

Analytical model of beam-column connections between steel beams and reinforced concrete columns

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ABSTRACT: An three-dimensional analytical model for predicting shear force-deformation relationship of beam-column joints of the frame structure composed of steel beams and reinforced concrete columns. Joint are modeled by four types of panel elements, inner element, outere elements, side elements and top/bottom elements. Each element consists of steel web panel or cover plate, as well as concrete panel. Supposing the effective cross-section and shear stress-deformation curves of elements, the shear force-deformation relationship of joint can be determined by considering the equilibrium conditions and compatibility condition of deformation. Results of analysis showed good agreement with test results. An equation for calculating the ultimate shear capacity of the joint is also proposed.

1 INTRODUCTION

Reinforced concrete columns provide us with fairly good rigidity and axial capacity. On the other hand, steel beams show a very good performance in terms of strength and ductility against bending and shear loads. These positive features make it reasonable to construct buildings with the reinforced concrete columns and steel beams. Authors have presented a new type of moment-resisting frame composed of steel beams and reinforced concrete columns in the previous paper(1988). The details of the beam-column joint are illustrated in Fig.1. The steel beams run through the reinforced concrete column, and the concrete in the joint is surrounded and tightly confined by thin steel plates named cover plates.

Buildings must be designed so that deformation in the horizontal direction is not so

much when an earthquake force acts. To calculate the deformation of buildings, deformation of beam-column joints must be calculated. However, a method for calculating deformation of steel beam-reinforced column joint has not been established yet. In this paper, an analytical model is presented which can predict the shear force-deformation relationship of the steel beam-reinforced concrete column joint. An equation for calculating the ultimate shear capacity of the joint is also proposed.

2 ANALYTICAL MODEL

In the model, the beam-column joint can be separated into four kinds of panel elements, as illustrated in Fig.2, inner element, outer elements, side elements and top/bottom elements.

Inner element consists of steel web panel which is the web of steel beam within the joint, and inner concrete panel encased by beam flanges. Outer element consists of outer concrete panel outside the flange width, and front cover plate which is parallel to steel beam. Side element consists of side cover plate attached normally to the steel beam, and a concrete layer adhered to the side cover plate. The top/bottom element is composed of top or bottom concrete panel, and flange of a transverse steel beam attached normally to the primary beam.

While models for dividing concrete into the outside and inside of flange width are proposed already by Deierlein (1989), a feature of this paper exists in considering new elements for connecting them, i.e., side element

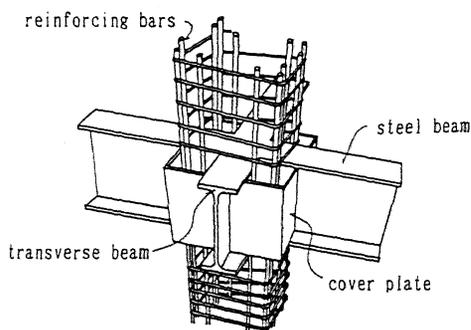


Fig.1 Details of beam-column joint

and top/bottom element. This is similar to explaining the torsional behavior of reinforced concrete members using a compression field in a concrete surface layer (see Mitchell (1974)).

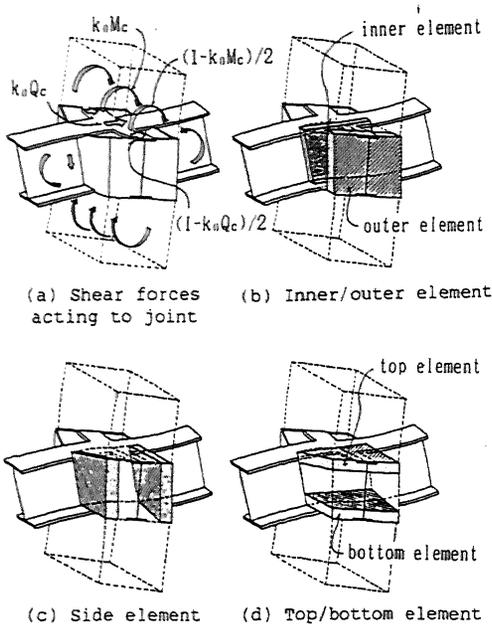


Fig. 2 Elements of beam-column joint

3 SHEAR FORCE AND DEFORMATION OF ELEMENTS

Shear force and deformation shared by the elements are shown in Fig. 3. Coefficients k_1 through k_6 represent effective cross sections (effective width or depth) of the elements.

