

## About the earthquake response of the flexible storage tanks

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**ABSTRACT:** The present paper treats the flexibility effect of structures on the earthquake response of the storage tanks. We have proposed a parametric parabola for the function of mode shapes, that takes into account in a better way the ratio between shear and moment. Several results regarding equivalent masses, maximum shear and moment are then given in the paper.

### 1. INTRODUCTION

The problem of earthquake response of the flexible storage tanks using the beam model was analysed by Veletsos and Yang [11]. The more sophisticated treatment of the tank as a shell structure was realised by Tani et al.[9]. The finite element approach has been applied to model the tank, (Haroun [6]) and to model both the tank and the fluid (Balendra et al.[1]). Fischer and Rammerstorfer [4] describe a fundamental concept of an iterative procedure for calculating the maximum dynamic pressure resulting from the common vibration of the elastic shell and the fluid. Rammerstorfer et al [8] have been developed a numerical algorithm using the "added mass concept" and the solution of the potential equation obtained for the specific boundary conditions corresponding to a liquid filled vibratory circular, cylindrical cantilever shell. Contributions have been brought by the others authors. The results obtained by means of quoted methods for usually tanks have been comparables.

### 2 ABOUT OF DYNAMICAL MODEL AND CONVECTIVE EFFECT

The hydrodynamic fluid pressure produced by a horizontal ground excitation which is exerted on the tank wall of a deformable tank is composed of three pressure contribu-

tions corresponding to the fundamental vibration modes [4][7][8][10][11]:

- $p_c$  - dynamically activated pressure produced by sloshing of the liquid,
- $p_{ir}$  - dynamically activated impulsive pressure produced by the horizontal rigid tank motion,
- $p_{if}$  - dynamically activated impulsive pressure produced by the interaction vibration of the structure tank and the liquid.

An equivalent dynamic model take into consideration the hydrodynamic effects by three masses  $m_c$ ,  $m_{ir}$ ,  $m_{if}$ , corresponding to  $p_c$ ,  $p_{ir}$ ,  $p_{if}$ , respectively (Figure 1).

The convective effect of the fluid is inexpressive influenced by flexibility of the tank. The values of attached masses  $m_c$ , are calculated by the relations (7):

$$m_c = 0,318 \frac{th 1,84 H/R}{H/R} m \quad H/R \leq 1,5 \quad (1)$$

$$m_c = 0,3154 \frac{R}{H} m \quad H/R > 1,5 \quad (2)$$

where H - height of the fluid in the tank,

R - tank radius,

m - total mass of the fluid.

The mass  $m_c$  is connected to the wall via a spring of stiffness  $k_c$  and represents the convective effect of the fluid. The spring stiffness  $k_c$  is

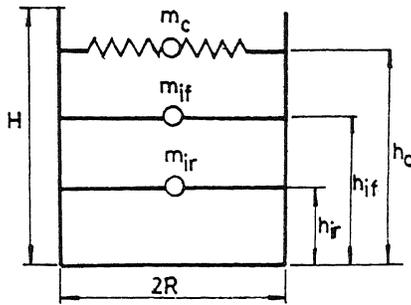


FIG.1. Dynamical model of flexible tank

such that the circular frequency of the vibrating fluid equals that of the spring-mass system given by

$$\omega_0 = \left[ \frac{1,84 g \operatorname{th} 1,84 \frac{H}{R}}{R} \right]^{1/2} \quad (3)$$

where  $g$  equals the acceleration due to gravity. The maximum forces exerted on the base of the tank by the mass  $m_0$  is,

$$P_0 = m_0 a_0 \quad (4)$$

where  $a_0$  = the maximum acceleration experienced by the sloshing mass, which may be found from the response spectrum of the earthquake by using the frequency given the equation (3). The maximum bending moment  $M_0$ , caused by the force  $P_0$  is:

$$M_0 = P_0 h_0 = m_0 a_0 h_0 \quad (5)$$

where for  $\frac{H}{R} < 1,5$

$$h_0 = H \left[ 1 - \frac{1}{1,84 \frac{H}{R} \operatorname{th} 1,84 \frac{H}{R}} + \frac{1}{1,84 \frac{H}{R} \operatorname{sh} 1,84 \frac{H}{R}} \right] \quad (a)$$

and for  $\frac{H}{R} > 1,5$  (6)

$$h_0 = H \left( 1 - \frac{0,409 R}{H} \right) \quad (b)$$

### 3 IMPULSIVE EFFECTS IN THE FLEXIBLE LIQUID STORAGE TANKS

We consider that the horizontal

acceleration is variable along the wall height [10]

$$a(x, t) = \psi(x) a(t) \quad (7)$$

where

$a(t)$  = the acceleration of the wall at the fluid level,

$\psi(x)$  = a dimensionless function defining the assumed variation of  $a(x, t)$  along the wall height. It follows that for  $x=H$ ,  $\psi(H)=1$ .

