

Design requirements for ductile RC frame structures

Y. Sakai

Earthquake Research Institute, University of Tokyo, Japan

H. Aoyama

Department of Architecture, University of Tokyo, Japan

ABSTRACT: An earthquake design spectrum and a deformation index to control deformation of ductile reinforced concrete frame structures designed by a simple method using a linear analysis are proposed. A dynamic magnification factor for column moment and shear force is discussed to ensure a beam yield collapse mechanism. Dynamic non-linear response analyses of reinforced concrete moment-resisting-frame structures and of corresponding single-degree-of-freedom systems are performed.

1 INTRODUCTION

In a design of a reinforced concrete moment-resisting-frame structure, it is desirable to use a "strong column-weak beam" system. In addition, it is necessary to ensure the strength and stiffness of structures to control deformation of structures. In this paper, the following design criteria were assumed for structures during an extraordinary intensity earthquake;

(1) A beam yield collapse mechanism should be ensured with yielding of the ends of all floor beams and at the base of first story columns.

(2) Response ductility factors at the ends of beams and at the bottom of first story columns should be less than 4 and 2, respectively.

(3) Story drift angles should be less than 0.01 rad.

Non-linear static and dynamic response analyses of mid- and high-rise frame structures designed by a simple method using linear analyses and of corresponding single-degree-of-freedom systems were carried out to investigate following three design requirements to meet the above three design criteria;

(1) design earthquake spectrum to control member ductility,

(2) a deformation index to control overall deformation of frame structures and

(3) dynamic magnification factor for column moment and shear force for the beam yield collapse mechanism.

2 FRAME STRUCTURES

Regular and symmetrical frame structures were analyzed. The outline of five prototype frame structures is shown in Table 1. The first character of the frame identification stands for the usage (Office or Apartment), and the next two digits indicate the number of stories. The member section dimensions and the concrete strength are shown in Table 2. High strength concrete was used at lower stories of the high-rise frame structures.

The stiffness of frame structures was varied by multiplying the member section dimensions by 0.8, 0.9, 1.0 (=prototype structures), 1.1, 1.2 to study a deformation index to control the story drift. The number of frame structures was 25 (5 story types and 5 cases of stiffness).

3 DESIGN OF FRAME STRUCTURES

Frame structures were designed under vertical and seismic lateral load. Design moments of members under lateral load were calculated by a linear analysis using reduced stiffness for beams and under the lateral force of a triangular distribution. A stiffness reduction factor, a ratio of the yielding to the initial elastic stiffness, was 0.3 for beams and 1.0 (not reduced) for columns. The design base shear coefficient was given

Table 1. Outline of prototype structures

ID	Usage	No. of Stories	Span (m)	Story Height (m)	Elastic Period (sec)
O-06	Office	6	8.0	3.7	0.54
A-08	Apartment	8	8.0	2.8	0.51
O-12	Office	12	8.0	3.7	0.78
A-16	Apartment	16	5.0	2.8	0.75
A-24	Apartment	24	5.0	2.8	1.06

Table 2. Member section dimensions and Concrete Strength

Column Depth (cm)	Beam Depth (cm)	Beam Width (cm)	Concrete Strength (MPa)
80~90	80~90	55~70	24
70~80	70~80	35~45	24
90~110	85~95	55~85	24~30
70~80	70~80	50~60	24~30
70~90	70~80	50~60	24~36

variably for each structures, because the design earthquake spectrum was not determined at this stage.

Design moments of columns, except at the bottom of first story columns, were obtained by amplifying the moments from the linear analysis by the ratio of an overstrength to the strength of a beam plastic hinge (denoted as "static magnification factor ϕ_s ") to ensure the beam-yielding collapse mechanism. The overstrength of a plastic hinge was calculated supposing that it was caused by the spread of the effective slab width with deformation. The strength at the ends of