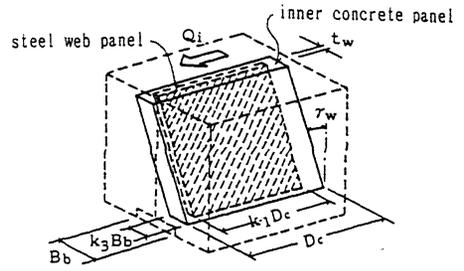
3.1 Total shear force acting on the joint

A typical distribution of moment in the frame subjected to lateral loads is shown in Fig. 4. Here, in the joint, a zone with longitudinal length of vh and lateral length of ul is considered. Where, v and u are coefficients, and h is column length, and l is beam length. vh is defined as the distance between the centroids of compressive stress and tensile stress at beam cross section. ul is assumed to be the distance between the reinforcing bars of column.

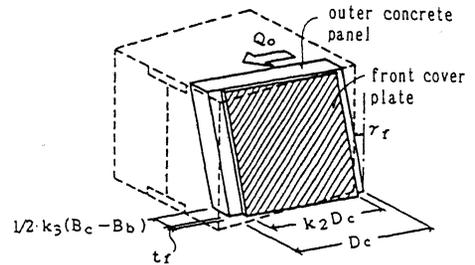
Total shear force acting on the joint is defined by the following equation:

$$Q_p = 2M_b / vh - Q_c \quad (1)'$$

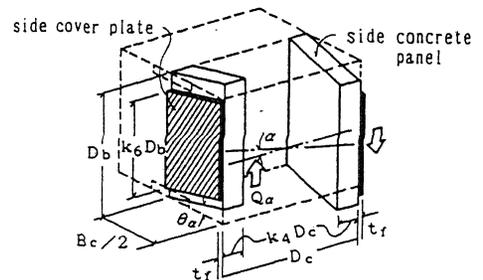
Where M_b is an average of moments of the right and left beams, and Q_c is the shear force of the columns. Eq.(1) can be rewritten to the following equations based on the equilibrium of moment in the frame.



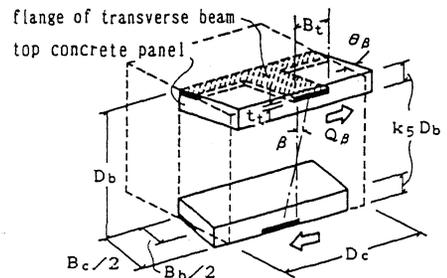
(a) Inner element



(b) Outer element



(c) Side element



(d) Top/bottom element

Fig. 3 Shear force and deformation of element

$$Q_p = [(1-u-v)/v] Q_c$$

$$Q_p = [2(1-u-v)/(1-v)vh] M_b$$

$$Q_p = [(1-u-v)l/vh] Q_b$$

$$Q_p = [2(1-u-v)/(1-u)vh] M_b$$

(2)

where Q_b is an average of shear forces of right and left beams, M_c and Q_c are moment and shear force of columns respectively.

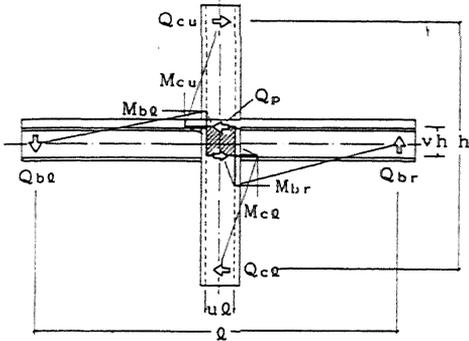


Fig. 4 Total shear force in joint

3.2 Relationship between total shear force in joint and shear forces in elements

As shown in Fig. 2(a), moment and shear force transferred from columns to inner element are defined as $k_0 M_c$ and $k_0 Q_c$, respectively. Consequently, moment and shear force transferred to outer elements are $(1-k_0)M_c/2$, and $(1-k_0)Q_c/2$, respectively. k_0 is assumed to be a ratio of flange width (B_b) and column width (B_c), i.e., $k_0 = B_b/B_c$.

