

Study of the vibratory characteristics of unanchored cylindrical liquid storage tank models

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ABSTRACT: This paper examines the behavior of ground supported metal cylindrical model tanks, which are based on a relatively stiff foundation and are left unanchored at the base, when these models are subjected to lateral loads. The first part of the paper studies the behavior at the base of model tanks when they are subjected to static lateral loads introduced by the hydrostatic pressures, which are generated by a static-tilt test experimental arrangement. Results obtained from such tests employing model metal tanks are presented in a summary form and discussed. Summary results are also presented from a correlation between these experimental results and predictions made by either a numerical solution or by certain simplified approaches. The second part of this paper investigates the dynamic behavior of one such model when it is subjected to lateral loads introduced by the hydrodynamic pressures which are generated on the tank wall from the horizontal base motions produced by an earthquake simulator. Again some of the results obtained from a variety of horizontal base motions are presented and discussed together with observations related to the parameters that seem to influence the dynamic characteristics of the studied model structure.

1 INTRODUCTION

During recent decades almost every strong earthquake has caused serious damage to ground supported cylindrical metal tanks. Figure 1 depicts an oil spil from the rupture of the bottom plate of an oil tank during the Coalinga 1983 earthquake (ref. 6) whereas figure 2 depicts the damage to the top and bottom of a wine tank during the San Juan Argentina 1977 earthquake (ref. 7).

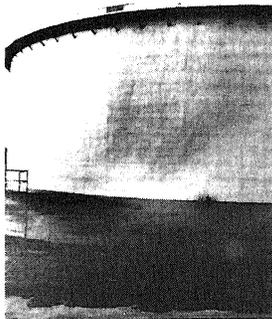


Fig. 1. Oil Spil

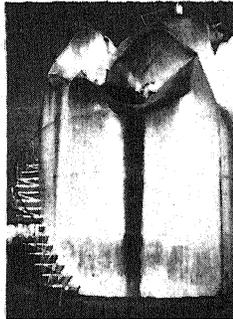


Fig. 2. Wine Tank Damage

A major reason that these structures are susceptible to earthquake damage is that they are designed to resist the primary hydrostatic loads efficiently. Thus, unless special attention is given to the seismic hazard, there is little extra capacity in the tank walls to resist the total different type of loading induced by the earthquake. During recent years a large amount of research has been done on the earthquake behavior of ground supported cylindrical tanks. Analytical procedures are now available to predict the stresses and deflections of tanks with fixed bases subjected to specific horizontal or vertical earthquake motions (refs. 12,13,16). These procedures can take into account of the flexibility of the

tank wall and even of initial geometric imperfections from the specified cylindrical form (Zui, 1987). Recent reasearch work made it possible to address the soil structure interaction effects for laterally or vertically excited tanks with fixed bases (refs. 17,18,20). However, the much more complicated behavior of unanchored tanks is not yet fully amenable to analytical treatment. Studies by ishida and Kobayashi (1988), Natsiavas (1989), Peek (1988), Manos and Talaslidis (1988) Rammerstorfer et.al. (1988), deal with certain aspects of the dynamic behavior of unanchored tanks or of the state of stress near the base from numerically analysing static "equivalent" simulations of the problem at hand.

This paper present in its first part experimental and numerical results from the study of static "equivalent" simulations of unachored tank earthquake response whereas at the second part experimental results are presented that aim to clarifying the dynamic response of unanchored tanks when subjected to a variety of horizontal ground motions.

2. BASE UPLIFT MECHANISM STATIC-TILT TEST SEQUENCE

Ground supported liquid storage tanks can either be anchored to the supporting foundation or else can be left unanchored having their base simply resting on the underlying supporting area. It is unrealistic to attempt to anchor structures that are larger than a certain size. In this case the hydrostatic forces are mainly transferred to the supporting media through the tank's bottom plate, which is usually in full contact with the foundation. However the tank base partially loses contact with the supporting foundation (base uplift) when the overturning forces generated from the hydrodynamic earthquake induced pressures exceed a certain limit. Overturning in this case is resisted by compression forces concentrated on one side of the shell as well as from resisting forces that develop at the uplifted part of the base plate. This is very clearly demonstrated by a loading arrangement known as the static-tilt test that simulates in a static way the earthquake induced overturning forces.