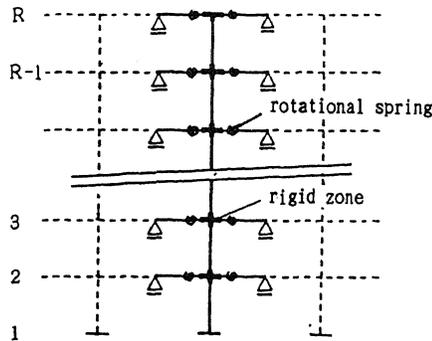
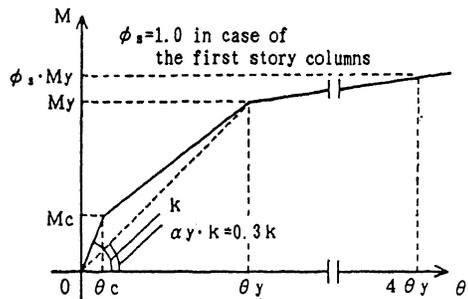
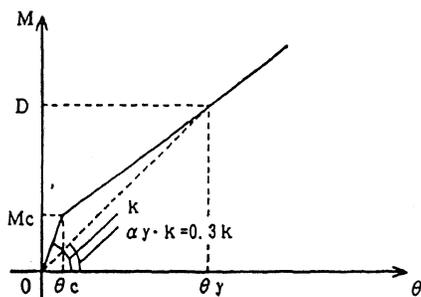


Figure 1. Analytical model



(1) Beams and the bottom of the first story column



(2) Columns except the bottom of the first story column

M_y : yield moment, θ_y : yield rotation
 M_c : crack moment, θ_c : crack rotation
 k : initial elastic stiffness
 D : amplified design moment

Figure 2. Skeleton curves of the hysteresis model

beams and at the bottom of first story columns was made equal to the design moments.

The strength of members was finally determined for the conditions that the amount of column reinforcement above and below a floor level was the same and that the minimum reinforcement requirement in AIJ Standard for Structural Calculation of Reinforced Concrete Structures was satisfied (the longitudinal reinforcement ratio of beams and the gross longitudinal reinforcement ratio of columns have to be larger than 0.4% and 0.8%, respectively).

4 STRUCTURAL IDEALIZATION FOR NONLINEAR ANALYSES

An analytical model, consisting of a continuous column with its both side beams, was removed from an original structure by cutting beams at their inflection points and by supporting beam ends by horizontal rollers (Fig.1). The members are idealized by an elastic element with two non-linear rotational springs (one-component model) with rigid zones at the ends.

The Takeda hysteresis model (Takeda et. al. 1970) is used for the rotational springs. Skeleton curves are shown in Fig.2. The cracking moment M_c is evaluated assuming the tensile strength of concrete to be $1.8\sqrt{F_c}$, where F_c : concrete strength. The ratio of the yielding to the initial stiffness of members is assumed to be 0.3 for both beams and columns. The ultimate resistance at a ductility factor of 4 was the yield strength multiplied by the static magnification factor ϕ_s . Since the columns, except at the bottom of the first story, were not assumed to yield, the stiffness after cracking was assumed to be constant (Fig.2(2)).

The mass of each story was assumed to concentrate at the floor level. The structure was assumed to be fixed at the base.

Table 3. Method to reduce multi-degree-of-freedom system

$[m] \{\ddot{x}\} + \{R(x)\} = -[m] \{1\} \ddot{x}_0$	(1)
$[m]$: mass matrix	
$\{x\}$: relative displacement vector	
$\{R(x)\}$: restoring force vector	
x_0 : ground motion	
$\{x\} = [u] \{q\}$	(2)
$[u]$: mode matrix	
$\{q\}$: function of time history	
$[m] [u] \{\ddot{q}\} + \{R(x)\} = -[m] \{1\} \ddot{x}_0$	(3)
$\{i, u\}^T [m] \{i, u\} = 0 \quad (i \neq j)$	(4)
$\{i, u\}^T [m] \{i, u\} \ddot{q} + \{i, u\}^T \{R(x)\} = -\{i, u\}^T [m] \{i, u\} \beta \ddot{x}_0$	(5)
β : participation factor	
$\{i, u\}^T [m] \{i, u\} \ddot{q} + \{i, u\}^T \{R(x)\} = -\{i, u\}^T [m] \{i, u\} \beta \ddot{x}_0$	(6)
$\overline{m} \ddot{x} + \overline{p}(x) = -\overline{m} \beta \ddot{x}_0$	(7)
$\overline{x} = \{i, u\}^T [m] \{i, u\} \ddot{q}$ (= top displacement)	
$\overline{m} = \{i, u\}^T [m] \{i, u\}$: equivalent mass	
$\overline{p} = \{i, u\}^T \{R(x)\}$: equivalent restoring force	