Taking into consideration the balanced state of forces as shown in Fig. 5, and from equilibrium condition on the line ① through the line ④, the following equations are obtained.

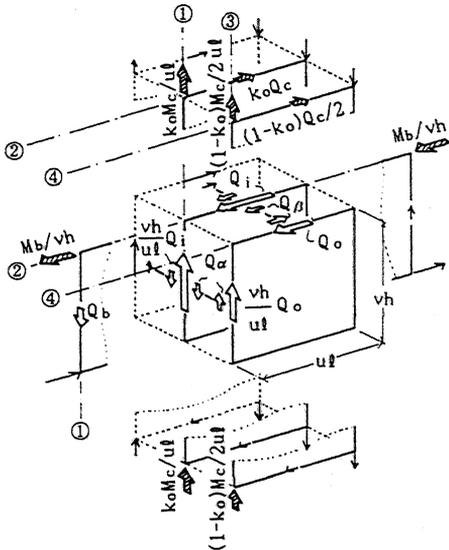


Fig. 5 Forces acting on elements

$$2k_0 M_c / (u l) - Q_b = (v h / u l) Q_i - 2 Q_\alpha$$

$$2 M_b / (v h) - k_0 Q_c = Q_i + 2 Q_\beta \quad (3)$$

$$(1 - k_0) M_c / (u l) = (v h / u l) Q_o + Q_\alpha$$

$$(1 - k_0) Q_c / 2 = -Q_o + Q_\beta$$

Eliminating Q_β from the second and fourth equations of Eqs.(3), and using Eq.(1), the following equation can be obtained.

$$Q_p = Q_i + 2 Q_o \quad (4)$$

As the left terms of Eqs.(3) can be expressed with respect to Q_p of Eq.(2), they are rewritten as follows:

$$m_1 Q_p = 2 k_0 M_c / (u l) - Q_b$$

$$m_2 Q_p = 2 M_b / (v h) - k_0 Q_c \quad (5)$$

$$m_3 Q_p = (1 - k_0) M_c / (u l)$$

$$m_4 Q_p = (1 - k_0) Q_c / 2$$

Where,

$$m_1 = (v h / u l) [k_0 (1 - v) - u] / [(1 - u - v)]$$

$$m_2 = (1 - u - k_0 v) / (1 - u - v) \quad (6)$$

$$m_3 = (v h / u l - m_1) / 2$$

$$m_4 = (m_2 - 1) / 2$$

Substituting $m_1 Q_p$ for the left term of the first equation of Eqs.(3), and using Eq.(4) and third equation of Eqs.(6), the following equation can be obtained:

$$Q_\alpha = m_3 Q_i - m_1 Q_o \quad (7)$$

Also the following equation can be obtained from the second of Eqs.(3), Eq.(4) and the fourth equation of Eqs.(6),

$$Q_\beta = m_4 Q_i + m_2 Q_o \quad (8)$$

3.3 Deformation compatibility condition

Deformations are supposed to be simple parallelograms as shown in Fig. 3. θ_α and θ_β represents shear deformation of inner element, outer element, side element and top/bottom element, respectively.

Here, rotational angle α and β , shown in Fig. 3(c),(d), are defined by the following equations:

$$\alpha = (B_c / D_c) \theta_\alpha \quad (9)$$

$$\beta = (B_c / D_b) \theta_\beta$$

Where B_c is the width of column and D_c is the

depth of column and D_b is the depth of beam.

The state of deformation of a frame is shown in Fig.6. In the Figure, solid lines represent deformation of steel beams and the inner element of the joint, and dashed lines show deformation of the reinforced concrete columns and outer element of the joint. Steel beams and the reinforced concrete columns turn as a fulcrum of scissors at the joint. An angle generated at the upper and lower faces of the joint is α , and an angle generated at the right and left faces is β .

