

Nonlinear modelling of the recorded response of Matahina Dam

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ABSTRACT: Features of the strong response of the 86 metre high Matahina earth dam recorded by an array of five accelerographs in the Edgcombe earthquake have been explained by a nonlinear model. The response was strongly nonlinear, as shown by the variation of the effective fundamental frequency from 1.14 Hz in the foreshock to 0.72 Hz during the strongest part of the mainshock motion, with accelerations showing spikes up to 0.77g. A model with a low strength zone near the crest reproduced the features of stronger accelerations at 2/3 height than at the crest, a concentration of fundamental mode and overall deformation in the top section of the dam, and an attenuation of the crest response in the 2-3 Hz range which dominates the response at 2/3 height. The response of a model with constant nominal yield strain was unable to represent these characteristics.

INTRODUCTION

Matahina Dam is an 86 metre high earth and rock fill dam located on the boundary of the Central Volcanic Region of New Zealand. The epicentre of the $M_L 6.6$ Edgcombe earthquake of 2 March 1987 was 22 km north of the dam, with the main surface fault trace approaching to within 11 km of the dam. The dam experienced very strong motions in the earthquake, with the 0.33g peak horizontal acceleration and 0.14g vertical acceleration recorded at the base of the dam producing a maximum recorded response of 0.77g horizontally (at site E) and 0.39g vertically.

Five film-recording strong-motion accelerographs which had been installed on the dam since 1967 recorded the $M_L 5.2$ foreshock and mainshock, and some aftershocks. Three of the accelerographs were located about 2 metres below the crest of the dam on the downstream face at the centre (site B) and quarter points (sites A and E), one at about 2/3 height in the centre of the downstream face (site C) and one on the ground only a few metres from the centre of the downstream face (site D).

This paper seeks to explain some of the features of the recorded transverse (upstream-downstream) mainshock motions, in particular marked differences in the recorded N07W transverse responses at the two instrument locations on the vertical centreline of the dam, at site B near the crest and at site C at about two-thirds height.

FEATURES OF THE RECORDED MOTIONS

Features of the recorded transverse motions at sites B and C included:

1. Stronger transverse accelerations (0.45g) were recorded on the face of the dam at 2/3 height (site C) than the maximum value of 0.35g recorded at the centre crest (site B), as shown in figure 1b.

2. The effective fundamental frequency in the transverse direction varied from 1.14 Hz in the foreshock (figure 2a) to 0.72 Hz during the strongest mainshock motions (figure 2b). The variation in natural frequency as the amplitude of the motion changed shows that the mainshock response was strongly nonlinear.

3. The centre crest transfer function for the mainshock had distinct peaks at 0.72 Hz and 0.98 Hz, rather than a single broad band fundamental mode peak (figure 2b). This suggested that stiffnesses associated with each of these frequencies were effective during substantial portions of the response, with sharp transitions in effective frequency between these two values.

4. The response at site C in the 0.7-1.2 Hz (0.8-1.4s) fundamental mode band was much reduced from the crest response (figures 2 and 3), instead of 0.6-0.8 times the crest response as expected from the fundamental mode shapes of various models commonly used to represent dams.

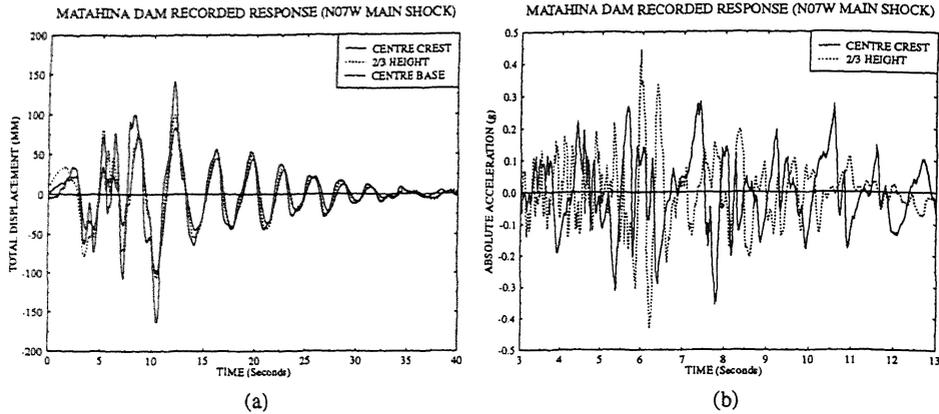


Figure 1. Displacements and accelerations of Matahina Dam from the recorded accelerograms.