Figure 3 depicts a model tank which is placed on a rotating platform located at the Laboratory of Strength of Materials of Aristotle University. From the tilting of the base of the tank, with respect to the horizontal, non-symmetric distribution of hydrostatic pressures is introduced on the tank wall that results in lateral loads resembling to a certain extent lateral loads arising from actual horizontal base motions. A number of researchers have used in the past this type of experimental arrangement because it offers the advantage that it can be performed relatively easy and it can provide results that can be used to check the approximation of analytical or numerical methods addressing the same phenomenon (refs. 1,3,5,11,14).

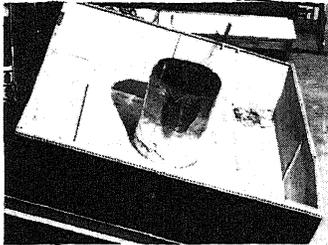


Fig. 3. Static Tilt Test of the Thessaloniki Model Tank

In the present test sequence a number of models have been constructed and tested. The results presented here were obtained with a model made of thin metal sheets (bronze alloy) with dimensions 0.3mx0.6m. Six of these metal sheets were used to form the tank wall and two for the bottom plate. The model is 0.5 in diameter and 0.6m high; it was filled with water during the test sequence at a depth of 0.5 m. Instrumentation was provided to measure the radial displacements at mid-height and top-rim cross sections of the structure by eight gages equally spaced around the tank's circumference. In addition, the uplift behavior of the base of the tank was also recorded during the tilting sequence by measuring the base uplift displacements at the tank's circumference on the tilt axis as well as at points located at 45 degrees either side from the tilt axis. In addition, a filler gage was used to measure the extent to which the bottom plate of the model tank is separated from the foundation and is indicated with the term "Uplift Penetration". Strain gages were attached both at the inside as well as the outside surfaces of the tank wall 10mm from the base on the excitation axis (NORTH) in order to record the axial (vertical) as well as the hoop (circumferential) strains that developed at this point during the tilting sequence.

2.1 Continuous Tilting

During this test the model tank with a liquid depth equal to 50cm was tilted with a constant speed from rest (0 degrees tilt) to 13 degrees tilt and back to rest recording continuously the strain response at North as well as the uplift displacement at the opposite side of the tank base on the excitation axis (South). This is shown in figure 4; the positive strains indicate extension and the positive uplift upward displacement of the base of tank from the foundation. It can be seen from this figure that the development of the bottom uplift at South does not increase linearly with the overturning moment, which varies almost linearly for relatively small tilt angles. Moreover, it can be seen that the nonlinearity is even more pronounced for the shell strains at North (near the base) because of the concentration of the compressive zone in a rapidly narrowing area, due to the spreading of the uplift. This is a phenomenon also observed and discussed in previous experimental works (refs. 1,2,3,4,14).

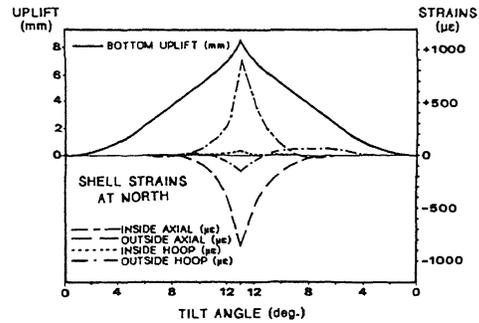


Fig. 4 Uplift Displacement and Shell Strains for continuous Tilting

2.2 Tilting at Distinct Angles

During this test sequence the tilt angle was introduced at four distinct stages with 4 degrees tilt angle steps from rest (0 degrees tilt) to 16 degrees. At each one of these distinct stages measurements of the radial and uplift displacement response were recorded simultaneously together with the recording of the "Uplift Penetration" as already described. The response measured in this way is plotted in figures 5 to 6. For all stages the plotted quantities represent the measurements from an initial reference condition which is the tank full of water (50cm depth) and resting on the rigid foundation with zero tilt. Figure 5 depicts the circumferential distribution of the measured radial displacement response at the top-rim section of the tank wall. This displacement response is plotted with dotted lines marked with special symbols to signify the four distinct tilt stages and it represents the magnified actual displacement response plotted as it physically occurs; the solid line is representing the undeformed cross-section prior to the tilting sequence. The response plotted in this figure has been normalized with respect to the tank radius, liquid height and shell thickness so that a non-dimensional parameter is finally obtained according to the following relationship:

