

Treatment of interaction effects in the seismic response of gravity dams

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ABSTRACT: In the seismic response analysis of gravity dam-reservoir-foundation systems, the wave absorption along the reservoir bottom can be accounted for by using either an approximate 1D wave propagation model or a rigorous analysis of interaction between the flexible soil along the base and the water. The rigorous approach requires enormous computational effort because of (a) cross coupling between the foundation of the dam and the soil below the reservoir, and (b) frequency dependence of the boundary condition along the fluid-foundation interface. The analysis can be simplified by neglecting the cross coupling and by using the approximate 1D wave propagation model. The effects of each of these two simplifications on the accuracy and computational efficiency of the procedure used for the response analysis of a dam subjected to random ground motion are examined in the present paper.

1. INTRODUCTION

The seismic response of gravity dams is strongly influenced by the interaction with the reservoir water and the underlying foundation. The reservoir induced hydrodynamic forces are in turn affected by radiation of waves towards infinity, wave absorption at the reservoir bed, and the cross coupling between the foundation below the dam and the reservoir bottom.

An analysis procedure that would take into account all of the foregoing interactions is generally quite complex and computationally demanding. The first level of simplification is achieved by neglecting the cross-coupling between the dam foundation and the reservoir bottom. Further simplification results when the wave absorption at the reservoir bottom is represented by an approximate 1D wave propagation model proposed by Hall and Chopra (1980).

The primary objective of the present work is to carry out a systematic study of the level of inaccuracies introduced by the various simplifications. It should be noted that the effectiveness of the method is best judged by determining how well it can predict the response to a random ground motion. Significant differences in complex frequency responses need not necessarily result in large differences in response to real earthquakes. The capability of the simplified 1D model to predict the seismic response of a dam-foundation-reservoir is investigated by comparing the results of simplified analysis with those obtained by rigorous approach. Analytical results used in the study are those for

the response of pine flat dam to the horizontal and vertical components of Taft ground motion.

2. METHOD OF ANALYSIS

The present work uses a boundary element method (BEM) for the seismic analysis of the dam - reservoir - foundation system. The dam is modeled by finite elements. The infinite reservoir is divided into two regions: a near field of finite extent and arbitrary geometry adjacent to the dam, and an infinite far field of uniform cross-section. The finite region of the reservoir is modeled by BEM. To improve the computational efficiency, a frequency independent fundamental solution is used (Tsai, 1987), so that the boundary element matrices do not vary with the excitation frequency. This, however, requires that a certain number of interior nodes be defined in addition to the boundary nodes. The infinite radiation of the reservoir far field is accounted for by modeling the interface between the near field and the far field by 1D finite elements. The near and far fields are coupled by ensuring compatibility of pressures and pressure gradients along the common boundary. The efficiency of the procedure is further improved by assuming that the reservoir bottom of the far field is rigid. This assumption can be justified provided the reservoir near field is of sufficient length. It is seen that, a near field length of about 2.5 times the reservoir depth combined with a far field having rigid bottom gives a response which is nearly equal to that obtained with

an infinite absorptive bottom. The approximation of rigid far field leads to a real and frequency independent eigenvalue problem associated with the reservoir far field.

The base soil, treated as a homogeneous, isotropic and viscoelastic half plane, is modeled by boundary elements (Abdalla 1984, Chandrashaker and Humar 1991) located along the dam-foundation interface and a short length of soil surface on either side of the dam. For a rigorous analysis, the fluid-soil interface at the reservoir near field also needs to be discretized. Compatibility and equilibrium are then enforced along these interfaces. It should be noted that, in a rigorous approach, the same soil half plane extends under the reservoir, as shown in Fig. 1, and the cross-coupling between the dam foundation and the reservoir bottom must be accounted for. A 2D plane stress idealization is adopted for both dam and the foundation analysis. In modeling the fluid-foundation interaction, it is assumed that only the flexible foundation below the reservoir is responsible for the wave absorption, and the effect of the sediment that could have deposited along the reservoir bed is ignored. Studies by other researchers have shown that this assumption is valid, especially if the foundation below the reservoir is flexible (Medina et al., 1990).