We shall assume that the deflection configuration on the tank at any time is prescribed by  $\psi(x)$ . The deflection mode depends on the relative magnitudes of flexural and shearing deformations of the fluid-filled tank during free vibration. These magnitudes depend on the ratios  $H/R$ ,  $R/t$ ,  $H/t$ , and on the relative weights of the roof system to the virtual mass of the contained fluid. For large  $H/R$ ,  $R/t$ ,  $H/t$ , and roof mass the mode  $\psi(x)$  will be more like a flexural type. On the contrary, for small  $H/R$ ,  $R/t$ ,  $H/t$  and roof mass, the mode  $\psi(x)$  will be more like a shear beam. For the deflection configurations of a flexible tank was suggested [10], [4] to take one of the following three forms (Figure 2).

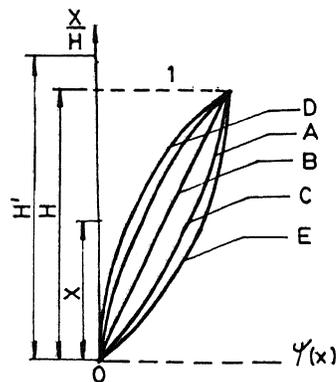


FIG.2. Mode shapes

$$A. \psi(x) = \sin \frac{\pi x}{2H}; \quad B. \psi(x) = \frac{x}{H};$$

$$C. \psi(x) = C \left( 1 - \cos \frac{\pi x}{2H} \right) \quad (8)$$

These forms for  $\psi(x)$  was used by others research workers which are

developed a fundamental concept of an iterative procedure for calculating the maximum dynamic response, resulting from the common vibration of the elastic shell and the fluid. The authors was extended the forms for  $\psi(x)$  with two parabolic function (Figure 2).

D. 
$$\psi(x) = \frac{x^2}{H^2}; \quad \text{E. } \psi(x) = -\frac{x^2}{H^2} + 2\frac{x}{H} \quad (9)$$

For large H/R, R/t, H/t ratios and large roof mass, the  $\psi(x)$  function will be closer to the assumed modes C and D representing flexural displacements. For small H/R, R/t, H/t ratios and small roof mass, the shear beam representation of  $\psi(x)$  in the forms A and D will be more realistic modelling of the displacements.

We consider for  $\psi(x)$  function a parametric parabola which containing D and E forms as particularly cases (Figure 3)

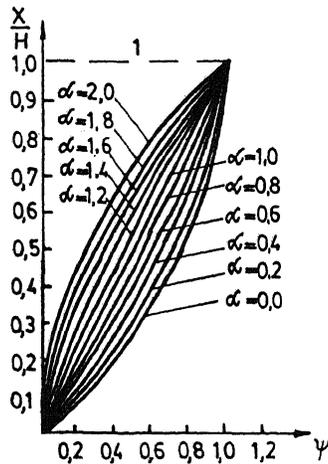


FIG.3. Parametric parabola of mode shapes

$$\psi(x) = (\alpha - 1) \frac{x^2}{H^2} + (2 - \alpha) \frac{x}{H} \quad (10)$$

where  $\alpha$  is the parameter and its values belong to the domain (0,2). For valuation the  $\alpha$ -parameter, may be used a coefficient r, defined as a function of shear and bending rigidity of the tank,

$$r = H \sqrt{GA^*/EI} \quad (11)$$

where  $A^*$  = shear area of cross-section of the tank,  $I$  = inertia of cross-section of the tank,  $E$  = Young modulus,  $G$  = shear modulus. For cylindrical steel tank this coefficient may be calculated by means of relation

$$r = 0,62 \frac{H}{R} \quad (12)$$

considering also the ratio 
$$\frac{\text{maximum radial displacement due to bending moment}}{\text{maximum radial displacement due to shear force}}$$

we have been obtained for  $\alpha$ -parameter following relations,

embedded wall of the base 
$$\alpha = 0,286 \frac{H}{R} + 0,857 \quad (13)$$

hinged wall of the base 
$$\alpha = -0,266 \frac{H}{R} + 1,133 \quad (14)$$

#### 4 THE HYDRODYNAMICS PRESSURES AND THE IMPULSE MASSES

For the determination of the hydrodynamic pressures have used the solutions proposed by Veletsos and Yang [11], respectively Fischer and Rammerstorfer [4], adopting like shape function for the vibration fundamental mode the parametric parabola. The Figure 4 indicates the hydrodynamic pressures corresponding to the mode shapes A, B, C, D, E and for the rigid tank. In the Figure 5 it presents the hydrodynamic pressures corresponding to the mode shapes generated from the parametric parabola, out of it results one good