5 DESIGN EARTHQUAKE SPECTRUM

5.1 Single-degree-of-freedom system

A single-degree-of-freedom system (denoted as "SDF" system) was used to compute a design earthquake spectrum, because vast calculation time was required when multi-degree-of-freedom system was used.

It is expected that a frame designed by the strong column-weak beam concept oscillates dominantly in the fundamental mode, especially after the formation of the beam-yielding mechanism. Therefore, an equivalent SDF system can be determined by reducing a multi-degree-of-freedom system (Table 3).

Equivalent displacement vs. equivalent restoring force skeleton curves were obtained from results of static analyses of frame structures. These curves were replaced by tri-linear shapes for non-linear response analyses. Results of the static analyses (solid line) and idealized tri-linear skeleton curves (dashed lines) of prototype frame structure O-12 are shown in Fig.3. The equivalent restoring force means overturning moment divided by an overall height of the structure and the equivalent displacement means the top displacement when the lateral force distribution is an inverted triangular shape.

The allowable ductility factor μ_A of a SDF system corresponding to the design criteria of structures was evaluated based on the replaced tri-linear curves. The ductility factor at the ends of beams became 4 (symbol O in Fig.3), before the ductility factor at the bottom of first story column became 2 (symbol X in Fig.3). This was similar in the case of any other frame structures. Thus, the allowable ductility factor μ_A of SDF system was determined corresponding to the point at which the ductility factor at beam ends became 4. The allowable ductility factor μ_A was found to be approximately constant in any structure, thus μ_A was determined to be constant (=2.8).

5.2 Design earthquake spectrum

Non-linear response analyses of SDF systems were carried out to evaluate a design earthquake spectrum. Damping was assumed to be proportional to the instantaneous stiffness. The input earthquake motions are three records normalized for a maximum velocity of 50 cm/sec (Table 4). Spectral characteristics of the motions are different from each other (Fig.4).

The base shear coefficient spectra which require that response ductility μ of a SDF system should be less than the allowable ductility factor μ_A (=2.8) is shown in Fig.5. The design earthquake spectrum is determined to envelope approximately the three spectra:

$$C_y = 0.18/T^{1.6} \quad (1)$$

if $C_y > 0.6$, $C_y = 0.6$, where C_y : design base shear coefficient, T: fundamental elastic period (sec). Eq.(1) is shown in Fig.5 in a bold line.

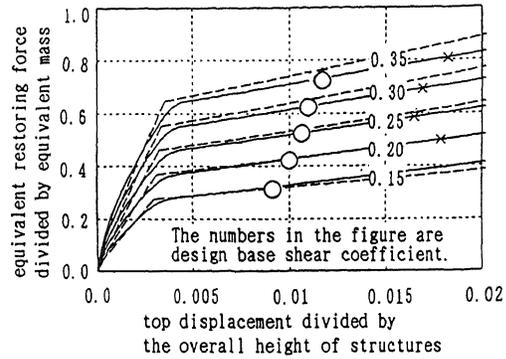


Figure 3. Results of static non-linear analyses and replaced tri-linear curves

Table 4. Input motions

ID	Name of Record and Earthquake	Maximum Accel.
E	El-Centro NS Imperial Valley 1940	511 cm/s ²
H	Hachinohe Kowan EW Tokachi-Oki 1968	256 cm/s ²
T	Tohoku Univ. NS Miyagiken-Oki 1978	357 cm/s ²

