

Probabilistic evaluation of the response of arch dam-reservoir systems to uncorrelated seismic excitation

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ABSTRACT: A probabilistic approach for the linear analysis of the seismic response of a 3D fluid-structure coupled system, discretized by Finite Elements, is reported. The seismic excitation acting non simultaneously along the foundation-structure boundary, is modelled as a non-stationary uniformly modulated process. An application to the seismic response of a real system, a concrete arch dam and its reservoir, is reported and discussed.

1 INTRODUCTION

Given the seismic characteristic of Italian territory, structural safety problems as a consequence of possible earthquakes are of utmost importance for ENEL (Italian Board of Electricity) which directly manages most of the largest electric power plants in Italy.

Recently ENEL recognized the opportunity to develop a probabilistic approach to the safety evaluation of the most important dam-reservoir systems and to this purpose a research program in cooperation with CISE and with the Department of Structural Mechanics of the Pavia University was defined.

As a first step a method for the analysis of a deterministic linear system, whose response randomness is due to the seismic input only, was set up; a finite elements model of a real dam-reservoir system was employed in the probabilistic approach.

2 THE FINITE ELEMENT SYSTEM MODEL

The equilibrium equations for dynamic analysis of three-dimensional systems taking into account fluid-structure interaction have been derived by assuming the following hypotheses:

- the structure has a linear elastic behaviour
- fluid velocity field is negligible so that the pressure field only is modelled
- sloshing effects are neglected

With these assumptions the fluid equilibrium is

described by the wave equation :

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad (1)$$

where p is the pressure field and c is the sound speed. The boundary surface S is subdivided (see Fig. 1) into four parts S_1, S_2, S_3, S_4 ; the conditions imposed on these surfaces are :

- free surface S_1 :

$$p|_{S_1} = 0.$$

- fluid-structure interface S_2 :

$$\frac{1}{\rho_f} \nabla p \cdot \mathbf{n}_2 |_{S_2} = -\ddot{\mathbf{u}}_{n_2}$$

where \mathbf{n}_i is the unit vector normal to S_i , ρ_f is the fluid density and $\ddot{\mathbf{u}}_{n_2}$ is the structure boundary surface acceleration,

- bottom surface S_3 :

$$\frac{1}{\rho_f} \nabla p \cdot \mathbf{n}_3 |_{S_3} = -\ddot{\mathbf{u}}_{gn_3}$$

where $\ddot{\mathbf{u}}_{gn_3}$ is the bottom surface acceleration vector normal to surface S_3 .

- upstream surface S_i :

$$\nabla p \cdot \mathbf{n}_i|_{S_i} = -\frac{1}{c} \frac{\partial p}{\partial t}$$

Using Galerkin method for the fluid domain and a classical formulation (virtual work based) for the structure, the equilibrium equations take the form :

$$\mathbf{k}_s \mathbf{d} + \mathbf{C}_s \dot{\mathbf{d}} + \mathbf{M}_s \ddot{\mathbf{d}} - \mathbf{H}^T \mathbf{p} = \mathbf{f} \quad (2a)$$

$$\mathbf{K} \mathbf{p} + \frac{1}{c} \mathbf{C} \dot{\mathbf{p}} + \frac{1}{c^2} \mathbf{M} \ddot{\mathbf{p}} + \rho_f \mathbf{H} \ddot{\mathbf{d}} = \mathbf{g} \quad (2b)$$

where \mathbf{d} is the displacement vector and a dot means time derivation.

The solution of the coupled problem was obtained by modal superposition technique implemented in the computer code INDIA-3 developed at CISE, Applied Mathematical Section (1986).

3 PROBABILISTIC MODELING OF THE SEISMIC GROUND MOTION

Seismic ground motion has non-stationary characteristics in time and in frequency; the representation of these processes proposed by Priestley (1967) was adopted:

$$a_g(t) = \int_{-\infty}^{+\infty} A(t, \omega) e^{i\omega t} dZ(\omega) \quad (3)$$

where $A(t, \omega)$ is a deterministic function of time t and frequency ω , $dZ(\omega)$ is a stationary orthogonal process. If the time evolution of the frequency content can be assumed as negligible, the input excitation process takes the form:

$$a_g(t) = A(t) a_g^*(t)$$

where $a_g^*(t)$ is a stationary process. This expression, usually used in seismic motion modeling, was assumed in this work. Several expressions have been proposed in literature for $A(t)$ function (see Fig. 2). Spectral density function was described by the Kanay-Tajimi spectrum:

$$G_g^*(\omega) = G_0 \frac{1 + 4\xi_g^2(\omega/\omega_g)^2}{(1 - (\omega/\omega_g)^2)^2 + 4\xi_g^2(\omega/\omega_g)^2} \quad (5)$$

where ω_g , ξ_g , G_0 are parameters to be defined according to the local site conditions.