$$\text{non-dimensional response} = \frac{\text{measured response} \cdot t}{R \cdot H}$$

t = shell thickness, R = tank radius, H = liquid height

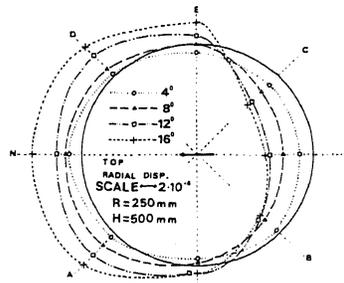


Fig. 5 Top Rim Displacement

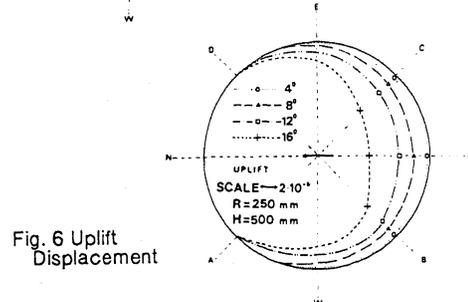


Fig. 6 Uplift Displacement

The tank bottom uplift is similarly normalized and plotted as shown in figure 5 for the four distinct tilt stages. In this case the solid line represents the intersection of the tank wall and the bottom plate whereas the dotted lines represent the magnified non-dimensional uplift displacement plotted inwards from the solid line.

The following summarise the most important observations :

-The uplift mechanism dominates the radial displacement response by introducing considerable rotations of the model structure at the base.

-The uplift displacement and "Uplift Penetration" vary nonlinearly with the tilting process. The uplift of the base exhibits initially a tendency to propagate in the circumferential rather than in the radial direction whereas this tendency is reversed at the later stages of the tilting sequence.

2.3. Study of the Uplift Behavior

A numerical study was performed using the finite element method together with an iterative approach, which attempted to simulate the behavior of a "broad" aluminium tank, which was examined during a tilting sequence performed at the Earthquake Engineering Research Center of the University of California at Berkeley, similar to the one described in 2.2, performed in Thessaloniki. The findings of this study and the correlation between numerical and measured results has been reported previously (ref. 9) and will be briefly repeated here. Figures 7 and 8 present a comparison between measured and predicted values for the base uplift displacement response when the tilt angle was 16 degrees whereas figure 9 depicts the axial membrane stress response for a horizontal section of the tank wall 25.4 mm from the bottom during the same tilt angle loading stage. As can be seen, there is reasonably good agreement for the base uplift displacement response whereas there is certain discrepancy for the stress response.

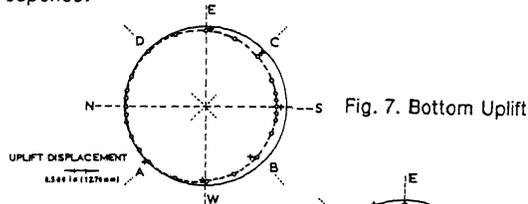


Fig. 7. Bottom Uplift

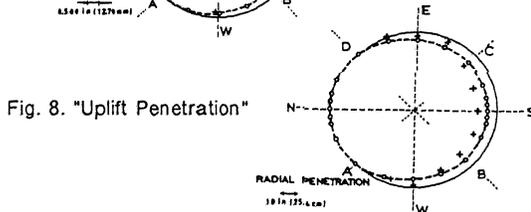


Fig. 8. "Uplift Penetration"

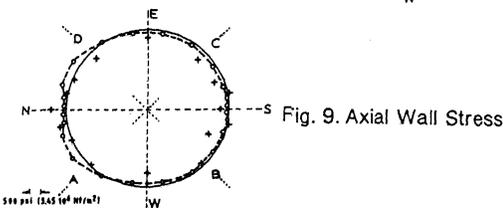


Fig. 9. Axial Wall Stress

In addition to the above numerical simulation the observed by the Thessaloniki tilting sequence base uplift behavior is compared in what follows with results obtained from two simplified approaches that try to describe the uplift phenomenon. The first approach, proposed by D. Clough (ref. 2), predicts the uplifted part of the bottom plate for a given overturning moment, without however deriving the maximum vertical (uplift)

displacement, whereas in the second approach, proposed by Ishida and Kobayashi (ref. 10), both the uplifted bottom plate area ("Uplift Penetration") as well as the maximum uplift displacement are derived. Figure 10 depicts the correlation between observed and predicted peak uplift response for the Thessaloniki tank for all the tilting sequence from 4 to 16 degrees.