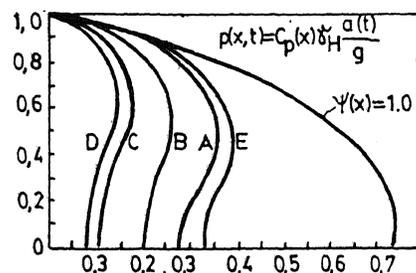


FIG.4. Hydrodynamic pressures for A,B,C,D,E, mode shapes

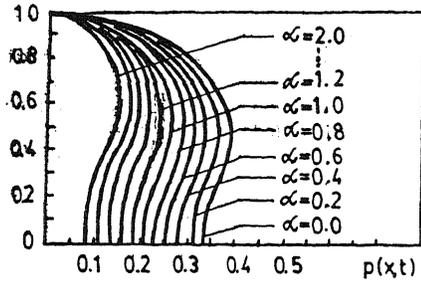


FIG.5. Hydrodynamic pressures for parabolic modes

covering of the real cases frequently met. Also through these diagrams it points out the effect of the shear, which enlarge the displacements.

Continuously it had determined the impulse masses  $m_{ir}$  corresponding to the rigid motion of the tank of the wall. For the tanks having  $H/R < 1,5$  it is possible to use, with one good approximation, for the determination of the masses  $m_{ir}$  and  $m_{if}$ , the relationships obtained by the authors, namely :

$$m_{ir} = (0,11 - 0,095\alpha) \frac{H}{R} m \quad (15)$$

$$m_{if} = (0,198 - 0,111 + 0,0174\alpha^2) \frac{H}{R} m \quad (16)$$

Also it was inferred the relationships for the impulse masses  $m_{ir}$ ,  $m_{if}$  of tank structure :

$$m_{ir}^i = \frac{1}{6} (4 - \alpha) m_w + m_a \quad (17)$$

$$m_{if}^i = \frac{16 - 7\alpha + \alpha^2}{30} m_w + m_a \quad (18)$$

with

$m_w$  - the mass of the tank wall

$m_a$  - the mass of the tank roof.

Knowing these masses it is possible to calculate the participation factor  $\xi_p$ , according with :

$$\xi_p = \frac{m_{ir}^i + m_{if}^i}{m_{ir}^i + m_{if}^i} \quad (19)$$

## 5 MAXIMUM BASE SHEAR AND MOMENT

Adding the hydrodynamic pressures it obtain the maximum shear produced by the liquid at the base of the wall,

$$Q_{ol} = \int_0^H \frac{H}{R} p(x,t) dx \quad (20)$$

The moment at the base of the wall produced by the impulse pressure is given by the relationship,

$$M_{ol} = \int_0^H \frac{H}{R} p(x,t) x dx \quad (21)$$

For the tanks with  $H/R < 1,5$  it give also the ultimate relations for the computation of the shear  $Q_{ol}$  and for the moment  $M_{ol}$ ,

$$Q_{ol} = (0,3118 - 0,0953\alpha) \frac{H}{R} \varepsilon_p S_a m \quad (22)$$

$$M_{ol} = (0,1412 - 0,0394\alpha) \frac{H}{R} \varepsilon_p S_a m \quad (23)$$

where  $S_a$  = the spectral acceleration determined from the acceleration spectrum depending on the fundamental vibration period of the system structure-liquid.

The base shear and momentum produced by the mass of the wall  $m_w$  and the mass of the roof  $m_a$ , noted with  $Q_{os}$ , respectively  $M_{os}$  are computed with the relations :

$$Q_{os} = (m_w + m_a) \varepsilon_p S_a \quad (24)$$

$$M_{os} = \left[ (5 - \alpha) \frac{m_w}{12} + m_a \right] H \varepsilon_p S_a \quad (25)$$

The total base shear  $Q_0$  and momentum,  $M_0$ , are given through the relations :

$$Q_0 = Q_{ol} + Q_{os} \quad (a) \quad (26)$$

$$M_0 = M_{ol} + M_{os} \quad (b)$$

It obtain for the tanks with  $\frac{H}{R} > 1,5$  relations more sophisticated, which moreover can be used for the total scale of the cylindrical tank.

## 6 CONCLUSIONS

The dynamic calculus model adopted for the first time by Veletsos [11], considering the tank like one system which are degree of freedom (SSDF) and which prescribed shape deformed represents one method relatively simple, used and checked up through many papers. In this paper it tries to bring some improvement to the determination of the real deformed shape. The use of the parametric parabola like shape function had intended this method to the tanks with various shapes, for which the computation is more near to the their real seismic behaviour. The parameter used considers the effect of shear and his percentage to the shape of the tank. The shear amplifies the displacements so that the consideration of the wall flexibility can be important also for the shorter tanks. Through the use of the parametric parabola to the description of the shape for the tanks which various geometries they are the possibilities of improvement of the solutions relatively simplified, which guide to the good results, lightly applicable in the designing and enough accurate.

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