From a figure shown on the upper left of Fig. 6, the following equations can be found easily.

$$\alpha = \gamma - \gamma_w \quad (10)$$

$$\beta = \gamma - \gamma_f$$

Where γ is the total shear deformation of joint calculated as the average difference in rotation between the beam and column. From Eq. (10),

$$\gamma = (\gamma_w + \gamma_f + \alpha + \beta) / 2 \quad (11)$$

or,

$$\gamma_w - \gamma_f + \alpha - \beta = 0 \quad (12)$$

are obtained. This equation is the deformation compatibility condition of elements.

Supposing that the shear force-deformation relationship of elements can be expressed linearly as the following equation, Eq.(12) can be expressed in terms of shear forces.

$$\begin{aligned} \gamma_w &= Q_i / K_i \\ \gamma_f &= Q_o / K_o \\ \alpha &= Q_\alpha / K_\alpha \\ \beta &= Q_\beta / K_\beta \end{aligned} \quad (13)$$

where, K_i , K_o , K_α and K_β are stiffness of inner element, outer element, side element and top/bottom element respectively.

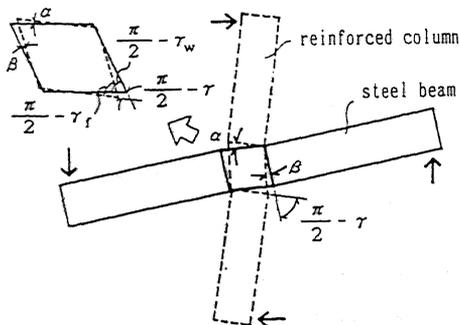


Fig.6 Deformation of joint

Substituting Eqs.(13) for Eq.(12), and using Eq.(7) and Eq.(8), the following equations are obtained:

$$Q_o = k_f (K_o / K_i) Q_i \quad (14)$$

$$\gamma_f = k_f \gamma_w$$

where,

$$k_f = \frac{1 + m_3(K_i / K_\alpha) - m_4(K_i / K_\beta)}{1 + m_1(K_o / K_\alpha) + m_2(K_o / K_\beta)} \quad (15)$$

4 SHEAR FORCE-DEFORMATION CURVE OF JOINT

4.1 Expression of shear force and deformation

Here, all shear forces and deformations are expressed with respect to γ_w . From the first and second equations of Eqs.(13) and from Eqs. (14), the following equations are obtained:

$$Q_i = K_i \gamma_w \quad (16)$$

$$Q_o = K_o \gamma_f = k_f K_o \gamma_w$$

Substituting these equations for Eq.(4),

$$Q_p = (K_i + 2 k_f K_o) \gamma_w \quad (17)$$

is obtained. And, substituting Eq.(16) for Eq.(7) and Eq.(8), the following equations are obtained:

$$Q_\alpha = (m_3 K_i - m_1 k_f K_o) \gamma_w \quad (18)$$

$$Q_\beta = (m_4 K_i + m_2 k_f K_o) \gamma_w$$

Substituting these equation for the third and fourth equations of Eqs.(13), the following equations are obtained:

$$\alpha = (m_3 K_i / K_\alpha - m_1 k_f K_o / K_\alpha) \gamma_w \quad (19)$$

$$\beta = (m_4 K_i / K_\beta + m_2 k_f K_o / K_\beta) \gamma_w$$

4.2 Calculation of shear force and deformation of joint by incremental representation

When the shear force-deformation relationships of each element are expressed with polygonal lines as shown in Fig.7, shear force and deformation of elements can be determined incremental representation. Suppose that each element stays at a point j (O marked in figures) on the shear force-deformation curve. Considering an increment $\Delta \gamma_w$ of γ_w within a range where the stiffnesses of elements do not vary, and corresponding increment of shear force and deformation of each element are calculated by Eqs.(16) through (19).

