

## MODELS OF RESIDENTIAL EARTHQUAKE LOSS BASED ON NON-LINEAR INSTRUMENTAL GROUND MOTION

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### SUMMARY

This study examines the relationship between observed residential loss during the 1994 Northridge earthquake and a range of linear and non-linear damage measures calculated from instrumental ground motion, to determine which instrumental measure is the best predictor of damage. The measure of predictive ability is the standard deviation of the prediction of observed loss, derived from least-square regression fits of observed loss to ground motion measures, compared to the estimation using Modified Mercalli intensity (MMI). Four measures of instrumental motion intensity are employed at each of two natural frequencies for a bi-linear structural response model. These are linear spectral response, ductility for a range of yield levels, “damage factor” (ratio of final to initial stiffness), and “normalized hysteretic energy” (ratio of non-linear to linear energy absorbed by the non-linear oscillator). Results show that 10 Hz models with a yield level of 0.2g give more accurate estimates for all three measures of non-linear damage, than MMI or elastic spectral acceleration. Ultimately the usefulness of any measure of ground motion must include the ability to predict that ground motion measure for future earthquakes. Current non-linear prediction methods yield slightly more accuracy than MMI-based methods, but future research in ground motion likely will reduce uncertainties further by accounting for source, path, and site effects.

### INTRODUCTION

Estimates of earthquake-induced losses to structures are important for emergency- and financial-planning purposes prior to future earthquakes, and are needed by insurers concerned about potential claims to insured properties. The Northridge, California earthquake (January 17, 1994) has abundant data in the form of observed losses to structures, recorded ground motions, Modified Mercalli intensities, and an understanding of the tectonic stress release that caused the earthquake. Thus this earthquake provides a combined data set that allows correlations among losses and ground motions to be examined, better than any other event in the US.

This study specifically examines the correlation between losses to residential structures during the Northridge earthquake and several measures of ground motion intensity. The ground measures examined are MM intensity and both linear and non-linear measures of ground motion amplitude, developed using both instrumental and synthetic sets of ground motions.

The main purpose of this study is to investigate methods of estimating residential losses as a function of ground motion. These methods can be used to pursue the development of better loss analysis methods and ground motion prediction tools that can be applied to future earthquakes. This includes developing the ability to estimate losses where a special tectonic condition exists (e.g. the buried thrust fault at Northridge) that would affect ground motions in the vicinity in a special way.

The measure of merit that is used here for the prediction of losses is the uncertainty in the final prediction. For the Northridge earthquake, one component of this uncertainty comes from the variability in losses from location to location, given the same fault distance and soil conditions. For future earthquakes additional components of

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uncertainty come into play, for instance from the predicted MM intensity or ground motion characteristics at sites given a future earthquake.

This paper presents a summary of results; further discussion is available in [5].

## RESIDENTIAL LOSS DATA

The residential loss database used here comes from surveys of losses to insured structures carried out by the California Dept. of Insurance [1]. These data were collected by the Dept. of Insurance for Los Angeles County by data calls to insurers in California. Residential losses accounted for 67% of insured losses during the Northridge event.

Note that these losses are as reported by insurers, so they are losses after deductibles on policies. We have no way of accurately correcting for the effect of deductible, which varied by insurer. The use of losses after deductible is acceptable for the main purpose of this study, which is to investigate correlations between losses and ground motion measures, both linear and non-linear.

Reported losses were available by postal ZIP code from the California Dept. of Insurance, and the damage ratio for all policies was used to determine losses, herein designated “observed losses.” It was assumed that losses occurred at the geographic centroid of each ZIP code, for the purposes of doing spatial correlations with ground motions. Only ZIP codes with more than 20 reported losses were included in the analysis, to avoid ZIPs with few reported losses and hence statistically unstable damage ratios in that ZIP. This resulted in a data base of 140 ZIPs with observed losses.

Figure 1 plots the observed losses vs. distance from the causative fault plane,  $D_f$ . For calculation of this distance, the source geometry derived by Wald and Heaton [10] was used. For later use, a regression was performed, regressing  $\ln(\text{observed loss})$  on  $\ln(D_f)$ . This resulted in the following equation:

$$\ln(\text{obs. loss}) = 11.965 + 8.056 \ln(D_f) - 1.776 (\ln(D_f))^2 \quad D_f > 9.7 \text{ km} \quad (1)$$

where loss is expressed as a fraction of policy value and where the quadratic form models saturation at close distances. The standard deviations of observations (of  $\ln[\text{obs. loss}]$ ) around the prediction is 0.885. Equation (1) is plotted in Figure 1; the term involving  $(\ln(D_f))^2$  was statistically significant and was retained.