3. ANALYTICAL FORMULATIONS

The equation of motion of the coupled dam-reservoir foundation system in frequency domain is given by

$$\{-\Omega^2 \mathbf{M} + (1 + \eta_d) \mathbf{K}\} \mathbf{U}^l(\Omega) = -\mathbf{E}^l + \mathbf{R}^l(\Omega) \quad (1)$$

where \mathbf{M} , \mathbf{K} represent respectively the mass and the stiffness matrices of the dam substructure including the dam-foundation interface nodes, Ω is the excitation frequency, η_d is the hysteretic damping factor of the dam material, \mathbf{U}^l the nodal displacement vector relative to the free field, $l = x, y$ the direction of excitation, $\{\mathbf{E}^x\}^T = \{m_1 \ 0 \ m_2 \ 0 \ \dots \ m_N \ 0\}$, $\{\mathbf{E}^y\}^T = \{0 \ m_1 \ 0 \ m_2 \ \dots \ 0 \ m_N\}$, and m_i is the lumped mass associated with the i th finite element node. Vector \mathbf{R}^l is the nodal load vector, having non-zero quantities corresponding to the upstream face and the base degrees of freedom (DOF) of the dam, which in turn comprises two vectors: \mathbf{R}_h^l denoting the hydrodynamic forces along the upstream face of the dam, and \mathbf{R}_b^l the forces along the dam-soil interface nodes.

3.1 Analysis using simplified approach

Analysis by simplified approach includes the dam-reservoir interaction, dam-foundation, and fluid-foundation interaction, but the cross coupling effect between the foundation below the dam and the reservoir bottom is ignored. Also, the fluid-foundation interaction effect is accounted for by

using the 1D approximate model for wave propagation in the foundation proposed by Hall and Chopra (1980).

For the analysis by simplified approach, the dam-foundation interface forces are given by

$$\mathbf{R}_b^l(\Omega) = -\mathbf{S}_{bb}(\Omega) \mathbf{u}_b^l(\Omega) \quad (2)$$

where \mathbf{S}_{bb} is complex valued soil impedance matrix, \mathbf{u}_b^l is the displacement vector corresponding to the DOF of the dam - foundation interface nodes.

The size of the problem represented by Eq. 1 can be reduced by using the Rayleigh Ritz method, in which the first N undamped mode shapes Φ of the dam-foundation system are used as Ritz vectors. The transformed equation becomes

$$\{-\Omega^2 \mathbf{I} + (1 + \eta_d) \Lambda + \Phi^T \mathbf{S}_{0f}(\Omega) \Phi\} \mathbf{Z}^l(\Omega) = -\Phi^T \mathbf{E}^l + \Phi^T \mathbf{R}_h^l(\Omega) \quad (3)$$

where

$$\mathbf{S}_{0f}(\Omega) = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{S}_{bb}(\Omega) - (1 + \eta_d) \mathbf{S}_{bb}^r(\Omega_0) \end{pmatrix}$$

Λ is the diagonal matrix of the squared frequencies ω_j^2 , $Z_j^l(\Omega)$ is the j th modal coordinate, ω_j and ϕ_j are the natural frequencies and mode shapes, and $\mathbf{S}_{bb}^r(\Omega_0)$ is the real part of the soil impedance matrix corresponding to a small frequency value Ω_0 .

3.1.1 Evaluation of hydrodynamic forces

The 2D wave equation governing the reservoir fluid vibration is given by

$$\nabla^2 p + k^2 p = 0 \quad (4)$$

where p is the hydrodynamic pressure, $k = \Omega/c$ the wave number, and c the wave velocity in water. By using the weighted residual technique, with the weighting function p_i^* chosen to be equal to the static solution for a unit source at point i , Eq. 4 can be transformed into an integral equation involving integrals on the boundary and the domain integral $k^2 \int \nabla^2 p \cdot p_i^* dA$. This domain integral can also be transformed to boundary integrals by assuming the following particular solution for p

$$p(x, y, \Omega) = \sum_{m=1}^M \alpha_m(\Omega) f_m(x, y) \quad (5)$$

in which $\alpha_m(\Omega)$ is an unknown coefficient, and $f_m = \nabla^2 \psi^m(x, y)$ is a suitably chosen harmonic function. By discretizing the near field boundary into a series of segments termed boundary elements, with the pressure and pressure gradients as well as ψ and its derivative $\eta = \partial\psi/\partial n$ assumed to vary in a prescribed manner over each element, the bound-

ary integral equations, can be represented in a discretized form as

$$\mathbf{H}\mathbf{p} - \mathbf{G}\mathbf{q} = k^2(-\mathbf{H}\psi + \mathbf{G}\eta)\{\alpha\} \quad (6)$$

where \mathbf{H} and \mathbf{G} are matrices of the boundary integrals involving $q_i^* = \partial p_i^*/\partial n$ and p_i^* respectively, \mathbf{p} is the vector of pressures, \mathbf{q} the vector of pressure gradients, and n the unit normal.

