STUDY ON FREQUENCY CHARACTERISTICS OF NON-LINEARITY OF STRESS AND STRAIN RELATIONSHIP IN SOIL DEPOSIT

Susumu NAKAMURA[1]

SUMMARY

The objective of this report is to make a model of stress and strain relationship for non-linear seismic response analysis in frequency domain. First of all, a new method based on Fourier transformation of hysteresis with respect to stress and strain in time domain was proposed to evaluate the relationship in frequency domain and was applied to the hysteresis calculated by seismic response analysis in time domain. Finally, a model of stress and strain relationship in frequency domain was proposed. As a results of these, it is found that a proposed method is useful to evaluate the stress and strain relationship in frequency domain and that the strain dependency is recognized in the deformation characteristics in frequency domain as well as that in time domain. Furthermore, it is made clear that the stress and strain relationship in frequency domain is able to be modeled to be similar with the stress and strain relationship in time domain.

INTRODUCTION

Evaluation of dynamic behavior of ground has been required for seismic design of civil engineering structures against strong earthquake motion so called as level 2 earthquake with revision of seismic design code based on serious damages based on the 1995 HYOGOKEN NANBU earthquake. In order to evaluate the behavior taking into account of strong non-linearity , not only seismic response analysis methods in time domain but also that in frequency domain have been used. However it has been pointed out the latter method is not available to evaluate the behavior under strong non-stationary condition caused by strong non-linearity of soil. Hence the follwing two processes correspond with the non-stationary condition. The first process is the change of shaking characteristics of ground with time due to the non-linearity of soil. The other one is the induced residual deformation with time. It is impossible to take into account of the latter non-stationary process for seismic response analysis method in frequency domain. However, it seems to be possible to consider the farmer process in the analysis method if the stress and strain relationship in frequency domain could be modeled under the stationary random process. In order to make the model, the stress and strain relationship in time domain has to be transformed to that in frequency domain under the stationaty random process. Furthermore, both relationship has to be associated with each other.

The objective of this report is to make a model of stress and strain relationship of soil in frequency domain as the stationary random process. First of all, a new method is proposed to transform the stress and strain relationship in time domain to that in frequency domain by use of Fourier transform technique. This method is applied to evaluate the frequency characteristics of the stress and strain relationship based on that obtained by non-linear seismic response analysis in time domain. Finally, the model of the stress and strain relationship in frequency domain was proposed.

[1] *Dept of Civil Engineering, College of Engineering, Nihon University, Koriyama, Japan Email:s-nak@civil.ce.nihon-u.ac.jp*

2.METHOD TO EVALUATE THE STRESS AND STRAIN RELATIONSHIP IN FREQUENCY DOMAIN

2.1 Comparison of Usual Stress and Strain Relationship in both Domains

As for discrete procedure of Navier's wave equation for time axis, incremental form has been used for the response analysis method in time domain. Therefore, the change of non-linearity of soil with time is considered by the analysis. On the other hand, wave equation has been formulated as matrix form based on multi-reflection theory at frequency. Therefore, non-linearity of soil has been considered as the equivalent linear method. In other words, the stress and strain relationship has been modeled as linear viscous elastic model. As a results of these, response of stress and strain have been obtained as non-stationary random process by seismic response analysis in time domain. However, those have been obtained as stationary process by the seismic response analysis in frequency domain.
According to the stochastic process, seismic responce analysis in frequency domain is very different from that in time domain due to the difference of discrete process of wave equetion.