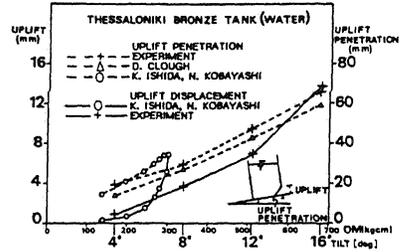


Fig. 10 Correlation for Uplift Response (4-16 deg. Tilt)

As can be seen from figure 10 there are certain discrepancies in the distribution of the "Uplift Penetration" between observed response and the one predicted by both approaches. Moreover, it must be noted that the second approach reaches a limit stage for a tilt angle equal to 8 degrees, whereby no further increase of the resisting overturning moment is possible by a further increase of the base uplift, thus indicating overturning of the model for this tilt angle.

3. EARTHQUAKE SIMULATOR TESTS TANK-WALL RESPONSE

Figure 11 shows the model tank, the same as the one used during the tilt-test sequence, being placed on the moving platform of the Earthquake Simulator of Aristotle University, which is capable of one horizontal direction of motion. Very light accelerometers (11 grams) were used to measure the tank wall acceleration response. Their signal was amplified and recorded in real time by a Dual Spectrum Analyzer whereby the various types of signal analyses were performed in the time as well as in the frequency domain.

The tests performed on the Earthquake Simulator can be divided into the following groups, although the objective of all the tests was basically the same, namely to investigate the dynamic characteristics of the liquid-filled model tank supported on the rigid platform of the Earthquake Simulator with its base left unanchored. The following outlines some of the details for each group of tests and gives the summary of the observed behaviour.

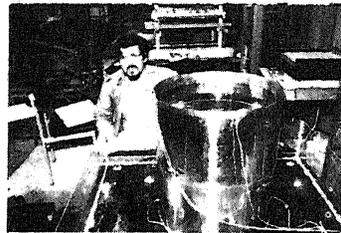


Fig. 11. Thessaloniki Model Tank on the Earthquake Simulator of Aristotle University

3.1 Sinusoidal Base Motion Tests

a) Low amplitude sinusoidal base excitation tests were performed with constant peak base displacement equal to 0.1mm and frequency of the base motion varying from

test to test in the range of 1Hz to 45Hz with steps of $\Delta f=0.25\text{Hz}$; the frequency step Δf was lowered (0.125Hz) for frequencies where distinct amplification of the measured response was observed.

During these tests the acceleration response was recorded at mid-height at the point of the excitation axis (NORTH). The amplification of the acceleration response is more predominant at two frequency ranges; the first one is at relatively high frequencies (peak at 37.5Hz), whereas the second is at relatively low frequencies (peak at 8.375Hz). The fundamental frequency of the fully anchored tank as derived by applying approximate procedures that take into account the flexibility of the tank wall was found to have values from 42.6Hz to 55.3Hz (Refs. 12,13,16). Thus it can be assumed that the relatively high frequency range where the observed response exhibits amplification it must be attributable to this mode of anchored tank response. However, the amplification that is observed for lower frequency values must be attributed to the uplift mechanism that significantly influences the observed behaviour.

b) A series of low amplitude tests with constant peak base displacement equal to 0.1mm and constant frequency equal to 8.375Hz were also performed. During these tests the acceleration was measured at the mid-height section of the model tank at 16 points equally spaced around the circumference of the tank wall at this cross section with angular intervals equal to 22.5° , including the point at the excitation axis (NORTH). By comparing the acceleration response measured at each one of the sixteen points at mid-height with that of the base motion the "frequency response function" magnitude was derived for each point for this type of base motion, which, as shown by the previous group of resonance tests, was the one where the maximum amplification was observed at this low frequency range. These magnitude values for each point were modified, using the value of the phase angle, in such a way that the obtained response represents concurrent amplitudes occurring simultaneously at a time when the acceleration response at North over that of the base is maximized. This is shown schematically by figure 12.