The spatial variation of the seismic motion, accounted for subdividing the structure-soil boundary into a five zones (see Fig. 3), gives rise to an excitation vector and a matrix of PSDF $\mathbf{G}_g^*(\omega)$.

4 SOLUTION OF THE DYNAMIC PROBLEM

Any quantity linearly dependent on the excitation, like displacement u and stresses σ , can be expressed by:

$$u(t) = \sum_{j=1}^{n_s} u^{(j)}(t) \quad (6)$$

where n_s is the number of acting seismic actions and $u^{(j)}(t)$ is the displacement response to excitation j , computed via modal superposition method:

$$u^{(j)}(t) = \sum_{k=1}^n \eta_k q_k^{(j)}(t) \quad (7)$$

where η_k is the k -th system eigenvector and $q_k^{(j)}(t)$ is the k -th normal coordinate, solution of the following equation :

$$q_k^{(j)}(t) = f_k^{(j)} \int_0^{+\infty} h_k(\tau) a_g^{(j)}(t-\tau) d\tau \quad (8)$$

where ν_k , ω_k are the k -th modal damping and frequency, $f_k^{(j)}$ is the k -th mode j -th earthquake excitation factor and $u_g^{(j)}(t)$ is the j -th ground motion. Substituting eqs. (3), (8) into eq. (7) finally we have:

$$u^{(j)}(t) = \int_{-\infty}^{+\infty} M^{(j)}(t, \omega) e^{i\omega t} d\hat{a}_g^{(j)}(\omega)$$

where

$$M^{(j)}(t, \omega) = \sum_{k=1}^n \eta_k q_k^{(j)} \int_0^t h_k(\tau) A^{(j)}(t, \omega) e^{-i\omega\tau} d\tau \quad (9)$$

$h_k(t)$ is the impulse response function of mode k and

$$E[\hat{d}a_g^{(i)}(\omega_1)\hat{d}a_g^{(l)}(\omega_2)] = \begin{cases} 0 & \text{se } \omega_1 \neq \omega_2 \quad \forall i, l \\ G_g^s(\omega)d\omega & \text{se } \omega_1 = \omega_2 = \omega \end{cases}$$

and the PSDF of displacement component u can be expressed as :

$$G_u(t, \omega) = m^T G_g^s(\omega) m \quad (10)$$

where

$$m^T = [M^{(1)}(t, \omega), \dots, M^{(n)}(t, \omega)]$$

A structural safety index (Vanmarcke 1975) can be evaluated with the so-called reliability function

$$L(t, \alpha) = e^{-\int_0^t v(\alpha, \tau) d\tau} \quad (11)$$

that is the probability that the absolute value of a certain quantity u does not exceed a fixed threshold in the time interval 0-t. In other words:

$$L(t, \alpha) = \text{Prob } E[|u| < \alpha] \text{ in } 0-t \quad (12)$$

where

$$v(t, \alpha) = \frac{2v_a(1 - e^{-\sqrt{\pi/2}\delta t})}{(1 - e^{-t/2})}$$

denotes the expected value of outcrossing rate of the threshold at time t and

$$\delta = \sqrt{1 - \lambda_1^2(t)/\lambda_0(t)\lambda_2(t)}$$

$$r = \alpha/\sigma_u$$

$$v_a = (1/2\pi)\sqrt{\lambda_2(t)/\lambda_0(t)}e^{-r^2/2}$$

λ_i are the spectral moments of the PSDF $G(\omega, t)$, σ_u is the Root Mean Square (RMS) of the displacement u. In an alternative definition, best suited in some cases, $L_u(t, \alpha)$ is the probability of the variable u of not exceeding the band of amplitude $\pm \alpha$ at a certain time t; in the following we refer to this definition of the reliability function.

6 APPLICATION

The first application of the methodology concerns a dam-reservoir system located in the north-eastern part of Italy assuming the two limit conditions of the reservoir: empty or completely full.

The finite element mesh of the dam, consisting of 152 parabolic brick elements, and the fluid discretization, consisting of 576 parabolic elements are shown in Fig(1). The total size of the resulting finite element system was 5500.