In the present formulation, p_i^* is given by

$$p_i^* = -(1/2\pi) \ln(r_{ij}/r'_{ij}) \quad (7)$$

and function f is chosen as

$$f_m(x_j, y_j) = r'_{mj} - r_{mj} \quad (8)$$

where r and r' are the distances to the field point j respectively from the source point and the mirror image of the source point. The solution p_i^* automatically satisfies the zero pressure condition on the free surface of the reservoir.

The vector $\{\alpha\}$ can be expressed in terms of \mathbf{p} and a matrix \mathbf{F} containing the values of f_m at the field points. Substitution in Eq. 6 gives

$$(\mathbf{H} - k^2\tilde{\mathbf{M}})\mathbf{p} = \mathbf{G}\mathbf{q} \quad (9)$$

where $\tilde{\mathbf{M}} = (\mathbf{G}\eta - \mathbf{H}\psi)\mathbf{F}^{-1}$, is the hydrodynamic mass matrix.

We now define $\mathbf{p}_o^l(\Omega)$ as the vector of pressures obtained by solving Eq. 9 with boundary conditions corresponding to the excitation of a rigid dam by a unit harmonic ground acceleration along the l th direction. The respective boundary conditions along the dam face and reservoir bottom are

$$\mathbf{q}_1^l = -\rho_w \mathbf{a}_n^l \quad (10a)$$

$$\mathbf{q}_2^l = -\rho_w \mathbf{b}_n^l - i\Omega\gamma\mathbf{p}_2 \quad (10b)$$

We next define $\mathbf{p}_j^f(\Omega)$ as the pressure vector obtained by solving Eq. 9, with boundary conditions corresponding to the motion of the dam-foundation system vibrating in its j th mode and no motion of the reservoir bed. The related boundary conditions along the upstream face of the dam and the reservoir bottom are

$$\mathbf{q}_1 = -\rho_w \{\psi_{jx} \mathbf{a}_n^x + \psi_{jy} \mathbf{a}_n^y\} \quad (11a)$$

$$\mathbf{q}_2 = -i\Omega\gamma\mathbf{p}_2 \quad (11b)$$

In Eqs. 10 and 11, ρ_w is the mass density of water, \mathbf{a}_n^l the acceleration vector of the dam face along the normal due to an unit excitation in direction l , \mathbf{b}_n^l is the similar acceleration vector corresponding to the reservoir bottom, γ the wave absorption parameter $= \rho_w/\rho_r c_r$, ρ_r and c_r the mass density and the compression wave velocity of the reservoir bottom, and \mathbf{b}_n the free field acceleration vector of

reservoir bottom along the normal. Suffixes 1 and 2 represent respectively the quantities corresponding to the upstream face of the dam and the reservoir bottom. In addition, ψ_{jx} and ψ_{jy} are the j th mode components along the horizontal and vertical DOF of upstream dam face nodes of the dam-foundation system. It should be noted that in Eqs. 10b and 11b, the wave absorption along the reservoir bed is accounted for by the simplified 1D model with γ as the controlling parameter. It is perhaps more meaningful to represent the wave absorption along the reservoir bottom through a wave reflection parameter $\tilde{\alpha}$ given by $\tilde{\alpha} = (1 - \gamma c)/(1 + \gamma c)$; a value of $\tilde{\alpha} = 1.0$ represents a nonabsorptive reservoir bottom.

The solution of Eq. 9 for either $\mathbf{p}_o^l(\Omega)$ or $\mathbf{p}_j^f(\Omega)$ requires the definition of the boundary conditions along the interface between the near and the far fields of reservoir. These boundary conditions are as described in the following paragraphs.