From Eq.(4) and Eq.(11), increments of total

shear force and deformation of the joint are obtained as follows:

$$\Delta Q_p = \Delta Q_i + 2 \Delta Q_o \quad (20)$$

$$\Delta \gamma = (\Delta \gamma_w + \Delta \gamma_f + \Delta \alpha + \Delta \beta) / 2$$

Among elements, one of which necessary increment for reaching the next nodal point (changing point of stiffness) is the smallest is selected (● marked in figure of γ_f). According to the smallest increment, each increment is adjusted by proportional calculation. Adding the increments to the current shear force and deformation, the next shear force and deformation state ($j+1$ point) is calculated. Repeating this procedure, the shear force-deformation curve of joint are calculated.

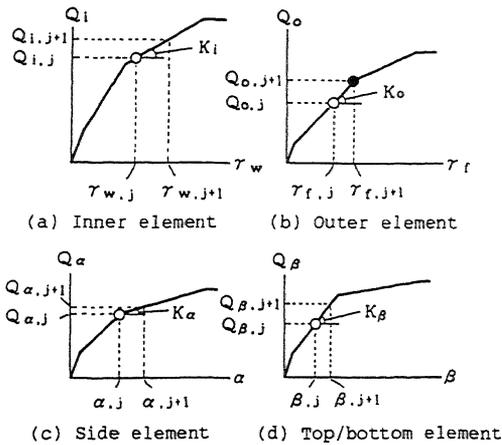


Fig.7 Shear force-deformation relationship of element

5 RESULTS OF TEST AND ANALYSIS

To determine the effective cross-section and stress-deformation relationship of elements, thirteen interior beam-column assemblages were tested. Here, outline of the contents which are related to this paper are discussed. For details, refer to the previous paper(1991).

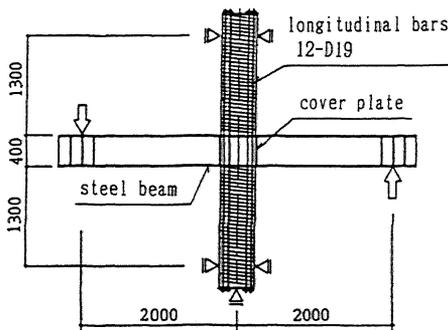


Fig.8 Test specimen

5.1 Outline of test

Variables in the test are dimensions of column cross section as shown in Fig.9, and with/without cover plates. Designations A, B, C, D, E and S of specimens indicate cross-sectional shape of the column, and numerals 1 and 2 indicate with/without cover plates (1 : equipped, 2 : not equipped).

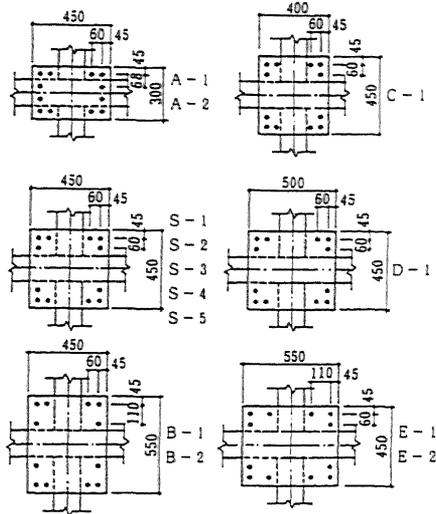


Fig.9 Column cross section of test specimen

5.2 Effective cross-section and stress-deformation relationship of elements

Here, shear stress τ_s and deformation γ_s relationship of steel web panel and cover plates is modeled as follows(Fig.10(a)):

- (1) The elastic stiffness is taken as the modulus of elasticity in shear,
- (2) The shear strength is reached at a point where $\tau_s = \sigma_y / \sqrt{3}$ (σ_y : yield strength) and $\gamma_s = 0.4\%$,
- (3) The stiffness begins to decrease at a point where $\tau_s = 0.65 \sigma_y / \sqrt{3}$.

On the other hand, shear stress τ_c and deformation γ_c relationship of of concrete panels is modeled as follows(Fig.10(b)):

- (1) The elastic stiffness is taken as the modulus of elasticity in shear,
- (2) The shear strength is reached at a point where $\tau_c = 0.3 \sigma_B$ (σ_B : compressive strength) and $\gamma_c = 0.4\%$,
- (3) The stiffness begins to decrease at a point where $\tau_c = 0.1 \sigma_B$.