Modal analysis of the coupled system was carried out with INDIA-3 code in the two limit reservoir conditions; for the empty reservoir condition the first 10 natural frequencies of the system were found in the 5-17(Hz) range while for the full reservoir situation (compressibility of the water included) the resulting range was 4-12 (Hz).

The parameters of the Kanai-Tajimi earthquake spectrum, chosen in order to best fit the six strongest italian seismic events (Casciati 1990), are as follows:

$$\omega_g = 3.61 \text{ [Hz]}$$

$$\xi_g = 0.267$$

$$G_0 = 174.0 \text{ [cm}^2/\text{sec}^3]$$

Window (b) of Fig.(2) with a duration t of 10 sec., was chosen to model the time evolution of the seismic energy.

Evolutionary PSDF of the displacements was obtained for empty and full reservoir conditions; for each of these hypothesis the the system response to an uniform and full uncorrelated multisupport excitation was computed. Time evolution of RMS of the upstream-downstream components of two nodes 121 and 131 (see Fig.(3)) in different excitation (single and multiple) and reservoir conditions (empty and full) are reported in Fig.4-5 respectively. From these two figures we can see that in the case of full reservoir and single excitation the values of RMS are about 2÷3 times with respect to the corresponding values in the empty reservoir condition. On the other hand multiple support excitation give not rise to analogous amplification of the response; in fact the response of node 121 (which is in the middle of the arch) is lower than in the case of single excitation (curve (b) with respect (a) of Fig.4) while the response of node 131 is not (curve(a) with respect to (b) of Fig.5). This is due to different mode of vibration involved in the response of the system by the spatial distribution of the excitation. These differences in RMS values give rise to differences in the reliability function of the same variables previously mentioned. In Fig.6-7 the reliability function at time t=2.5, 5.0, 10. seconds are reported. Also in this case the response of node 121 shows that multiple support excitation give rise to a

more safe condition (with respect to the single excitation case) while response of node 131 is less safe in this condition.

7 CONCLUSIONS

This work is a part of a long term research program sponsored by ENEL-CRIS. First step of this research program was to implement and validate a software module for random vibration analysis of 3D coupled fluid-structure systems. However in view of a systematic application of the developed methodology it is necessary to investigate about limit states of these coupled systems.

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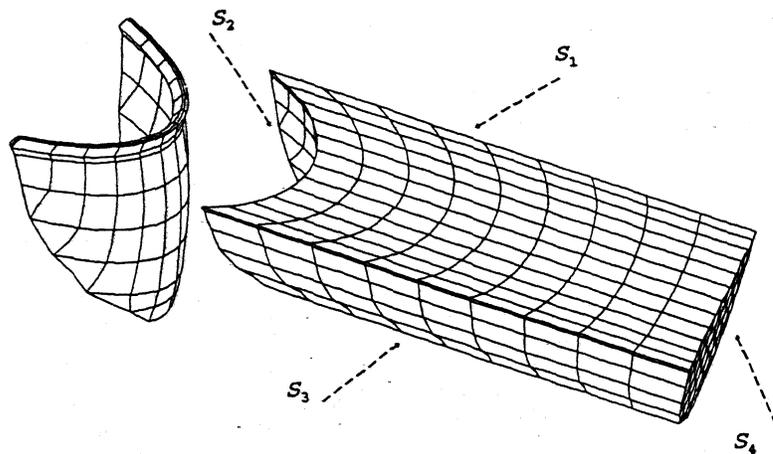