2.2 Proposal of the Method to Evaluate the Stress-Strain Relationship in Frequency Domain

Hysteresis of stress and strain is specified on the coordinate system with respect to stress and strain axis which are perpendicular to each other. A point on the stress and strain plaine at time t is defined as the hysteresis vector from the origin as shown in Figure.1. Hence, the vector is able to be expressed as complex plain by equation(1). Fourier transformation of the complex time history as hysteresis vector is performed by use of the technique proposed by [Nakamura,1995]. The hysteresis of stress and strain at arbitrary circular frequency is obtained to be elliptic as shown in Figure.2. The shape is represented by the amplitude on the major and minor axis. Those amplitudes are expressed by equation(2). Hence, $P_{\tau}(\omega)$ and $P_{\gamma}(\omega)$ represent power spectrum of stress and strain, respectively. $K_{\tau\gamma}(\omega)$ represents co-spectrum. These spectrum are obtained by the treatment of Parzen window whose band width is 0.2Hz.
° This stress and strain relationship at arbitrary circular frequency is obtained by considering the non-stationary random process in time domain to be stationary random process and is defined as the stress and strain relationship in frequency domain.
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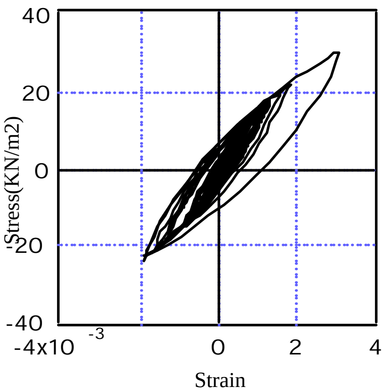


Figure.1 Definition of Hysteresis Vector

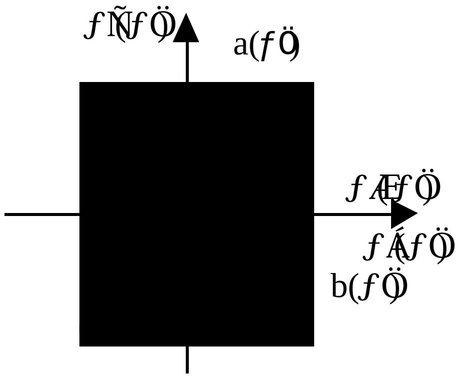


Figure.2 Hysteresis of Stress and Strain obtained by Fourer Transformation at Arbitary Circular Frequency

$$t_S(t) = \gamma(t) + \tau(t)i \qquad (1)$$

$$\left.\begin{aligned} a(\omega) &= 2\sqrt{\frac{2\pi A(\omega)}{T}} \ , \quad b(\omega) = 2\sqrt{\frac{2\pi B(\omega)}{T}} \\ A(\omega) &= \frac{P_{\tau}(\omega) + P_{\gamma}(\omega) + \sqrt{\{P_{\tau}(\omega) - P_{\gamma}(\omega)\}^2 + 4K_{\tau\gamma}(\omega)^2}}{2} \\ B(\omega) &= \frac{P_{\tau}(\omega) + P_{\gamma}(\omega) - \sqrt{\{P_{\tau}(\omega) - P_{\gamma}(\omega)\}^2 + 4K_{\tau\gamma}(\omega)^2}}{2} \end{aligned}\right\} \qquad (2)$$

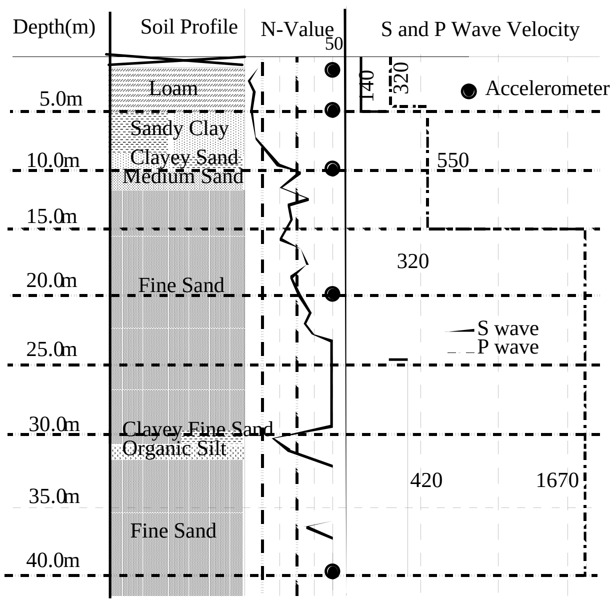
This relationship is considered to be equivalent to the relationship for linear viscous elastic model. Then, shear modulus and damping constant are obtained for the strain amplitude in frequency domain based on the