Vectors \mathbf{p} and \mathbf{q} along the interface between the reservoir near field and the far field, also referred to as transmitting boundary of the reservoir are given by (Hall and Chopra 1980)

$$\mathbf{p} = \tilde{\Phi} \mathbf{v} + \rho_w \tilde{\Phi} \mathbf{K}^{-2} \tilde{\Phi}^T \mathbf{d} \quad (12a)$$

$$\mathbf{q} = -\tilde{\Phi} \mathbf{K} \mathbf{v} \quad (12b)$$

where $\tilde{\phi}_i$ and λ_i are respectively the i th eigenvectors and eigenvalues of the far field; $\tilde{\Phi}$ the matrix of eigenvectors; \mathbf{v} is a vector of modal coordinates, \mathbf{K} a diagonal matrix with elements $\kappa_1, \kappa_2, \dots, \kappa_L$; L the number of finite element nodes on the interface boundary; $\kappa_i = \sqrt{\lambda_n - k^2}$, and \mathbf{d} a vector with only one non-zero entry equal to an input acceleration \mathbf{a}_y corresponding to the base node. The far field eigenvalue problem is complex valued and frequency dependent for an absorptive far field reservoir bottom, but real valued and frequency independent otherwise. Hence confining the bottom absorption only to the near field will increase the efficiency of the procedure.

The hydrodynamic force vector \mathbf{R}_k^l in Eq. 3 can now be expressed as

$$\mathbf{R}_k^l = \mathbf{R}_o^l - \Omega^2 \sum_{j=1}^N \mathbf{Z}_j^l \mathbf{R}_j^f \quad (13)$$

where \mathbf{R}_o^l , \mathbf{R}_j^f are the nodal forces along dam's upstream face equivalent to the hydrodynamic pressures $\mathbf{p}_o^l(\Omega)$ and $\mathbf{p}_j^f(\Omega)$ respectively. It is evident from Eq. 13 that the terms representing cross-coupling between the foundation below the dam and the reservoir bottom are excluded in the formulation by simplified approach.

3.2 Analysis by rigorous approach

The model based on rigorous approach eliminates some of the key assumptions made in the simplified approach. The major difference between the two models lies in the inclusion of the cross coupling between the foundation below the dam and the reservoir bottom, and in the treatment of the fluid-foundation interaction. As stated earlier, the simplified approach ignores the cross coupling between the foundation below the dam and the reservoir, while accounting for the wave absorption along the flexible reservoir bed through an approximate 1D wave propagation model. On the other hand, the model based on a more rigorous approach considers the cross coupling effect. In addition, it accounts for the fluid-foundation interaction effect by modeling the foundation soil below the reservoir, as a halfplane using a boundary element discretization of the fluid-foundation interface.

For the analysis by rigorous approach, the force displacement relation for the soil substructure can be expressed using the complex valued, frequency dependent soil impedance matrix \mathbf{S} as:

$$\begin{pmatrix} \mathbf{S}_{bb} & \mathbf{S}_{bf} \\ \mathbf{S}_{bf}^T & \mathbf{S}_{ff} \end{pmatrix} \begin{pmatrix} \mathbf{r}_b^l \\ \mathbf{r}_f^l \end{pmatrix} = \begin{pmatrix} \mathbf{R}_b^l \\ \mathbf{Q}_f^l \end{pmatrix} \quad (14)$$

where suffix b denotes DOF along the foundation below the dam, and suffix f indicates those corresponding to the reservoir bottom. Vector \mathbf{r} represents the displacement of the soil, \mathbf{R}_b^l the interaction forces in the soil along the dam-foundation interface nodes, and \mathbf{Q}_f^l the interaction forces along the fluid-foundation interface.

After simplification and application of the equilibrium and compatibility conditions along the dam-foundation interface, the expression for the forces along that interface reduces to

$$\mathbf{R}_b^l = -\mathbf{S}_{bf}\mathbf{S}_{ff}^{-1}\mathbf{Q}_f^l - \tilde{\mathbf{S}}_{bb}\mathbf{u}_b^l \quad (15)$$

in which $\tilde{\mathbf{S}}_{bb} = (\mathbf{S}_{bb} - \mathbf{S}_{bf}\mathbf{S}_{ff}^{-1}\mathbf{S}_{bf}^T)$