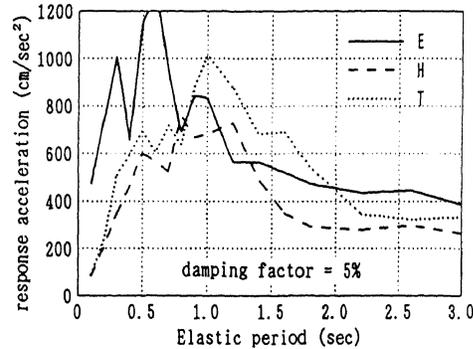


Figure 4. Elastic response acceleration of input motions

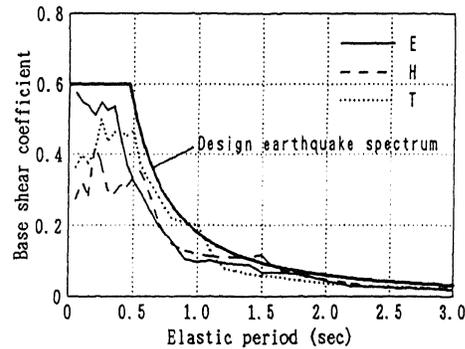


Figure 5. Required base shear coefficient spectra and design earthquake spectrum

6 DEFORMATION INDEX TO CONTROL STORY DRIFT OF FRAME STRUCTURES

As an index for a strength of structures, base shear coefficient is generally used (also used in this study in section 5). But, there is no adequate index to express the stiffness of structures to control story drifts of structures. The index is investigated in this section.

Consider a skeleton curve of restoring force vs. deformation of a SDF system shown in Fig.6 to study the index to control story drifts (denoted as "deformation index Di"). The response drift angle of the SDF system βR is given in Eq.(2).

$$\beta R = \beta d_m / H = \beta \epsilon \mu d_y / H \quad (2)$$

where β : participation factor, H : the height to the roof level of the structure, $\epsilon \mu$: response ductility of the SDF system. The yield deformation of the SDF system d_y is given in Eq.(3).

$$d_y = Q_y / (\alpha k) = C_y m g / \{ \alpha m (2\pi/T)^2 \} = g C_y T^2 / (4\pi^2 \alpha) \quad (3)$$

where Q_y : yield base shear force, α : a ratio of yielding to initial elastic stiffness, k : initial elastic stiffness, m : mass of structure, C_y : yield base shear coefficient, T : fundamental elastic period (sec), g : gravity acceleration. Eq.(4) is obtained by substituting Eq.(3) into Eq.(2).

$$\beta R = \{ \beta g / (4\pi^2 \alpha) \} (T^2 / H) C_y \epsilon \mu \quad (4)$$

Eq.(4) means the relation between the response drift angle βR and the response ductility factor $\epsilon \mu$ of the SDF system.

Assuming the mode of an inverted triangular shape, β is slightly varied from 1.36 to 1.48 when the story of structure is varied from 5 to 30, and assuming α is approximately constant in the case of reinforced concrete structures, $\beta g / (4\pi^2 \alpha)$ on the right side of Eq.(4) is constant. Therefore, the response drift angle βR on the left side is proportional to $T C_y \epsilon \mu / H$ on the right side. Thus, the deformation index Di to control drift angles is defined as:

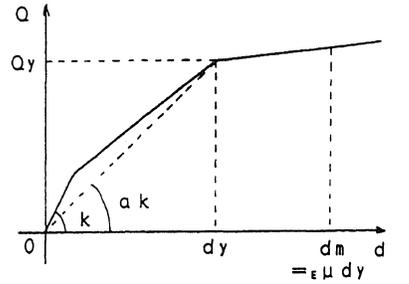
$$Di = T^2 C_y \epsilon \mu / H \quad (5)$$

T^2 / H is determined only from member dimensions of structures. On the contrast, $C_y \epsilon \mu$ is determined for an intensity of earthquake and for the designed structure. Assuming the property of constant displacement, from the earthquake spectrum to make $\epsilon \mu$ constant (=2.8), the deformation index Di is expressed as the smaller one of Eq.(6) and Eq.(7) by substituting Eq.(1) into Eq.(5) and 2.8 into $\epsilon \mu$.

$$Di = 0.54 T^{0.4} / H \quad (6)$$

$$Di = 1.8 T^2 / H \quad (7)$$

Next, the relation between the proposed deformation index Di and response drift angles is investigated. Assuming $\beta=1.4$, $\alpha=0.3$ and response story drift angles are constant in any floor level in the case of regular and symmetrical structures, the maximum response story drift angle R is given in Eq.(8) from Eq.(4) and Eq.(5).