$$\left.\begin{aligned} G(\omega) &= \frac{\{a(\omega)^2 - b(\omega)^2\}\sin\{2\theta(\omega)\}}{2\{a(\omega)^2\cos^2\theta(\omega) + b(\omega)^2\sin^2\theta(\omega)\}} \\ h(\omega) &= \frac{a(\omega)b(\omega)}{\sin 2\theta(\omega)\{a(\omega)^2 - b(\omega)^2\}} \end{aligned}\right\} \qquad (3)$$

definition of the model and are expressed as equations(3) which are specified by the major and minor amplitude and the angle between major axis and strain axis.

3. STRESS AND STRAIN RELATIONSHIP IN TIME

° The stress and strain relationship in time domain was calculated by use of non-linear seismic response analysis code「YUSAYUSA」developed by [I.Tohata and N.Yoshida,1991]. Ground structure model at Chiba Experimental station of Institute of Industrial Science, University of Tokyo was used for the analytical model as shown in Figure.3 because high density array seismic observation has been carried out since 1982. Hence, Ramberg-Osgood model was used for the stress and strain relationship. And seismic records observed at the depth with GL-40m on the 1987 Chibaken-Tohooki earthquake whose magnitude was 6.7 was used as input motion. Hence, maximum acceleration was modified to be three times to the original amplitude.



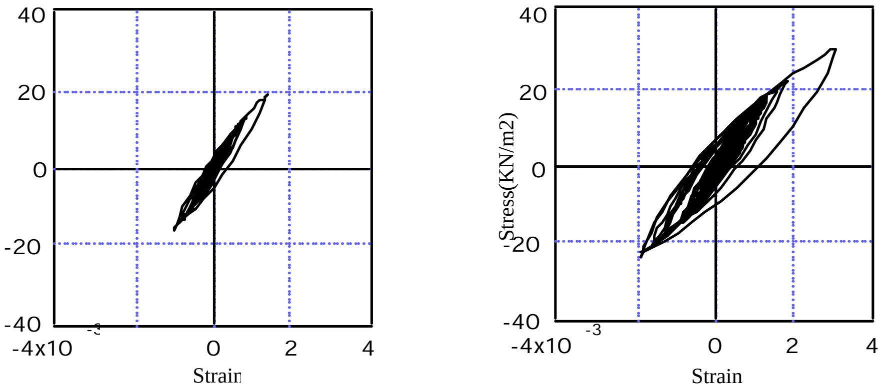
° **e**

Figure.3 Ground Structure at Chiba Experimental Station of Institute Science, University of Tokyo

The stress and strain hysteresis at two layers in surface ground is shown in Figures.4. Hence, maximum strain amplitudes at the two layers are 1.4×10^{-3} at the depth between GL-2m and GL-3m, 3.0×10^{-3} at the depth between GL-4m and GL-5m, respectively. Fourier spectrum of stress and strain in the layers are shown in Figure.4. It is found that both Fourier amplitude of strain and that of strain are predominant around the frequency between 1 and 6.5Hz and that the Fourier amplitude at the other frequency is remarkably smaller than those around the frequency mentioned above.

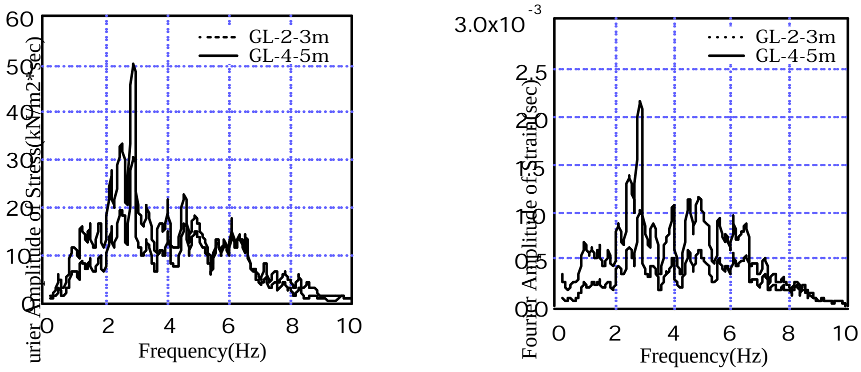
4.STRESS AND STRAIN RELATIONSHIP AND DYNAMIC DEFORMATION CHARACTERISTICS IN FREQUENCY DOMAIN

