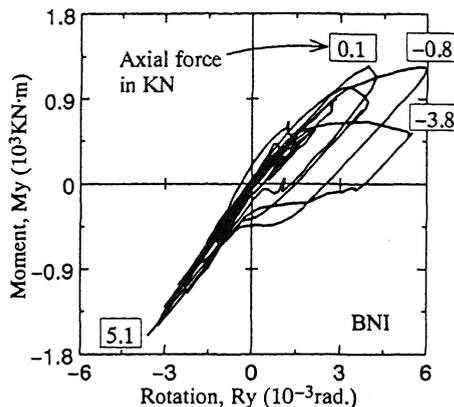
##### 4.1 Effect of N-M interaction on member response

For the response considered the N-M interaction, flexural yielding in columns took place in some middle story columns as well as the first story column base (Fig.8). The N-M interaction caused the column an unsymmetrical bending moment-rotation hysteresis behavior (Fig.9b), i.e. the flexural strength and stiffness decay and yielding occurred as the column under tension varying axial force, but regained elasticity as under compression varying axial force. By considering the multi-spring column model as a substructure and by the method of reduction of degrees of freedom, the unbalanced axial forces of the column could be corrected successfully and the axial forces in the linear element and two multi-spring element of the column model were exactly same as shown in Fig.9a.

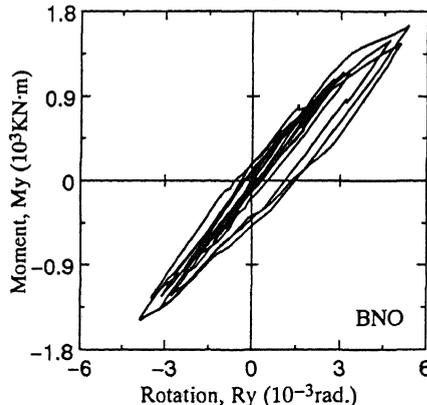
Flexural yielding occurred in all beams except some at the roof level for both responses with and without N-M interaction. The maximum ductility demands of



(a) Axial forces in 3-element of the first story column, C7



(b) Biaxial response with N-M interaction



(c) Biaxial response without N-M interaction

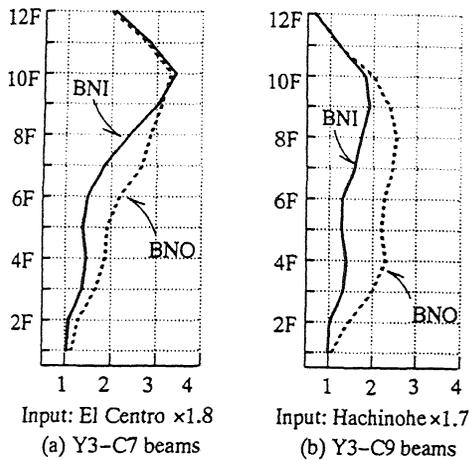
Fig.9 Axial force and base moment-rotation relation of first story column, C7 (input: El Centro NS,EW  $\times 1.8$ ).

beams were reduced as show in Fig.10 especially at lower-middle stories because the N-M interaction caused yielding damage on those columns.

Comparing the results indicated by the response with and without the N-M interaction as shown in Figs. 8-10, it is clearly that the interaction affect significantly the member response.

##### 4.2 Torsional response

In spite of symmetrical frame configuration in both X and Y directions (no initial structural eccentricity),



BNI: Biaxial response with N-M interaction  
 BNO: Biaxial response without N-M interaction  
 Fig.10 Maximum ductility factor of beams.

some torsional oscillation was induced (Fig.11) during the earthquake response with the N-M interaction. This can be attributed to the interaction, which caused the stiffness and strength decay on one-side exterior and corner columns under tension varying axial force. The additional lateral displacement by the torsional response at the external frame was less than 5% in total response in the analysis frame model. For a multi-span real frame structure, such torsional response would be decreased and have little effect.

#### 4.3 Frame's overall response

The absolute maximum value (took place in any direction) of inter-story displacement angle response were 1.3% to story height under El Centro motion, and 1.1% under Hachinohe motion. The response without N-M interaction caused slight increment by maximum of 5% in absolute maximum value of story shear (Fig.12), and little change in floor lateral displacement as shown in Fig.13 for roof level compared with the response took into account the N-M interaction.

For the frame model, the beam yielding mechanism could be formed and the column did not yield during the column subjected to a compression varying axial load. In the first story, the column shear forces decreased in the side of columns under tension varying axial force while they increased in the other side under compression varying axial force. In upper stories, however, the column shear must be in balance with the moment transferred by the beams conjoined to the column, and be within the limits of the beam yielding strength. Therefore, the N-M interaction did little effect on the story shear response.

#### 4.4 Comparison of biaxial and uniaxial responses

The uniaxial response to one component of the biaxial input motion was calculated and compared with the biaxial response.