Q_y : yield strength, d_y : yield deformation
 d_m : response displacement
 k : initial elastic stiffness
 α : the ratio of the yield to the initial elastic stiffness

Figure 6. Skeleton curve of the reduced SDF system

$$R = 1.16 Di \quad (8)$$

The member dimensions can be determined so that the deformation index Di is below 0.01 in order to make story drift angles within 0.01rad.

7 RESPONSE ANALYSES OF FRAME STRUCTURES

Dynamic response analyses of frame structures were performed in order to

- (1) confirm the adequacy of the proposed design spectrum,
- (2) confirm the adequacy of the proposed deformation index and
- (3) investigate dynamic magnification factors.

Frame structures were redesigned according to the proposed design earthquake spectrum. Damping was assumed to be proportional to the instantaneous stiffness. Input motions in Table 4 were used.

7.1 Design earthquake spectrum

Response ductility factors at the ends of beams in the case of prototype structures are shown in Fig.7. The maximum ductility factor is generally constant for all structures and approximately satisfies the design criteria (The response ductility factor at the ends of beams should be less than 4.). This was similar in the case of any other structure. Thus, the design earthquake spectrum calculated according to the fact that the first mode is predominant in structures at the beam-yielding collapse mechanism was found to be adequate.

7.2 Deformation index

Story drift angles in the case of frame O-12 are shown in Fig.8. Story drift angles are larger for larger deformation index Di . This was similar in the case of other frame structures.

The relation of maximum story drift angles ("maximum" means maximum in whole stories) vs. deformation index Di is shown in Fig.9 in the case of all structures. Close correlation is found between maximum

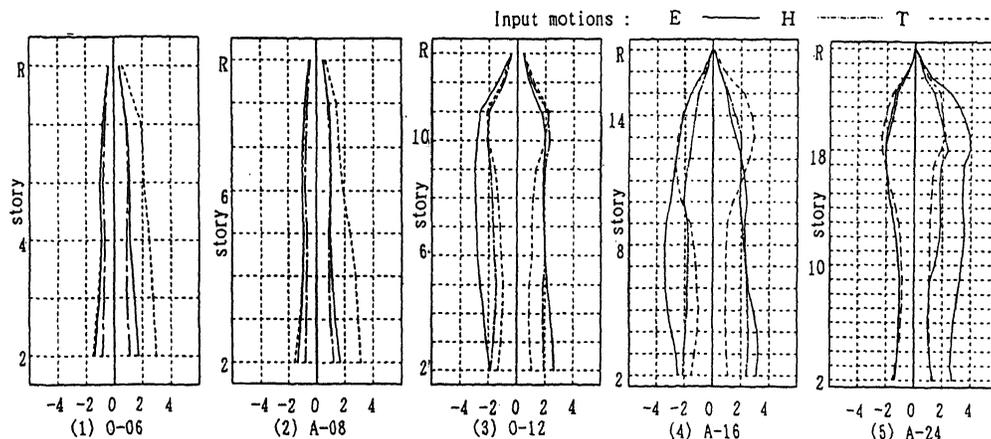


Figure 7. Beam response ductility factors (prototype structures)