Using Eq. 15 and transforming to the Ritz coordinates, Eq. 1 reduces to the form

$$\begin{aligned} & \{-\Omega^2\mathbf{I} + (1 + i\eta_d)\mathbf{A} + \Phi^T\tilde{\mathbf{S}}_{0f}(\Omega)\Phi\}\mathbf{Z}^l(\Omega) \\ & = -\Phi^T\mathbf{E}^l + \Phi^T \begin{pmatrix} \mathbf{R}_h^l(\Omega) \\ \mathbf{S}_{bf}(\Omega)\mathbf{S}_{ff}^{-1}(\Omega)\mathbf{Q}_f^l(\Omega) \end{pmatrix} \end{aligned} \quad (16)$$

where

$$\tilde{\mathbf{S}}_{0f}(\Omega) = \begin{pmatrix} 0 & 0 \\ 0 & \tilde{\mathbf{S}}_{bb}(\Omega) - (1 + i\eta_d)\tilde{\mathbf{S}}_{bb}^r(\Omega_o) \end{pmatrix}$$

3.2.1 Evaluation of hydrodynamic forces

The hydrodynamic pressure along the reservoir bottom can be related to corresponding forces in soil

along the fluid-foundation interface by conditions of equilibrium giving

$$\mathbf{Q}_f = -\mathbf{A}\mathbf{p}_2 \quad (17)$$

where \mathbf{A} denotes the area matrix relating the forces and pressures at the fluid-foundation interface.

The hydrodynamic force vector \mathbf{R}_h^l can be expressed as

$$\mathbf{R}_h^l = \mathbf{R}_o^l - \Omega^2 \sum_{j=1}^N \mathbf{Z}_j^l (\mathbf{R}_j^f + \mathbf{R}_j^b) \quad (18)$$

where \mathbf{R}_o^l , \mathbf{R}_j^f , and \mathbf{R}_j^b are the nodal forces along dam's upstream face, statically equivalent to the hydrodynamic pressures $\mathbf{p}_o^l(\Omega)$, $\mathbf{p}_j^f(\Omega)$ and $\mathbf{p}_j^b(\Omega)$ respectively.

The procedure for the evaluation of the hydrodynamic pressures is similar to that described in Section 3.1.1, except for a modification in the boundary conditions as given below. The boundary conditions along the upstream face of the dam, and the reservoir bottom for the evaluation of \mathbf{p}_o^l are

$$\mathbf{q}_1 = -\rho_w \mathbf{a}_n^l \quad (19a)$$

$$\mathbf{q}_2 = \rho_w \{-\mathbf{b}_n^l + \Omega^2 \mathbf{S}_{ff}^{-1}(\Omega)\mathbf{A}\mathbf{p}_2(\Omega)\} \quad (19b)$$

The corresponding boundary conditions in the evaluation of \mathbf{p}_j^f are given by

$$\mathbf{q}_1 = -\rho_w \{\psi_{jx} \mathbf{a}_n^x + \psi_{jy} \mathbf{a}_n^y\} \quad (20a)$$

$$\mathbf{q}_2 = \rho_w \Omega^2 \mathbf{S}_{ff}^{-1}(\Omega)\mathbf{A}\mathbf{p}_2(\Omega) \quad (20b)$$

Pressure vector \mathbf{p}_j^b is the solution of the governing equation with boundary conditions corresponding to that of vibration of the fluid-foundation interface in the j th mode χ_j , with no motion of the dam, i.e.

$$\mathbf{q}_1 = 0 \quad (21a)$$

$$\mathbf{q}_2 = \rho_w \{\Omega^2 \mathbf{S}_{ff}^{-1} \mathbf{A} \mathbf{p}_2 - (\chi_{jx} b_n^x + \chi_{jy} b_n^y)\} \quad (21b)$$

where χ_{jx} and χ_{jy} are the mode shape components corresponding to the horizontal and vertical DOF of reservoir bottom vibrating in the j th mode, defined by

$$\chi_j(\Omega) = \mathbf{S}_{ff}^{-1}(\Omega)\mathbf{S}_{bf}^T(\Omega)\phi_j^b \quad (22)$$

In Eq. 22 ϕ_j^b is the j th mode shape corresponding to the DOF of nodes along the dam-foundation interface. Some researchers have used the static stiffness matrix of foundation to obtain the mode shape components for reservoir bottom, namely, $\chi_j(0) = \mathbf{S}_{ff}^{-1}(0)\mathbf{S}_{bf}^T(0)\phi_j^b$. The correct formulation is to use $\chi_j(\Omega)$ which in turn is related to frequency

dependent impedance matrices as in Eq. 22.