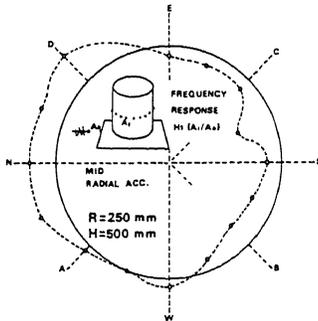


Fig. 12 Acceleration Response at Mid-Height

c) In order to further investigate the response at mid-height of the model tank for the sinusoidal horizontal base motion with frequency 8.375Hz the following additional study was performed.

The acceleration response was measured at North and at one of the other fifteen points simultaneously; this was done till all 15 pairs (North and each of the other 15 points) were completed. For each one of these tests the acceleration response was sampled and analysed in the frequency domain by means of the Fourier Spectrum. In order to study the displacement rather than the acceleration response, because the former would be more representative of the expected modal contributions, the Fourier Spectrum for the acceleration response was transformed to a Fourier Spectrum corresponding to the displacement response for each of the sixteen points at mid-height. Because the accelerometers were attached on the shell of the model tank with their axis coinciding with the radial direction at

this point it is obvious that the displacement response Fourier Spectrum, obtained in this way, corresponds to the "radial" displacement for each one of the sixteen points at mid-height. This Fourier spectrum displacement response for each one of the sixteen points at mid-height was studied in terms of frequency and amplitude (RMS). At the end of each test by comparing the displacement response Fourier Spectrum of one of the 15 points with that of the North (both of them recorded simultaneously) the phase angle was also recorded for these spectral components that corresponded to the six dominant frequency values, namely 2.0Hz, 3.125Hz, 4.25Hz, 5.25Hz, 6.25Hz, 8.375Hz. This response corresponds to certain "modes", with modal frequencies the ones already mentioned, and it is plotted with circles in figures 13a to 13f for each one of the sixteen points. Plotted together is a $\cos\theta$ type distribution (dashed line) that best approximates the displacement response recorded in the way described. At each one of these plots the frequency that these "modal" displacement contributions correspond to is also indicated. It must be remembered that they all represent displacement response at mid-height for base sinusoidal excitation at 8.375Hz. The following summarize the main observations :

- The modal contributions correspond to harmonics (8.375Hz) as well as subharmonic (5.125) rocking response ($\cos\theta$).

- The ovaling mode is also excited ($\cos 2\theta$) with considerable contribution at a frequency well below the excitation frequency (3.125Hz).

- There are also contributions in the $\cos 3\theta$ in two subharmonic frequencies (2.0Hz and 4.25Hz) as well as a $\cos 4\theta$ contribution for 6.25Hz.

In order to obtain the degree of the dominance of each one of the circumferential Fourier modes ($\cos n\theta$) for each one of the displacement response depicted in figures 13a to 13f this response was numerically treated by applying a Fourier decomposition approach based on the following relationship .

$$R(\theta) = \sum_{i=0}^8 A_i \cdot \cos(i\theta) + \sum_{j=1}^7 B_j \cdot \sin(j\theta)$$

where $R(\theta)$ is the measured response quantity as a function of the angular coordinate θ varying from 0 to 360 degrees.

and A_i, B_i = the Fourier Coefficients that describe the amplitude of the corresponding $\cos i\theta$ or $\sin i\theta$ circumferential displacement response.

The following are again the main observations:

- The approximation attempted by a $\cos n\theta$ distribution for plots 13a to 13f is quite successful thus indicating that the corresponding A_i term is the most predominant.

- The most "pure" contributions are these of the rocking A_1 subharmonic response ($\cos\theta$, 5.125Hz) and the ovaling A_2 subharmonic response ($\cos 2\theta$, 3.125Hz). It must be pointed out that these contributions are also the most significant as their amplitude is 0.67mm and 1.4mm respectively, for a sinusoidal base peak displacement of 0.1mm at 8.375Hz.

- The harmonic rocking A_1 contribution ($\cos\theta$, 8.375Hz) is both less "pure" and less significant (with $A_1=0.33\text{mm}$) than the corresponding subharmonic rocking contributions, ($\cos\theta$, 5.125Hz, $A_1=0.67\text{mm}$).