5. The Fourier and response spectra plots showed that site C at 2/3 height responded strongly in the 2-3 Hz band, which provided the major contribution to the acceleration response at this location. The spectra from the three crest locations showed little response in this frequency band. This is inconsistent with linear shear-beam models for which the strongest response occurs at the crest in all modes.

6. The absolute displacement responses calculated from the recorded accelerations are dominated by components at 0.3 Hz at all locations. There are no transfer function peaks corresponding to the strong Fourier spectra peaks at this frequency. This frequency corresponds to a ground motion Fourier spectrum peak, with the dam responding to this motion with only slight amplification (figure 1).

7. Simple two-dimensional shear beam modelling with a constant shear modulus produced modal frequencies at many of the peaks of the transfer functions if the shear wave velocity of the dam was chosen to match the fundamental mode peak at about 1 Hz.

However, there was a significant number of peaks which did not correspond to the modal frequencies of the simple shear beam models.

SHEAR BEAM MODELS

The modal frequencies $\omega_{n,r}$ for the transverse response of a uniform modulus triangular shear beam model of a dam of height H , length L and shear wave velocity V_s are given by

$$\omega_{n,r} = \frac{V_s}{H} [\beta_n^2 + (r\pi H/L)^2]^{1/2} \quad (1)$$

The vertical profiles of the transverse mode shapes, corresponding to different values of n , are Bessel functions, with $J_0(\beta_n) = 0$, while r gives the number of half sine waves in the across dam variation of the mode shape. Frequencies are generally grouped according to the value of n , with frequencies corresponding to several values of r occurring before the next ($n, r=1$) frequency.

With a nominal height of 86 m and a length of 314 m, the fundamental frequency of 0.98 Hz obtained from the transfer function between the crest site B and the base D corresponds to a shear wave velocity of 207 m/s for the uniform modulus shear beam model.

A number of peaks of the ratios between various sites of the transverse N07W mainshock Fourier spectra (B/D, C/D and C/B) shown in figure 2 can be associated with modal frequencies of the shear beam models, as designated in the figure by the mode number $(n,r)_e$, where e refers to the elastic phase as discussed below. It is not surprising that such a simple model fits the modal frequencies well, in that various power-law distributions of modulus with depth give similar modal frequency ratios to those of the constant modulus model. In addition, a transfer function peak at 1.53 Hz has been associated with the first heaving mode (H1) of a three-dimensional model. A strong peak occurs at 1.53 Hz in the vertical transfer functions also, confirming the coupled horizontal and vertical nature of the mode at this frequency. Of the major peaks of frequencies less than 2 Hz, these associations leave the transfer function peaks at about 0.72 Hz unidentified, along with some higher frequency peaks in the 2-4 Hz range which are difficult to associate confidently with particular modes. The peaks around 0.72 Hz in figure 2 have been denoted $(1,1)_y$, corresponding to a post-yield stiffness distribution of the $(n=1, r=1)$ mode, which