It should be noted that when the terms representing cross-coupling between the foundation below the dam and the reservoir bottom are ignored the expression for hydrodynamic force vector R_h^i is given by Eq. 13 rather than Eq. 18, which is equivalent to neglecting the contributions from the $p_j^b(\Omega)$ pressure vector.

4. ANALYSIS OF PINE FLAT DAM

The non-overflow section of Pine Flat Dam, California is analyzed for its response to the Taft ground motion. The cross section dimensions and the finite element mesh for the analysis of this dam are identical to those given in Fenves and Chopra (1984). For the dam material, the Young's modulus of elasticity $E_d = 22500$ Mpa, unit weight $= 2483$ kg/m³, Poisson's ratio $= 0.2$. The hysteretic damping factor for the dam η_d and for the foundation soil η_s were both taken equal to 0.1. The properties of the foundation material are $E_f = 22500$ Mpa, unit weight $= 2643$ kg/m³, and Poisson's ratio $= 1/3$. The full reservoir has a constant depth of $H = 115.5$ m. The pressure wave velocity is 1440 m/sec, and the unit weight of water is 1000 kg/m³. The length of the reservoir near field is $L_f = 2.5 \times H$. For the 1D simplified wave absorption model and the selected properties of the foundation soil, the value of wave reflection coefficient $\bar{\alpha}$ works out to be 0.6853.

The S69E and vertical components of Taft earthquake ground motion of 1952 are treated as the free-field input. Response analysis is performed by considering the first ten mode shapes of the associated dam-foundation system. For the frequency domain analysis, 2048 time steps of 0.02 s are used. The first 20 s of the excitation is taken as equal to the ground acceleration values for the Taft motion, the remaining period is made up of a grace band of zeros to avoid the aliasing errors inherent in discrete Fourier transforms.

To demonstrate the effect of cross-coupling between the foundation below the dam and the reservoir bottom, time history analyses are carried out by the rigorous approach once including the $p_j^b(\Omega)$ pressure terms, and next excluding this pressure term, and the corresponding results are plotted in Fig. 2. It is quite clear from this plot that the cross-coupling effect causes small differences in peak values, but these differences are not very significant, for either components of ground motion.

The second set of analysis compares the performance of the two models based on the simplified and rigorous approach, under both transverse and vertical components of ground motion. Figure 2 presents the first 15 s of time history of the horizontal crest displacement. The results reveal that the rigorous and approximate 1D model yield responses that are in close comparison to each other. The small differences in some peak values can be

attributed to the fact that the cross coupling is neglected in the simplified approach. The 1D model is therefore satisfactory in the evaluation of response due to both components of ground motion.

5. CONCLUSIONS

The following conclusions can be drawn from the present study:

(1) The results obtained by using rigorous model to represent the fluid-foundation effect are not much different from those obtained by the simplified approach described in this paper. On the other hand the rigorous model involves much greater computational effort.

(2) The effect of cross-coupling between the foundation of the dam and the foundation soil below the reservoir is not very significant in the response to either component of ground motion, and hence could be conveniently ignored thereby improving the computational efficiency of the analysis procedure.

Taken together the above conclusions imply that in the analysis of the seismic response of a dam located on a flexible foundation and impounding a reservoir, the simplified approach presented in this paper is found to be fairly accurate.

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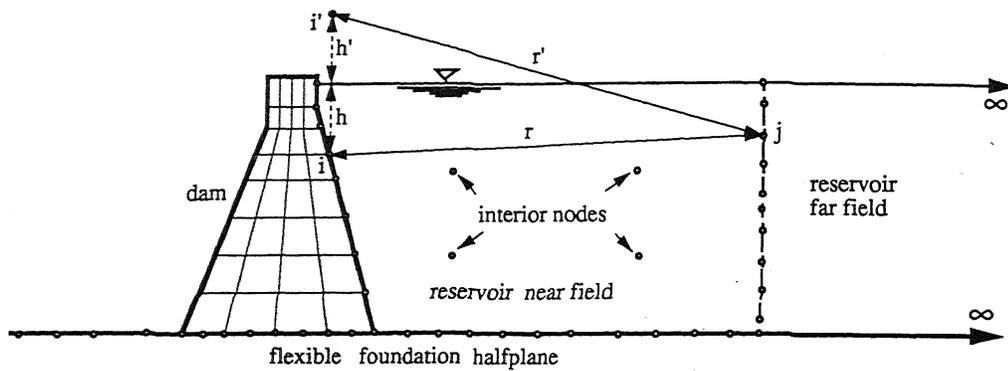