- The higher order contributions ($\cos 3\theta$, for 2Hz and 4.25Hz and $\cos 4\theta$ for 6.25Hz) are also both less "pure" and less significant than the rocking and ovaling response

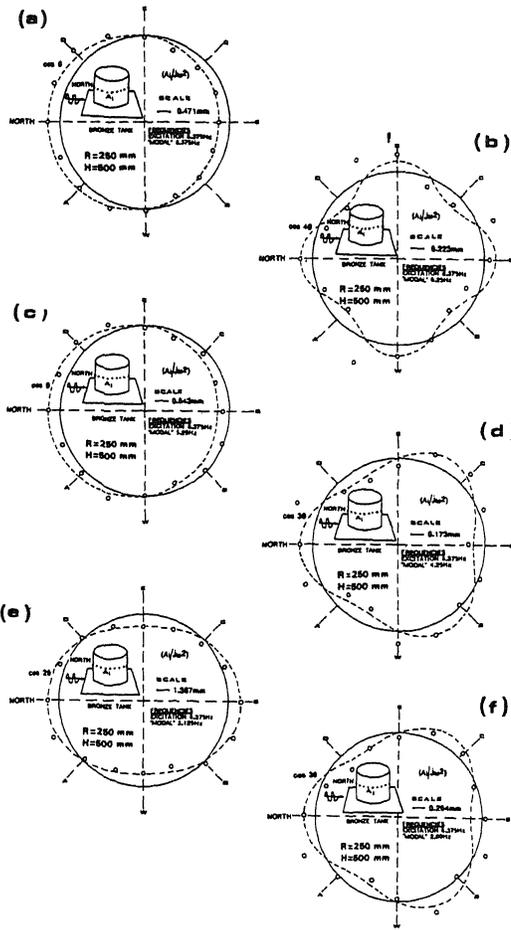


Fig. 13a-13f Modal Response at Mid-height of Tank Wall for Sinusoidal Horizontal Base Motion of 8.375Hz

3.2 Random White Noise Tests

Another group of tests was performed with base horizontal excitation a random white noise limited in the frequency range from 1Hz to 100Hz. A number of tests was performed in this case each with progressively increasing intensity in five distinct stages as described by the parameter SPAN (higher SPAN values indicate more intense tests). The acceleration response was recorded at North point of the mid-height cross-section and it was compared with the base acceleration in terms of "frequency response function" magnitudes.

The "frequency response function" magnitude obtained from these tests, which was derived from the acceleration response at NORTH as well as that at the base, is shown in figures 14a to 14e. It is interesting to confirm the findings of the two previous groups of tests this time as a function of intensity by the following observations:

-For low intensity tests the relatively high frequency range exhibits the most predominant amplification.

-As the intensity of the base motion is increased the frequency range of the most predominant amplification is continuously shifted to low frequency values. This must

again be attributed to the bottom uplift of the tank, a highly non linear phenomenon as was also shown by previous studies, that makes the total displacement response intensity dependent not only in amplitude but also in the dominant response frequencies.