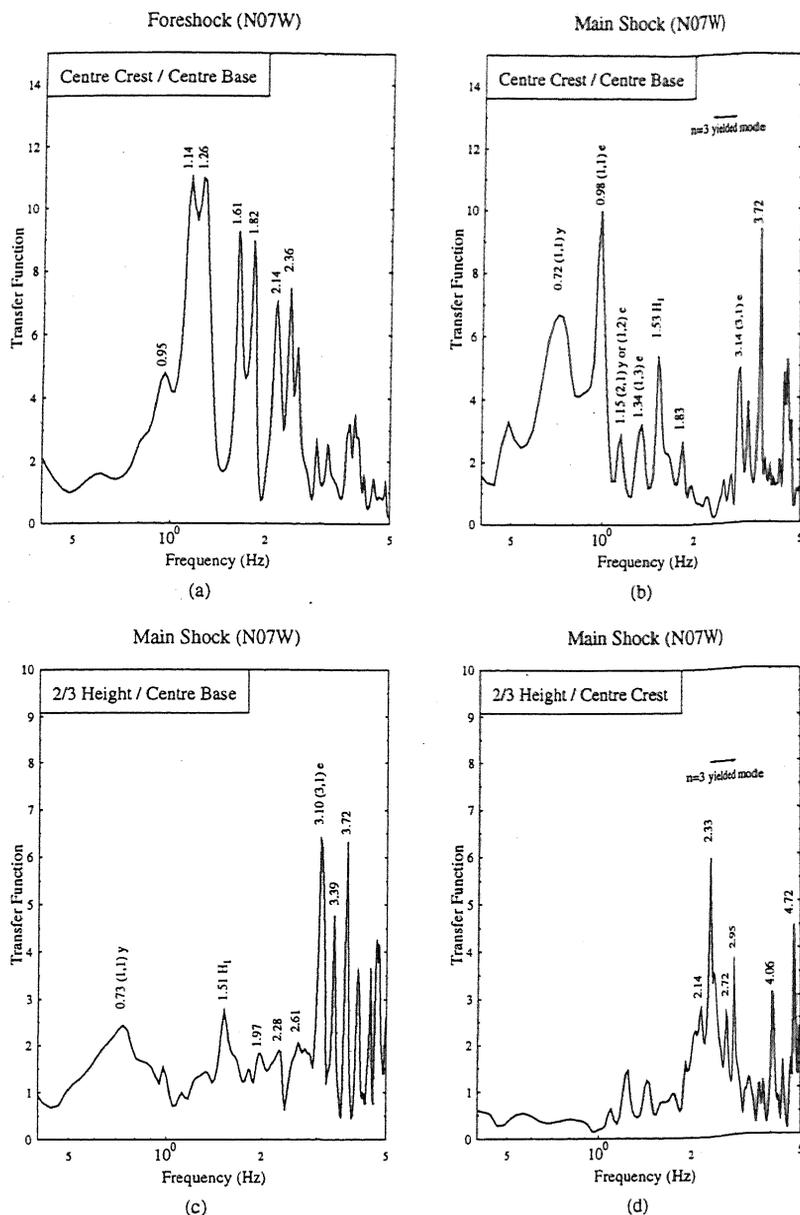


Figure 2. Ratio of Fourier amplitude spectra of the recorded motions of Matahina Dam in the foreshock (a), and mainshock (b,c,d). Frequencies and mode numbers are given for various peaks.

is explained below. Note that there are no well-defined transfer function peaks in the 2-3 Hz band, which should be occupied by various of the $n=2$ modes. It is demonstrated below that it is likely that the crest response was attenuated by nonlinear action in this frequency range. The site C to site B ratio shows a

strong peak in this range.

The profile of the fundamental mode shape down the vertical centre-line of the dam was far different from that expected from the uniform modulus triangular shear beam theory. The values of the C/B ratio at the elastic frequency (0.98 Hz) and yielded frequency (0.72 Hz)

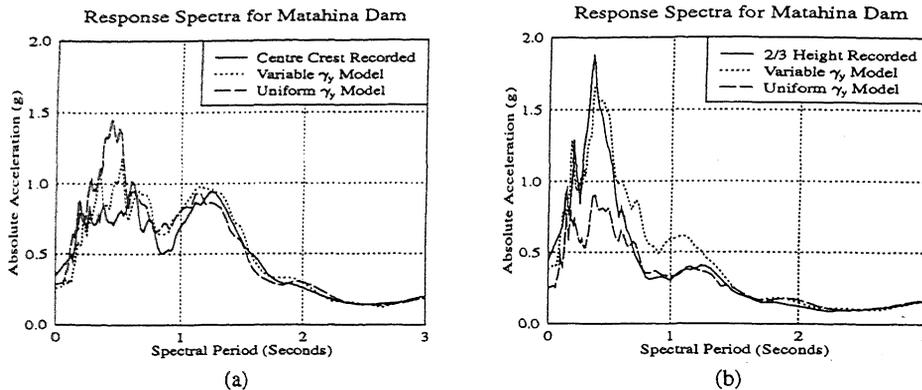


Figure 3. Comparison of 5% damped response spectra of recorded and modelled transverse motions of Matahina Dam at centre crest and 2/3 height.