By use of the method mentioned above, the stress and strain relationship was calculated. The hysteresis at the frequency, 1.6,2.44 and 2.88Hz was shown in Figures.6. In order to verify the accuracy, the hysteresis were compared with the hysteresis calculated by the filter treatment of narrow band window as shown in Figures.7. It is found that both hysteresis are good agreement each other. Then, the method is useful to evaluate the stress and strain relationship in frequency domain.



a)GL-2-3m b)GL-4-5m

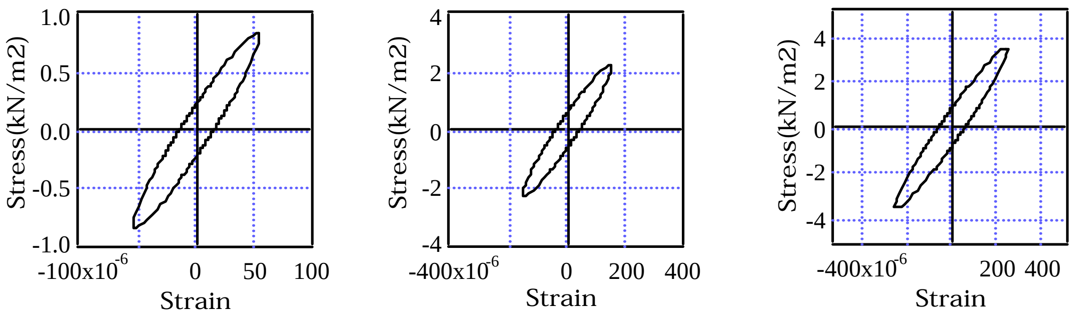
Figure.5 Stress and Strain Relationship in Time Domain



a)Shear Modulus b)Damping Constants

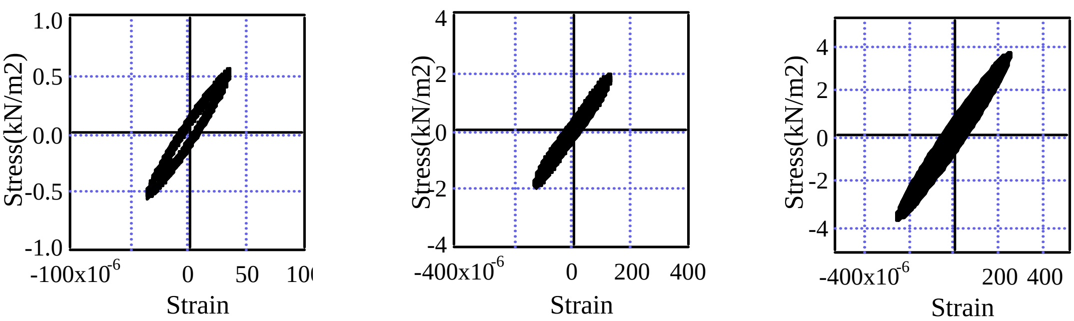
Figure.6 Frequency Characteristics of Deformation Characteristics of Soil

Next, the frequency characteristics of shear modulus and damping constant are shown in Figure.8. In comparison with the characteristics of strain as shown in Figure.4, it is found that shear modulus is smaller at the frequency which strain amplitude becomes to be large and that damping constant is larger at the frequency which strain amplitude becomes to be small.