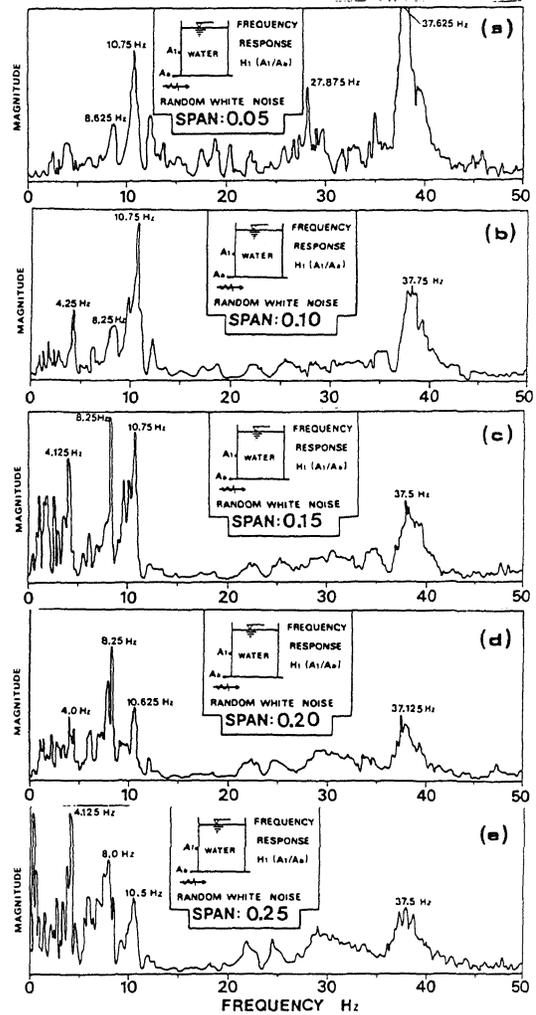


Fig. 14a-14e. Acceleration Response at Mid-Height

4. DISCUSSION OF RESULTS

Shaking table studies (refs. 1,2,3,4) have shown that the base fixity has a controlling influence on the dynamic behavior of cylindrical tanks. If the tank wall is rigidly attached to the foundation, which is typical of small "tall" tanks (height greater than the radius), and if influences arising from the deformability of the anchoring system as well as that of the foundation are ignored, then the response mechanism is relatively simple and rather easily predicted. On the other hand, if the tank is not anchored but merely resting on the foundation, which is typical of large capacity "broad" tanks (height less than radius) or if the deformability of the anchoring system will lead to significant displacements during strong ground motions, then the response is a very complex non-linear mechanism involving uplift at one side of the base as the inertial loads are directed towards the opposite side. The following summarize the most

important observations of the earthquake simulator test sequence presented above.

4.1 Tank-wall response

High order contributions to the tank wall displacement response have been attributed in the past for anchored tanks to imperfections of the circular shape of the horizontal cross sections as well as to irregularities of the anchoring arrangement in achieving perfect fixity conditions. Zui et. al. (1987) has investigated the Fourier coefficient contributions by both sinusoidal as well as earthquake excitation tests of an aluminium model tank 1600mm in diameter and 1400mm high supported either by a fixed base or free to uplift from a flexible foundation. Manos and Clough (1982) and Clough et.al. (1979) have also studied the response of relatively large aluminium models either "broad" or "tall" subjected to earthquake excitations. The "broad" tank model studied by Manos and Clough (1982) was supported by either a fixed base or by a base free to uplift from either a rigid or a flexible foundation. By studying the Fourier coefficient contributions during that research it was concluded that the uplift phenomenon of cylindrical tanks with relatively flexible wall and bottom plate structural members (as is the case for large metal liquid containers) introduces significant "rocking" and "ovaling" radial displacement response, that is also accompanied by the higher order Fourier coefficient contributions. As was shown by the present study this is also confirmed with the examined tank model behaviour where uplifting response is far from a pure rocking but also involves higher order modal contributions, as already described by Manos and Clough (1982, 1983). The recent study also examined the dynamic characteristics of those radial displacement modal contributions generated by the uplift mechanism. It is of interest to observe that the most significant of those contributions are subharmonic. They are not excited to a significant degree when the base excitation frequency coincides with their "modal" frequency but they are excited instead by an excitation frequency higher than these "modal" frequencies.

5. CONCLUSIONS

1. The studied static uplift behaviour showed that this non-linear mechanism when combined with a rigid base introduces a highly non-linear concentration of strains at the bottom of the tank accompanied with a displacement response that is dominated by this uplifting behaviour.
2. The procedure using the finite element formulation that was employed in simulating the base uplift behavior of the Berkeley "broad" tank yields reasonably good agreement for the displacement uplift response. However, there is a certain degree of discrepancy in simulating the tank wall stress response.
3. Simplified approaches that try to describe this static uplift behaviour yield results that exhibit discrepancies with the observed behaviour, especially for relatively large overturning moments (tilt-angles).
4. The dynamic characteristics of the liquid-tank system are also influenced significantly by the base uplift behaviour even for low levels of base motion, yielding significant response amplifications for frequency values well below the ones predicted for fully anchored tanks.
5. The observed acceleration and displacement response was shown to be intensity dependent in amplitude as well as in the frequency content. Larger intensity base motions cause a lowering of the dominant response frequencies. They also introduce higher order "modal" contributions that as was found vibrate in a subharmonic fashion.