were 0.18 and 0.37 respectively, compared to the value of 0.83 for the Bessel function mode shape of a uniform modulus shear beam. For various non-constant distributions of shear modulus with depth, the theoretical ratios are less than that for a uniform modulus shear beam, but not as low as those obtained from the earthquake records. Plane-strain finite element models give mode shape values on the faces of a dam less than those at the centre-line, but again the reduction is insufficient to account for the low ratio calculated from the recorded earthquake response.

A related feature of the response of Matahina Dam is the stronger response at site C than at site B in the frequency band of about 2-3 Hz (figures 1b, 2 and 3), which gave the unexpected result of a stronger recorded maximum acceleration response at site C part way down the dam face than at site B near the crest.

The observed distributions of response for various frequencies could possibly be explained by a marked reduction in the effective stiffness in the upper section of the dam, producing a fundamental mode with an accentuated crest displacement and some higher modes with their greatest displacements part way down the dam with near nodal crest displacements. This behaviour is similar to the response of a structure with a low frequency appendage attached at the top. The reduced effective stiffness could arise from a combination of less than assumed initial stiffness in the top section of the dam aggravated by softening from yielding during the earthquake response. The remainder of this paper shows that this possibility offers a plausible explanation of the observed behaviour of Matahina Dam in the Edgcombe mainshock.

SIMPLE MASS AND SPRING MODELS

Mass and spring computer models equivalent to two-dimensional shear beam models of a dam were developed to investigate the effects of nonuniform variations of shear modulus down the depth of the dam, particularly for models with low modulus values near the crest of the dam. The model produced frequencies for $n=1,2,3$ and $r=1,2,3$ of acceptable accuracy compared with the theoretical values for a constant modulus shear beam. In an attempt to reproduce the "post-yield" transverse fundamental mode frequency of Matahina Dam and to obtain mode shapes compatible with the stronger response at site C than at the crest in the 2-3 Hz range, the effects of softening the springs in the top section of the model were investigated. All springs connected to the top line of masses had their stiffness changed to 1/12th of their value in the uniform shear modulus model. This stiffness distribution was intended to represent the post-yield state of a model with bilinear hysteretic springs connected to the top line of masses in which all the bilinear springs have yielded. With this model, the fundamental transverse mode frequency was reduced from 0.98 Hz to 0.74 Hz, very close to the "elastic" and "post-yield" fundamental mode periods identified from the transfer function peaks. The $(n=1, r=1)$ and $(n=3, r=1)$ post-yield modes also have shapes which help explain the observed ratios of the site C to site B responses in the 0.7-1.0 Hz and 2-3 Hz bands. The ratio of the $(1,1)$ post-yield mode shape at site C to site B is 0.18, much reduced from the elastic mode shape ratio and in line with the amplitude ratios of 0.18 at the elastic mode frequency and 0.37 at the post-yield fundamental mode frequency measured in the mainshock response. The $(3,1)$ post-yield mode

produces a site C to site B mode shape ratio of 7.9 at its natural frequency of 2.42 Hz (or 2.36 Hz if multiplied by the ratio 0.72/0.74 of the observed and modelled post-yield frequency). This is more than sufficient to account for the ratio of 4.8 between the site C and site B responses at 2.33 Hz. All the shear beam models for the various modulus distributions considered earlier gave a site C to site B mode shape ratio of less than one.