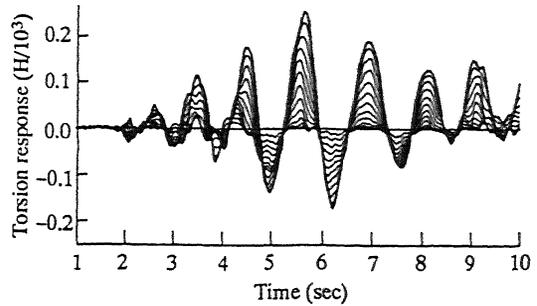
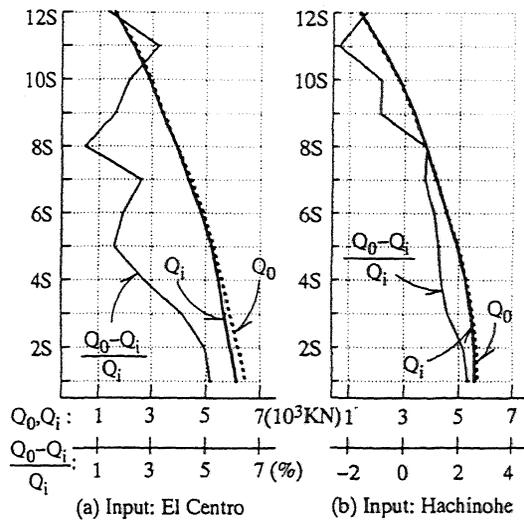
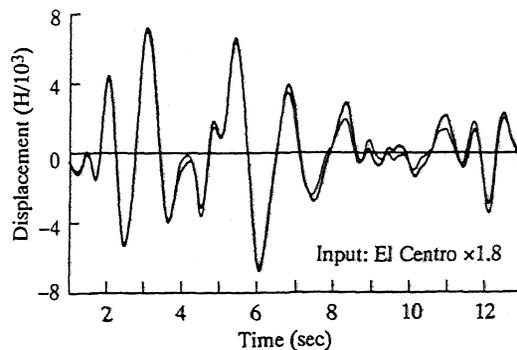


Fig.11 Additional displacement in external frame by torsional response (input: El Centro NS,EW  $\times 1.8$ ).



$Q_i$ : Biaxial response with N-M interaction  
 $Q_0$ : Biaxial response without N-M interaction  
 Fig.12 Maximum story shear response.



Bold: Response with N-M interaction,  
 Thin: without N-M interaction,  
 Fig.13 Lateral displacements at roof level in Y-direction.

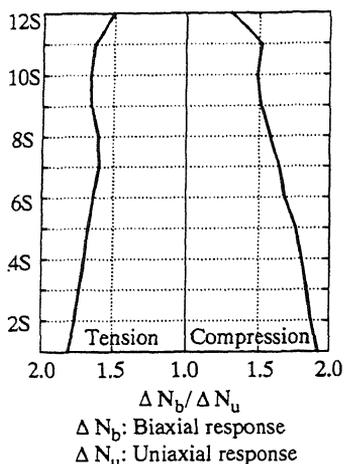


Fig.14 Maximum varying axial force in corner column.

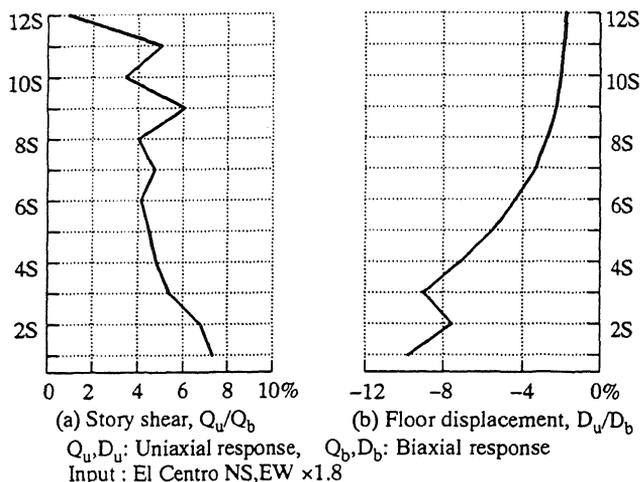


Fig.15 Comparison of uniaxial and biaxial overall responses.

In uniaxial response, the exterior and corner columns were subjected to a varying axial force caused by only one-directional overturning moment on the structure as well as subjected to uniaxial bending. Therefore, the columns almost remained elastic (Fig.8c). The maximum varying axial force in the corner column by biaxial response ranging from 1.5 to 1.8 to the uniaxial response (Fig.14). In other words, the beams conjoined to a row of columns from the base to the top story did not yield simultaneously in biaxial response. The maximum ductility factor of beams indicated by biaxial response was decreased as compared with that by uniaxial response, because more damage on columns took place during the biaxial response.

The maximum values of floor lateral displacement and story shear represented by the root of the square sum of the uniaxial responses in X and Y directions led in smaller displacement and larger base shear than biaxial responses (Fig.15). The maximum differences between them were less than 10%.

## 5 CONCLUSIONS

The interaction among varying axial force and bi-directional bending moments affects significantly the response of individual members, and causes an extra stiffness and strength eccentricity on structures and induces some torsional response. Therefore, the interaction should be considered to estimate the member response and to determine the yielding mechanism of frame structures. The interaction, however, does not affect much on the overall response of the structure dominated in beam yielding mechanism.

By uniaxial response can approximately estimate the overall response of the moment-resisting frame within acceptable error as compared with the biaxial response, but may make notable difference in indicating the member response and yielding mechanism.

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