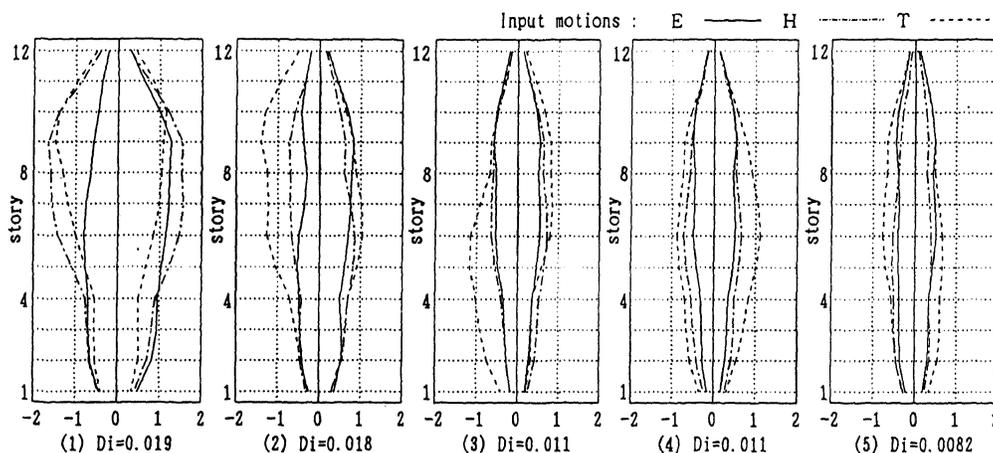


Figure 8. Response story drift angles (frame O-12)

story drift angles and the deformation index D_i . Thus, the deformation index D_i was found to be adequate to control story drift angles.

The relation given in Eq. (8) is shown in Fig. 9 in solid line. This line adequately gives the upper value of the response drift angles, because the design earthquake spectrum was safely evaluated to envelope three required spectra (Fig. 5).

7.3 Dynamic magnification factor

A dynamic magnification factor for column moments and shear forces was investigated. The dynamic magnification factor ϕ_D was defined as Eq. (9).

$$\phi_D = R/D \quad (9)$$

where R : dynamic response, D : design moment or shear force amplified by the ratio of the overstrength to the strength of beam plastic hinges (equal to $\phi_s E$), E : moment by the result of a linear analysis.

In the case of moment, D is a moment determined for the conditions that the amount of column reinforcement above and below a floor level is the same and that the minimum reinforcement requirement in AIJ Standard for Structural Calculation of Reinforced

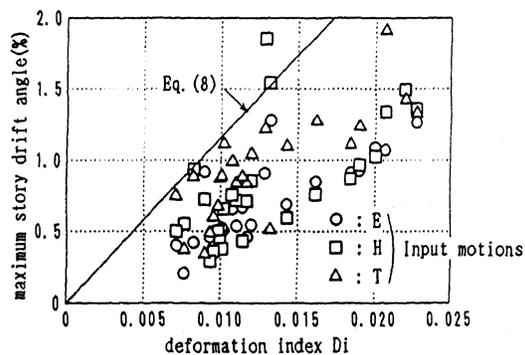


Figure 9. Relation between maximum story drift angles and deformation index D_i

Concrete Structures (previously described) should be satisfied.

Dynamic magnification factors of column shear force in the case of prototype structures are shown in Fig. 10. These are rather small and slightly larger than 1.0 in upper and lower stories. This was similar in the case of other frame structures.

Dynamic magnification factors of column moment in the case of prototype structures are shown in Fig. 11.

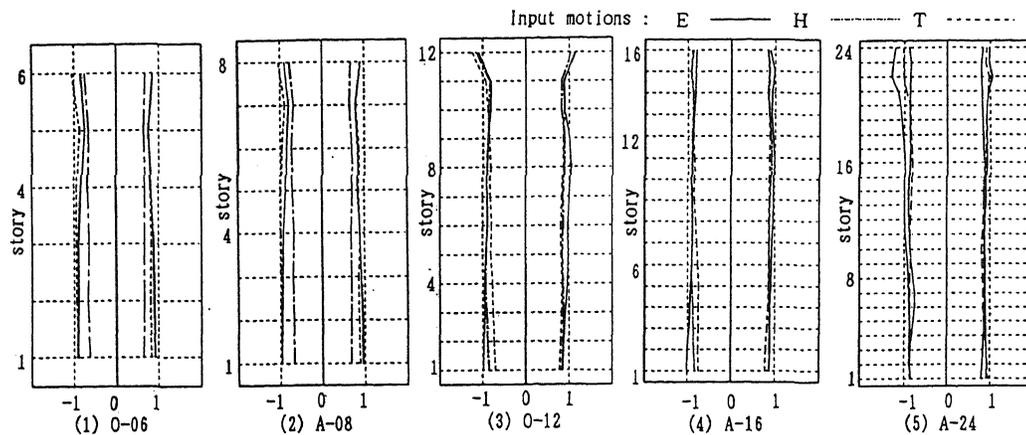


Figure 10. Dynamic magnification factors of column shear force (prototype structures)