Fig.1 Dam-reservoir system finite element model

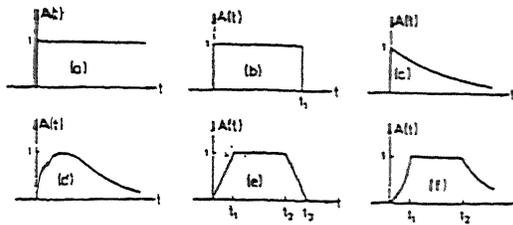


Fig.2 Time modulating functions

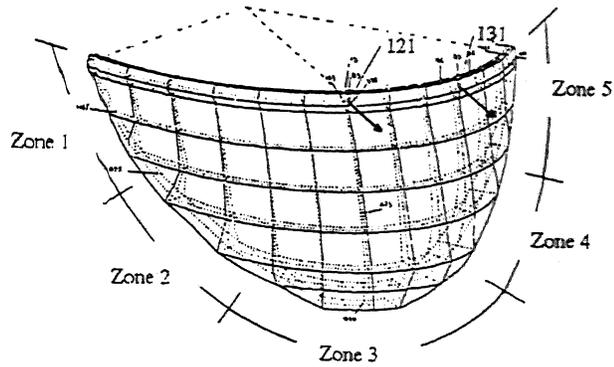


Fig.3 Spatial distribution of the seismic excitation

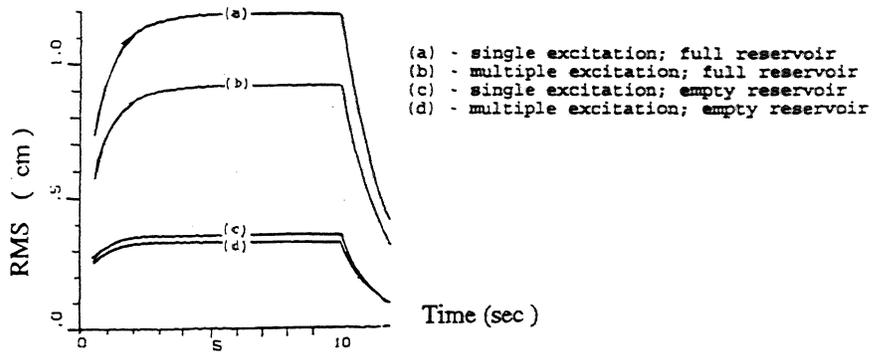


Fig.4 Time evolution of RMS of the upstream-downstream component of node 121

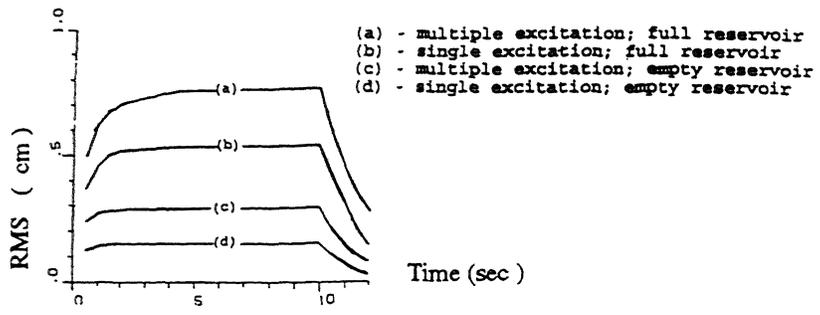


Fig.5 Time evolution of RMS of the upstream-downstream component of node 131

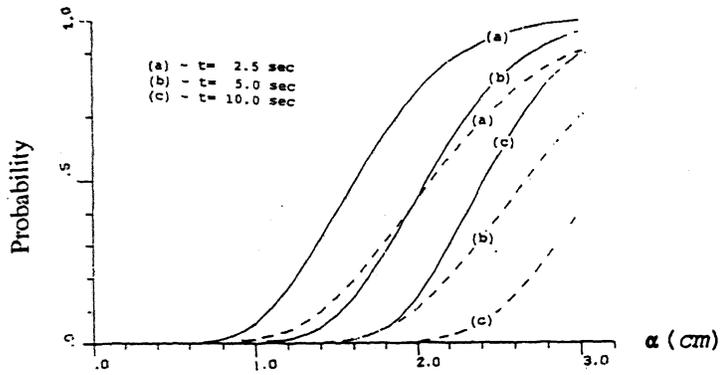


Fig.6 Reliability function of the upstream-downstream component of node 121

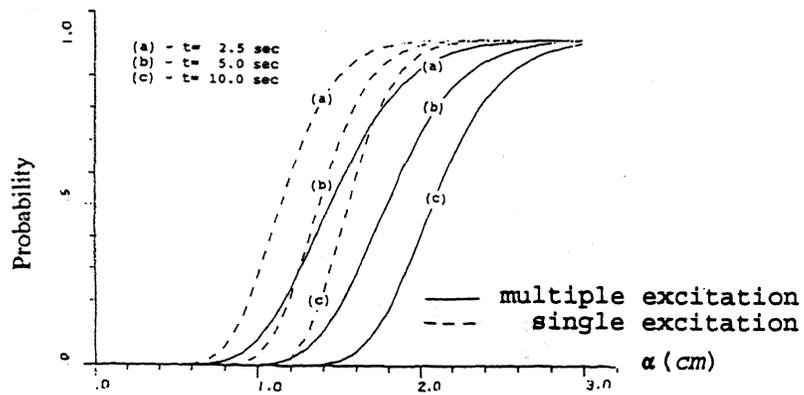


Fig.7 Reliability function of the upstream-downstream component of node 131