Superposing these $\tau - \gamma$ relationships, shear force-deformation curves of elements are calculated.

The effective cross-section of elements are obtained from the shear strength and shear force acting on elements at ultimate stage. The test results gave the coefficients k_1 through k_6 as follows:

$$k_1 = k_2 = 0.9, \quad k_4 = k_5 = 0.16, \quad k_6 = 0.8 \quad (21)$$

$$k_3 = 0.32 D_c / D_b + 1.45 B_b / B_c - 0.36$$

When cover plates are not equipped, 0.9 is to be multiplied to k_1 and k_3 .

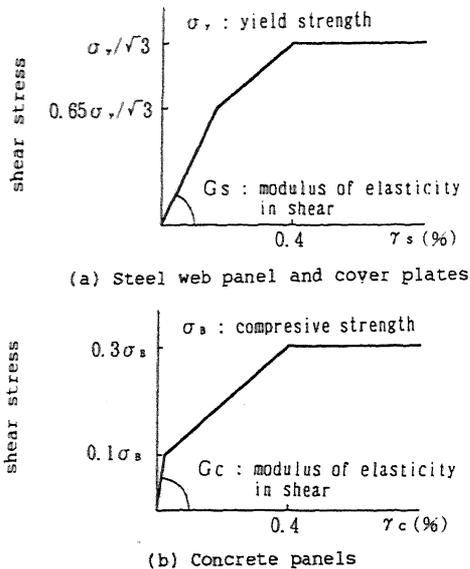


Fig.10 Shear stress-deformation relationship of element

5.3 Comparison of calculated results and test results

Shear force-deformation curves of joint obtained from the tests are shown in Fig.11 compared with the calculated results. The

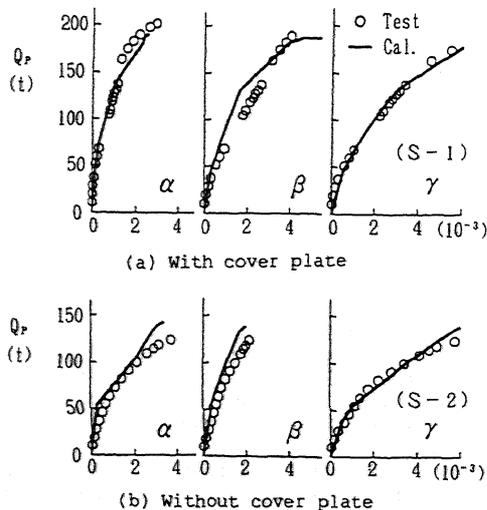


Fig.11 Shear force - deformation relationship of joint

figure shows that the calculated values agree well with the test results.

From Eq.(4), the ultimate shear capacity of joint can be expressed as

$$Q_{pu} = (k_1 \sigma_{wy} / \sqrt{3}) t_w + 2k_2 \sigma_{fy} / \sqrt{3} t_f + k_3 (0.3 \sigma_s) B_c D_c \quad (22)$$

Where σ_{wy} and σ_{fy} are yield strength of steel web panel and cover plate respectively, and t_w and t_f are thickness of steel web panel and cover plate respectively. The ultimate shear capacities of test specimens are plotted in Fig.12 against the predicted values based on Eq.(22). The predicted values and test results show favorable correspondence.

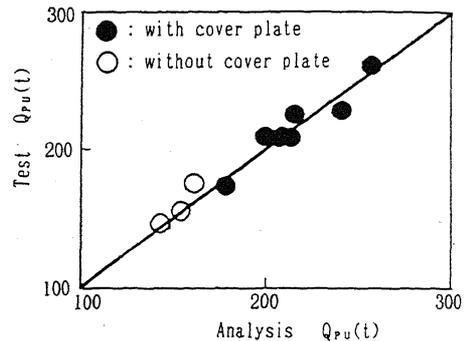


Fig.12 Shear capacity of joint

6 CONCLUSIONS

A new method for calculating the shear force-deformation curves of steel beam-reinforced concrete column has been presented. An equation for predicting the ultimate shear capacity of joint has been also proposed. The proposed method can be confirmed to best describe both the ultimate shear capacity and shear force-deformation curve of joint.

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