REFERENCES

1. Clough, R.W., Niwa, A., and Clough, D.P. (1979) "Experimental Seismic Study of Cylindrical Tanks", Journal of the Structural Division, ASCE, Vol. 105, ST12, pp. 2565-2590.
2. Clough, D.P., (1977) "Experimental Evaluation of Seismic Design Methods for Broad Cylindrical Tanks", EERC Report No. 77/10, UBC, California.
3. Manos, G.C. and Clough R.W. (1982) "Further Study of the Earthquake Response of a Broad Cylindrical Tank Model", EERC Report No. 82/07, UBC, California.
4. Manos, G.C. and Clough, R.W. (1983), "The Measured and Predicted Shaking Table Response of a Broad Tank Model", ASME PVP, Vol.77, pp. 14-20.
5. Shih, Choon-Foo, (1981) "Failure of Liquid Storage Tanks Due to Earthquake Excitation", Thesis submitted to California Institute of Technology.
6. Manos, G.C. and Clough, R.W., (1985) "Tank Damage during the May 1983 Coalinga Earthquake", Earthq. Engin. Struct. Dynamics, Vol. 13, pp. 449-466.
7. Manos, G.C. (1991) "Evaluation of the Earthquake Performance of Anchored Wine Tanks during the San Juan, Argentina 1977 Earthquake, Earthq. Engin. Struct. Dynamics, Vol. 20, pp. 1099-1114.
8. Peek, R. (1988) "Analysis of Unanchored Liquid Storage Tanks under Lateral Loads", Earthq. Engin. Struct. Dynamics, Vol. 16, pp. 1087-1100.
9. Manos, G.C. and Talaslidis, D. (1988) "Response of Cylindrical Tanks Subjected to Lateral Loads", 9WCEE, Vol. VI, pp. 667-672, Tokyo-Kyoto, JAPAN.
10. Ishida, K., Kobayashi, N. (1988) "An Effective Method of Analysing Rocking Motion for Unanchored Tanks Including Uplift", J. PVP, Tran. ASME, Vol. 110, pp. 76-87.
11. Manos, G.C. (1989) "Study of Cylindrical Liquid Storage Tanks when Subjected to Lateral Loads", PVP Vol. 157, in sloshing and Fluid Structure Interaction, pp. 75-81, ASME, U.S.A.
12. Shibata, H. (1986) "Seismic Design of Industrial Facilities", ISBN-4-621-03079-5C3058, Maruzen Co. Ltd., Tokyo, Japan (In Japanese).
13. Veletsos, A.S. (1984) "Seismic Response and Design of Liquid Storage Tanks", Guidelines for the Seismic Design of Oil and Gas Pipeline Systems, ASCE ISBN 0-87262-248.5
14. Sakai F. et. al. (1988) "Experimental Study of Uplifting Behaviour of flat Based Liquid Storage Tanks without Anchors", 9WCEE, Vol. VI, pp. 649-654, Tokyo-Kyoto, JAPAN.
15. Zui H., et. al. (1987) "Experimental Studies on the Earthquake Response Behaviour of Cylindrical Tanks", J. PVP, Trans. ASME, VVol. 109.
16. Haroun, M. A. and Housner, G.W. (1981) "Seismic Design of Liquid Storage Tanks", J. Tech. Councils ASCE 107, pp. 191-207.
17. Haroun, M. and Abdel-Hafiz, E.A. (1986) "A Simplified Seismic Analysis of Rigid Base Liquid Storage Tanks under Vertical Excitation with Soil-Structure Interaction", Soil Dyn. Earthq. Engin., Vol. 5, No. 4, pp. 217.
18. Shimizu, N. (1989) "Seismic Design of Cylindrical Tanks on Elastic Soil", ASME PVP-JSME, Vol. 157, pp.83.
19. Natsiavas, S. (1989) "Simplified Model for the Dynamic Response of Tall Unanchored Liquid Storage Tanks", ASME PVP-JSME, Vol. 157, pp. 15-21.
20. Veletsos, A.S. and Tang, Y. (1990), "Soil-Structure Interaction Effects for Laterally excited Liquid Storage Tanks", Earthq. Engin. Struct. Dynamics, Vol. 1990, pp.473-496.
21. Zui, H. and Shimke, T. (1984) "Seismic Response Analysis of Cylindrical Tanks with Initial Irregularities on Side Walls", ASME PVP, Vol. 77.
22. Rammerstorfer, F.G., Fischer, F.D., Scharf, K. (1988), "A Proposal for the Earthquake Resistant Design-Results from the Austrian Research Project", 9WCEE, Vol. VI, pp. 715-720.