## NORTHRIDGE EARTHQUAKE GROUND MOTIONS

Three categories of data were used to represent Northridge earthquake ground motions: MM intensity data, instrumental strong motion records obtained during the event, and synthetic ground motions (time histories) estimated using representative fault geometry, source characteristics, crustal structure, and site conditions. These data are described below, along with processing to estimate non-linear measures of damage.

**MM intensity data.** MM intensity (MMI) data were obtained from the US Geological Survey for ZIPs with observed losses. A regression equation was fit to these data as follows:

$$\text{MMI} = 4.673 + 3.324 \ln(D_f) - 0.805 (\ln(D_f))^2 \quad D_f > 7.9 \text{ km} \quad (2)$$

where  $D_f$  again is distance to the fault in km. The standard deviations of MMI observations around the equation was 0.66, and the  $(\ln(D_f))^2$  term was again found to be significant. Correlations of observed loss and MMI were used for comparison purposes, as described in the next Section.

**Instrumental records.** The Northridge earthquake was well recorded by strong motion instruments, and we used a database of 141 records (282 horizontal components of motion) from instruments maintained primarily by the California Division of Mines and Geology and the University of Southern California. All records were from free-field sites or basements of small buildings.

To estimate ground motions at ZIP code locations, the records were first processed to obtain linear and non-linear measures of damage, then interpolated spatially to obtain damage estimates at ZIP locations. Details of the interpolation method are described in [5]. Note that no correction of instrumental records for soil conditions was made, as we have no simple way to correct time histories or non-linear measures of damage to account for different soil types.

**Synthetic motions.** Synthetic ground motions were generated by W. Silva for 78 ZIP locations using a source model specific to the Northridge earthquake. These were the 78 ZIPs closest to the epicenter with observed losses greater than 0.06, the purpose being to concentrate on the most damaging motions and use these synthetic motions to check the instrumental results.

To develop these simulated ground motions, the location of each ZIP centroid was determined on a digitized map of surficial soil conditions, and ZIPs were categorized by USGS soil type A, B, or C. Shear-wave velocity profiles and rock/soil characteristics appropriate for these site categories were used for the near-surface materials. The computational scheme employed to compute the site response used an equivalent-linear approach employing random vibration theory (RVT). In this approach the control motion power spectrum was propagated through the one-dimensional soil profile using the plane-wave propagators of Silva [9]. In this formulation only SH waves were considered.

Soil parameters that were varied randomly in developing synthetic motions were the shear-wave velocity profile, depth to competent material, modulus reduction curves, and hysteretic damping curves. The result was a set of synthetic time histories of motion at each ZIP centroid, that represent the range of ground motions that might have occurred during the Northridge earthquake.

#### **Record processing to calculate damage measures.**

To measure quantitatively the damage potential of ground motion records, both the instrumental data set and the synthetic data set were processed to calculate responses of non-linear, single-degree-of-freedom (SDOF) oscillators subjected to the ground motions. Each SDOF oscillator was specified in terms of its natural frequency  $f_n$ , hysteretic damping (assumed to be 5% throughout this study), and yield point (specified by a spectral acceleration at frequency  $f_n$ ). A bilinear model was used with a strain-hardening ratio of 0.1.

Four measures of ground motion (one linear and three non-linear) were used to characterize the strength of shaking. First was the elastic spectral acceleration at  $f_n$ . Second was the ductility  $\mu$ , i.e. the maximum displacement of the SDOF oscillator divided by the yield displacement. Third was the “damage factor,” herein designated DF, which is the ratio of the initial stiffness to the secant stiffness for the maximum displacement of the SDOF oscillator. Fourth was the “normalized hysteretic energy,” herein designated NHE, which is defined as follows:

$$NHE = E_{total}/(2E_{el}) \quad (3)$$

where  $E_{total}$  is the total energy dissipated by the oscillator through elastic and plastic deformation, and  $E_{el}$  is energy dissipated during elastic response only. The first of these ground motion measures has units of g, and the last three are ratios.

## **RESULTS OF ANALYSIS**

### **Correlations of loss with MMI**

The first correlation of observed losses is with MMI. Figure 2 shows this correlation. A least-squares regression of observed losses L on MMI results in the following equation:

$$\ln(L) = -14.156 + 1.366 \text{ MMI} \quad (4)$$

with a standard error of  $\ln(L)$  of 0.880. Figure 2 indicates that some of the observations at MMI=5 might be censored by our exclusion of zero losses or very low losses (caused by fewer than 20 reported claims in a ZIP code). A second regression including only the data for MMI>6 yields the following equation:

$$\ln(L) = -14.777 + 1.449 \text{ MMI} \quad (5)$$

with a standard error of  $\ln(L)$  of 0.856. The latter equation is preferred.