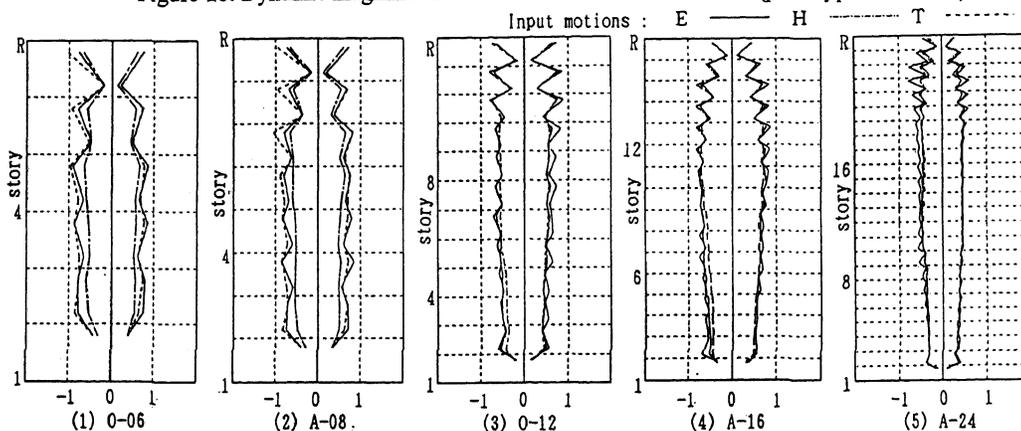


Figure 11. Dynamic magnification factors of column moment (prototype structure)

These are also rather small and constant in any floor level. This was similar in the case of other frame structures.

The reason why dynamic magnification factors were rather small is that structures had enough strength to satisfy the design criteria and column moments were amplified by the ratio of the overstrength to the strength of beam plastic hinges. Thus, the design dynamic magnification factor of column design action can be 1.5 in upper stories and 1.2 in lower stories for shear force and 1.2(constant) for moment.

8 CONCLUSIONS

Non-linear static and dynamic response analyses of mid- and high-rise frame structures designed by a simple method using a linear analysis and corresponding single-degree-of-freedom (SDF) systems were carried out. The following conclusions were obtained;

(1) A design earthquake spectrum was proposed by non-linear response analyses of SDF systems. Dynamic response of frame structures during the strong motions was adequately evaluated by dynamic response analyses of SDF systems due to that the first mode in the response of frame structures at "strong column-weak beam" system was predominant.

(2) A deformation index to control story drifts of structures was proposed and a value of a deformation index to satisfy the design criteria was calculated.

(3) Dynamic magnification factors for column action were small, when structures had enough strength to satisfy the design criteria and column moment was amplified by the ratio of an overstrength to the strength of beam plastic hinges.

REFERENCES

- Aoyama, H., Otani, S., Kubo, T. and Kabeyasawa, T. 1987. Earthquake resistant design of ductile reinforced concrete buildings (in Japanese) : *Proceedings of the Japan Concrete Institute Vol.9 No.2* : 447-452.
- Architectural Institute of Japan 1988. *All Standard for structural calculation of reinforced concrete structures* (in Japanese).
- Shiohara, H., Otani, S. and Aoyama, H. 1981. Inelastic Response Analysis of Building with Reduced Model (in Japanese) : *Journal of Structural Engineering, Vol.28* : 101-112.
- Takeda, T., Sozen, M.A., and Nielsen N.N. 1970. Reinforced concrete response to simulated earthquakes *Journal, Structural Division, ASCE, Vol.96, No.ST5*: 2557-2573.