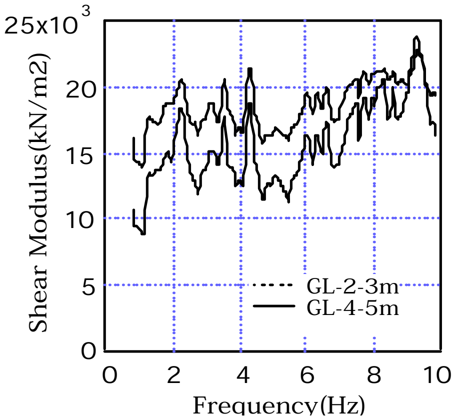
a)1.6Hz b)2.44Hz c)2.88Hz

Figure.6 Hysteresis obtained by Proposed Method

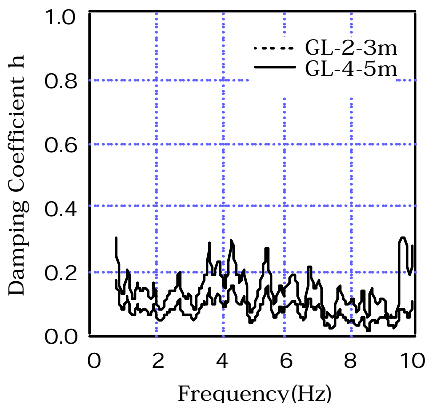


a)1.6Hz b)2.44Hz c)2.88Hz

Figure.7 Hysteresis obtained by Band Pass Filter Treatment



a)Shear Modulus

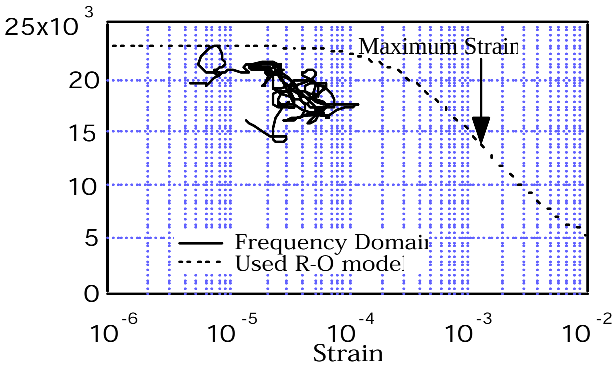


b)Damping Constants

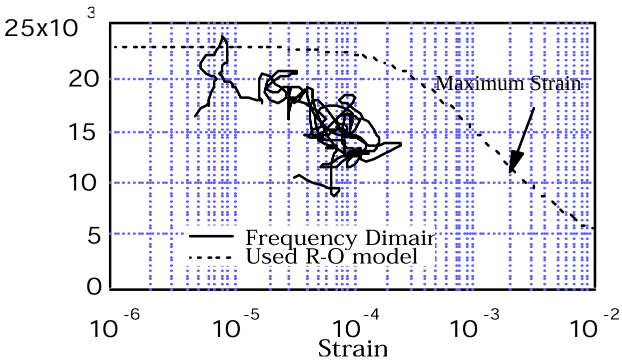
Figure.8 Frequency Characteristics of Deformation Characteristics of Soil

5.COMPARISON OF STRESS AND STRAIN RELATIONSHIP IN TIME AND FREQUENCY DOMAINS

As the comparison of stress and strain relationship in frequency with that in time, the relationship between strain and such a deformation characteristics of soil as shear modulus and damping constant are shown in Figure.9 and 10. The strain dependency of the deformation characteristics of soil used in seismic response analysis in time domain is also shown in these figures. It is found that each deformation characteristics of soil in frequency domain is depend on the strain as well as that in time domain and that stiffness reduction ratio in frequency domain is almost same with that in time domain. However, maximum strain in frequency domain at which maximum stiffuness reduction was observed is 10% to that in time domain.