Although this simple model explains the observed "modal" periods and shapes in the "elastic" and "post-yield" states, it fails to explain why severe softening appears to initiate in the top portion of the dam. Most previous studies of the earthquake response of earth dams have indicated that the maximum stresses and strains occur at about one-third height from the base of the dam, corresponding to the position where the maximum values occur in the fundamental mode response of a uniform shear modulus model. The maximum stresses for the second and third modes of uniform modulus shear beam models occur towards the top of the dam, at about one-third height from the crest. The acceleration response spectrum values of the Matahina Dam base motion are considerably stronger for some of the higher mode periods than at the fundamental mode period, leading to a relatively greater contribution of these modes and moving the maximum shear strains nearer the crest.

High strains in the top section of the dam in the elastic phase response provide a mechanism for severe softening to initiate there, leading to a considerable modification in the distribution of the response during the strongest motions from that expected on the basis of linear elastic modelling. Even so, it appears that the yield stresses must also be less in the upper section to produce sufficient softening of the top section relative to the body of the dam to produce the observed features of the response.

To investigate some of these points, a nonlinear one-dimensional model with more realistic stress-strain characteristics than the bilinear model is studied in the next section.

ONE-DIMENSIONAL NONLINEAR MODEL WITH NONUNIFORM MODULUS AND STRENGTH

A one-dimensional nonlinear hysteretic model was developed in which the backbone stress-strain curve was given by the commonly used hyperbolic relation.

$$\tau = \frac{G_{\max} \gamma}{1 + |\gamma/\gamma_r|} \quad (3)$$

The low-strain shear modulus G_{\max} was taken as increasing as a linear function of depth from a nonzero

value at the surface (figure 4b), and the nominal yield strain or reference strain γ_r also increased with depth (figure 4c). The variation of γ_r with depth was found by trial and error to produce a reasonable match at sites B and C between the 5% damped acceleration response spectra of the recorded motions and those of the calculated motions for the model subjected to the recorded base motion. The resulting variation of secant shear modulus with strain for materials at various depths is shown in figure 4a.

The general features of the response at both locations are reproduced by the model with non uniform yield strain (figure 3), in particular with the response at 2/3 height greatly exceeding the crest response in the 0.25s to 0.5s period band and with the crest response dominating around the range of 1.0s to 1.5s corresponding to the fundamental mode. The nature of the calculated response was much different from that for a uniform yield strain model, also shown in figure 3 in which the crest response is stronger than that at site C in both period bands.

Hysteresis loops calculated for the non-uniform yield strain model for a location 11m below the crest and at a depth of 50m are shown in figures 4d and e. The much fatter loops from the near-crest location, corresponding to greater nonlinearity of the response there, are obvious.

Figure 4f shows the variation of maximum shear stress down the dam. This distribution is not greatly different than that expected for the fundamental mode of a linear uniform modulus model, except that the uniform model has a slight drop in maximum stress over the bottom third of the dam. However, the effect of the low strength layer hypothesized in the top third of the dam is shown dramatically by a plot of maximum shear strain with depth (figure 4g). For a constant modulus model, the shear strain distribution is proportional to the shear stress distribution, but for the model used here the low strength region near the surface concentrates the maximum shear strains in that region. This behaviour produces a large increase in displacement over the top third of the dam and attenuates the higher mode response in this portion of the dam, as observed in the recorded responses. The distribution of strength with depth appears to be at least as significant as the distribution of low amplitude shear modulus in determining the overall response of the dam in strongly nonlinear motions.

CONCLUSIONS

The recorded earthquake motions on the 86 metre high Matahina earth dam in its strongly nonlinear response in the Edgecumbe earthquake showed distributions of acceleration and displacements down the centreline of the dam much different from those expected on the