Figure 1. Dam-reservoir-foundation system for rigorous analysis

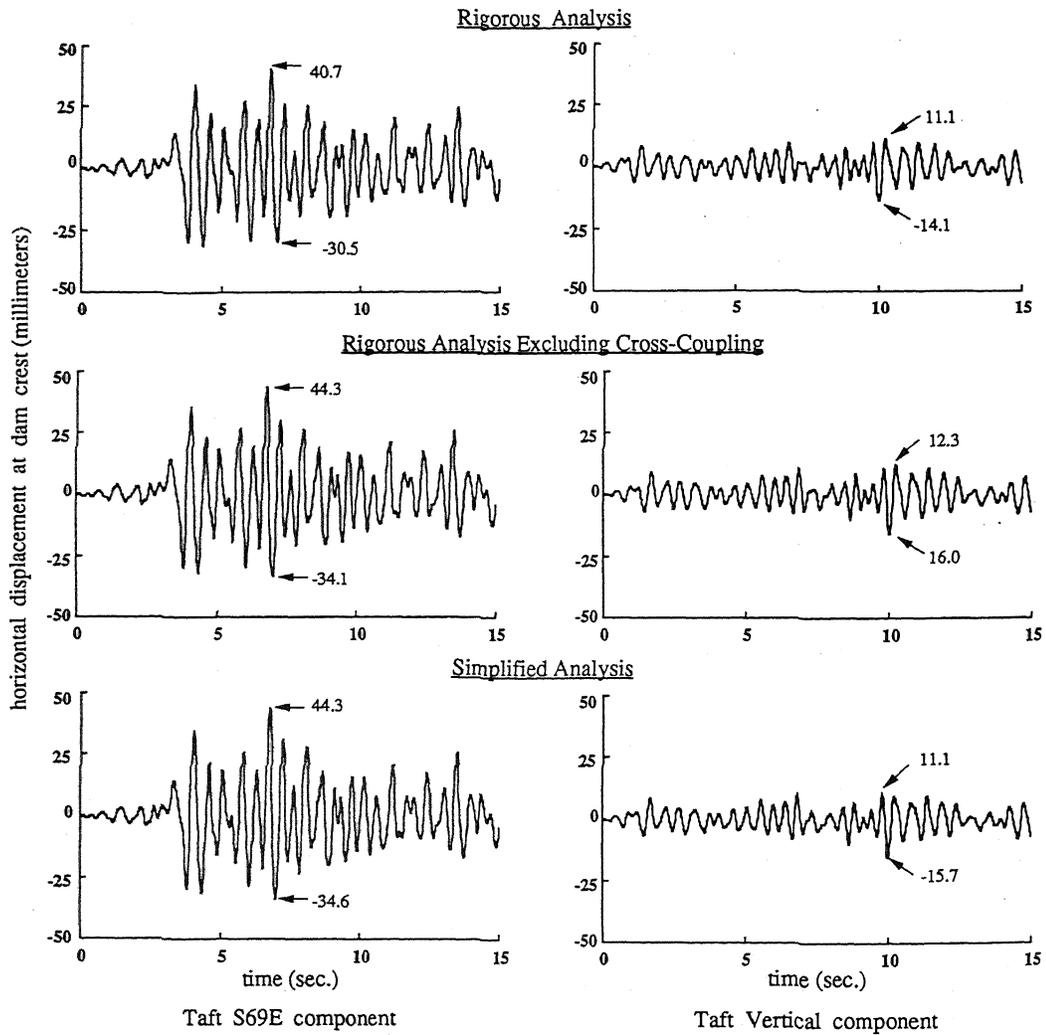


Figure 2. Displacement history of pine flat dam-reservoir-foundation system