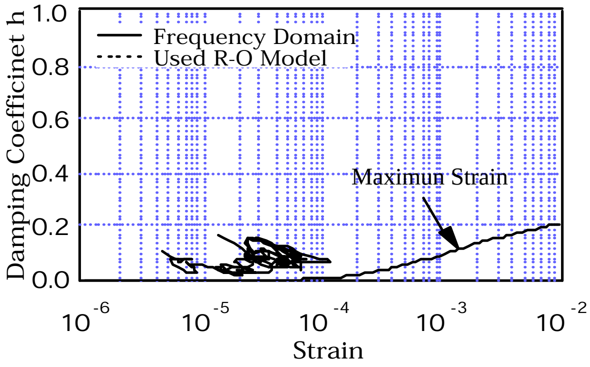


a)GL-2-3m

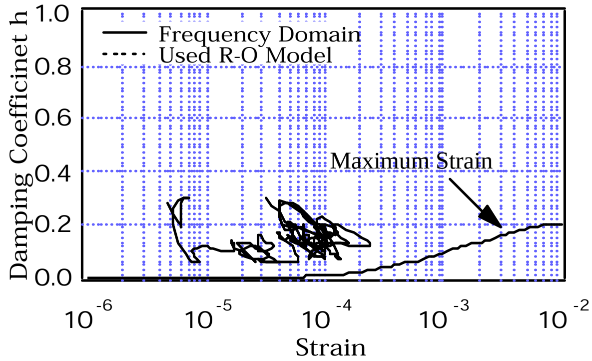


b)GL-4-5m

Figure.9 Comparison of Strain Dependency of Shear Modulus



a)GL-2-3m



b)GL-4-5m

Figure.10 Comparoson of Strain Dependency of Damping Constants

6.MODEL OF STRESS AND STRAIN RELATIONSHIP IN FREQUENCY DOMAIN

The deformation characteristics shown in the figure.9 indicates that the deformation characteristics in frequency domain is possible to be expressed by the characteristics in time domain based on reducing the strain in time domain. Therefore, stress and strain relationship in frequency domain is considered the similar function with that in time domain as shown in Figure.11. Hence, strain γ_f in the function of frequency domain has to be reduced as shown in equation(4) to strain γ_t in time domain.

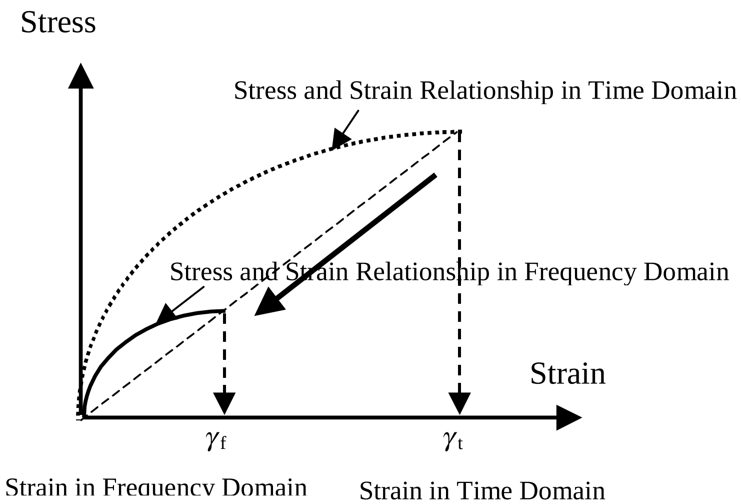


Figure.11 Basic Idea for Modeling Stress and Strain Relationship in Frequency Domain from that in Time domain

$$\gamma_f = c\gamma_t \tag{4}$$

The reduction coefficient is able to be evaluated by the following two ideas. The first one is as followings; If the stress and strain relationship in frequency domain is similar to that in time domain, the coefficient have to be depend on the reduction ratio of shear modulus. Then, the coefficient is determined by the strain ratio at the same reduction ratio between shear modulus in time domain and that in frequency domain. The other one is as followings; the coefficient is determined by the ratio between the maximum strain in frequency domain and that in time domain because stress and strain relationship in frequency domain is controlled by the hysteric in time domain. The coefficient based on the above two idea was calculated using the stress and strain relationship in frequency and time domain and was shown in Figure.12 . Hence, maximum value of the coefficient, 0.2 is used for the coefficient based on the same reduction ratio of shear modulus. And the idea for evaluating the coefficient is named as model 1. The idea for evaluating the coefficient based on the maximum strain ratio is named as model 2. Furthermore, the comparison of the stress and strain relationships based on the two idea for the coefficient is shown in Figure.13. In this figure, the relationship in frequency and time domain is also shown as white circle and solid line, respectively. Although the stress and strain relationship obtained by model 2 seems to be correspond with the mean value of the calculated relationship as shown in white circles, the relationship falls off around maximum strain. On the other hand, the relationship obtained by model 1 has the characteristics as the function along the maximum stress at any strain and is good agreement with the relationship in frequency domain around maximum strain. Then, model 1 seems to be better for evaluating the coefficient.