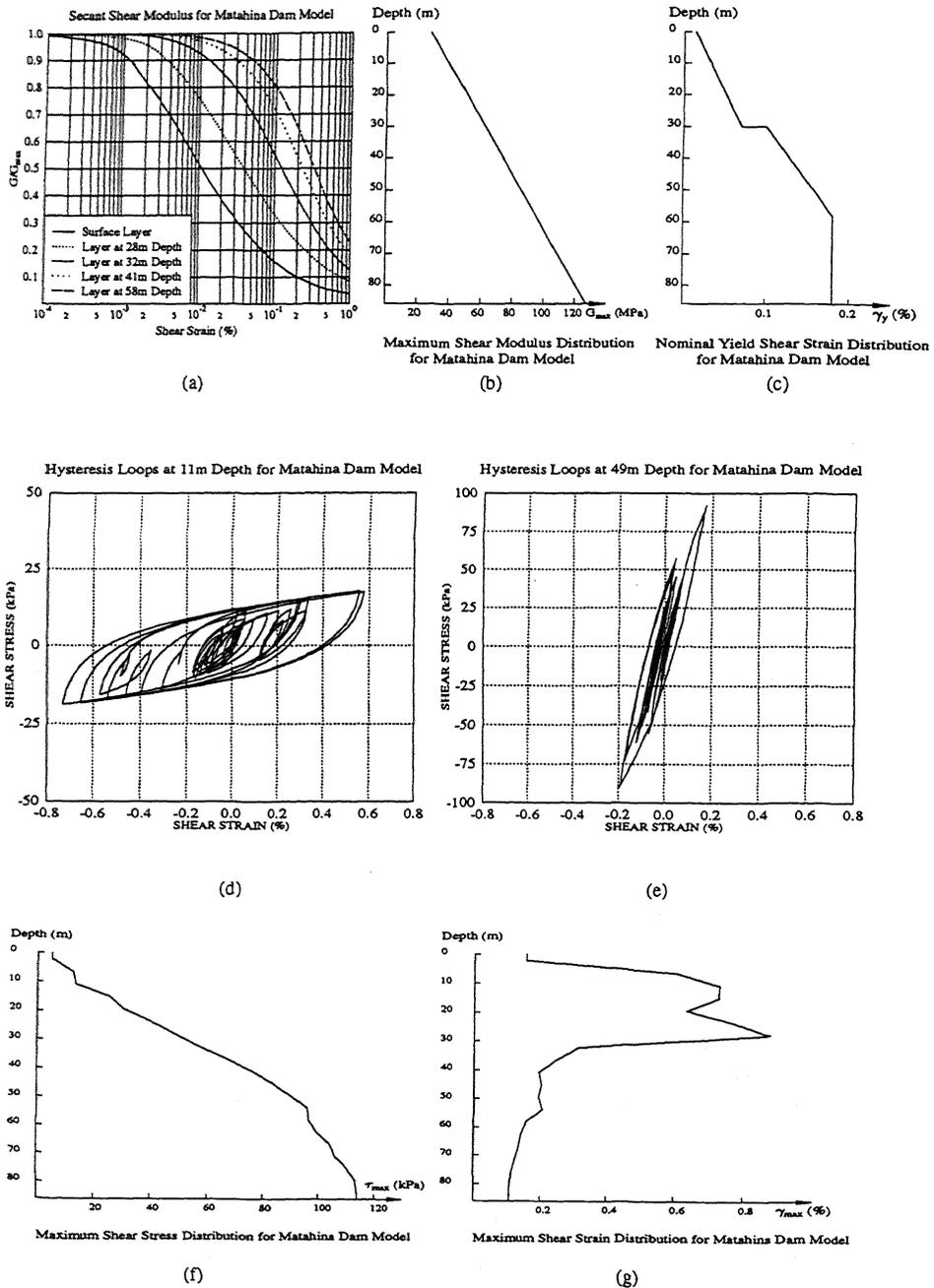


Figure 4. Parameters and results of the nonlinear model study.

basis of linear models. Nonlinear models with much reduced shear strengths near the crest than deeper in the dam explained the initiation of severe softening near the top of the dam. This allowed acceleration and deformation distributions which gave the observed concentration of deformation within the top third of the

dam, and stronger accelerations at 2/3 height than on the crest of the dam. These strong accelerations were associated with the 2-3 Hz frequency band, modelled as corresponding to the $n=2$ and $n=3$ modes, in which there was little response at the crest of the dam.