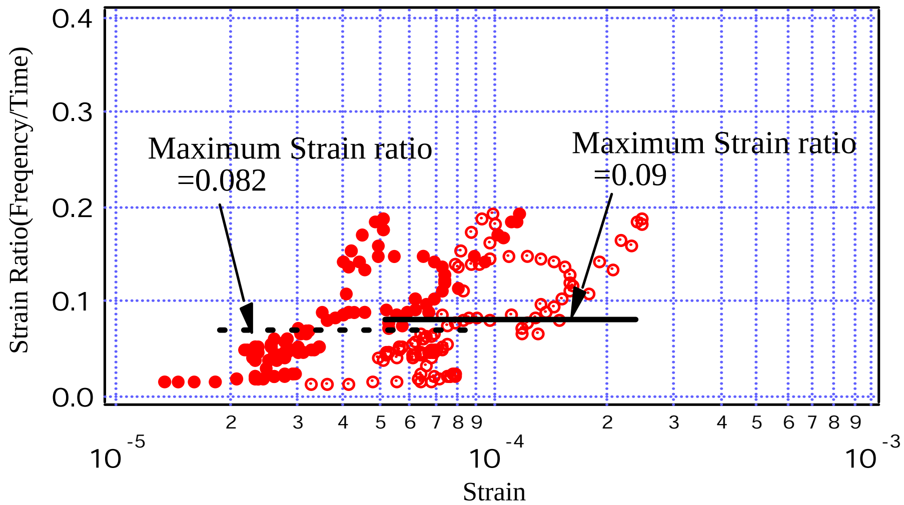


Figure.12 Comparison of the coefficient for two model

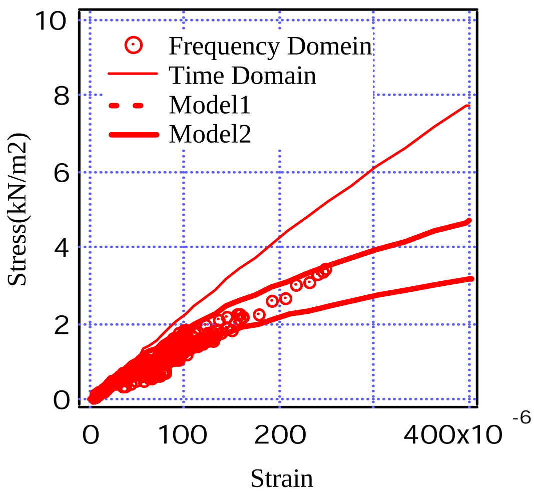


Figure.13 Comparison of stress and strain relationship obtained by two model for the coefficient between strain in frequency domain and that in time domain

7. CONCLUDING REMARKS

The new method based on Fourier transformation of hysteresis with respect to stress and strain in time domain was proposed to evaluate the relationship in frequency domain and was applied to evaluate that by use of hysteresis calculated by seismic response analysis in time domain. Furthermore, the model of the stress and strain relationship in frequency domain was proposed in comparison of the relationship in time with that in frequency domain.

As a results of these, it is found that the proposed method is useful to evaluate the stress and strain relationship in frequency domain and that the strain dependency is recognized in the deformation characteristics in frequency domain as well as that in time domain. Furthermore, it is made clear that the stress and strain relationship in frequency domain is able to be modeled to be similar with the stress and strain relationship in time domain. Hence, strain in frequency domain is specified as reduced strain for that in time domain. The accuracy of the model will have to be verified by seismic response analysis based on the new model.

ACKNOWLEDGEMENT

This study was financially supported by Grand-in-Aid for scientific research No.11115101 from The Ministry of Education of The Japanese Government.

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