Some perspective is appropriate here. MMI is the traditional way in which losses have been predicted (e.g. [2]), although peak acceleration has received some use in recent years (e.g. [6]). Thus for evaluation and comparison purposes, the correlation of  $\ln(L)$  with MMI and its residual standard error of 0.856 is appropriate. It is important to note that this is not the standard error to apply for *future* predictions of loss, as *observed* MMI values are used in Figure 2 and in developing equations (4) and (5). For future earthquakes we will have uncertainties on actual MMI values at each location (ZIP code) and these must be integrated with the standard errors discussed above.

### **Correlations of loss with instrumental ground motion.**

In developing correlations of loss with damage measures from instrumental ground motion, the frequency of the SDOF oscillator and its yield spectral acceleration (SA) must be specified. Choices examined here are:

- 10 Hz oscillator: yield SA of 0.05, 0.10, 0.20, 0.30g
- 5 Hz oscillator: yield SA of 0.1g
- 1 Hz oscillator: yield SA of 0.02, 0.05, 0.10, 0.20, 0.30g

The residential structures represented by the observed losses are primarily low-rise, woodframe buildings (single family residences, apartment buildings, condominium complexes) that will have high natural frequencies (around 10 Hz). The other frequencies were examined to gain a broader perspective on the relationship between observed losses and frequency content of the ground motion.

Results of regression  $\ln(L)$  on the four damage measures are summarized in Table 1 for the 10 Hz oscillator. This table gives regression coefficients for  $\ln(L)$  vs  $\ln(\text{ground motion measure})$  in the form:

$$\ln(L) = \text{const.} + c_1 \ln(\text{ground motion measure 1}) + c_2 \ln(\text{ground motion measure 2}) \quad (6)$$

where in some cases  $\ln(L)$  is regressed on one ground motion measure and in some cases on two, or on the square of  $\ln(\text{ground motion measure 1})$ . Figure 3 illustrates the correlation of observed loss with DF for 0.2g SA at yield.

The conclusion from Table 1 is that the non-linear measures of ground motion give more precise estimates of observed loss than does elastic SA. For 10 Hz the yield SA of 0.20g gave the most precise regression, indicating a residual standard error of 0.801 on  $\ln(\text{ductility})$  and 0.788 on  $\ln(\text{DF})$ . Adding SA to the regression on ductility or DF improves the precisions of the regressions only slightly. Interestingly, the coefficient of elastic SA for these two-parameter regressions is negative, meaning that for a given ductility or DF, a high elastic response spectrum causes less damage than a lower elastic response. This is logical: if a structure is driven to a given ductility, more damage will occur if it goes through several cycles to get to that ductility, than if the structure goes through only one large cycle of non-linearity.

Results of regressing  $\ln(L)$  on 5 Hz and 1 Hz damage measures are not reported here. These regressions indicated generally larger standard deviations than for 10 Hz. [1]

### **Correlations of loss with synthetic ground motion.**

As described in Section 4, synthetic ground motions were generated at 78 of the ZIP code locations, six random horizontal motions were processed for damage measures, and these measures were averaged to obtain the measures for that ZIP code. Observed losses were regressed on damage measures, but the slope of the correlations had significantly less statistical precision than for instrumental ground motion due to the smaller range of ground motion values (and observed losses) considered by the 78 sets of synthetic ground motions. Also, the synthetic motions are based on source mechanism, path, and site conditions only, and do not account for actual unexplained variations in real motion. Thus we do not present these regressions and rely on comparisons with instrumental results to draw conclusions.

In general the results from synthetic motions were similar to those from instrumental motions (see Figure 3), indicating that synthetics are a rational way to predict earthquake damage. The synthetic motions did not identify areas of large damage, but these would be difficult to predict by any method.

### PREDICTION OF LOSSES FOR FUTURE EARTHQUAKES

The prediction of losses for future earthquakes involves predicting an intermediate ground motion measure such as MMI, SA(10 Hz), or SA(1 Hz), then predicting the loss given that measure of motion. To assess the best prediction method we must account for the precision with which we can predict the intermediate ground motion measure, as well as the precision of the loss estimate given that measure.

We can represent the loss  $L$  with a linear equation on a ground motion measure  $G$  (which might be MMI,  $\ln(\text{SA at 10 Hz})$ , etc.):

$$\ln(L) = a_1 + b_1 G + \epsilon_L \quad (7)$$

The ground motion measure is a quadratic function of  $\ln(\text{fault distance } D_f)$ :

$$G = a_2 + b_2 \ln(D_f) + b_3 [\ln(D_f)]^2 + \epsilon_G \quad (8)$$

Substituting (8) into (7) gives an estimate of  $\ln(L)$  as a function of  $\ln(D_f)$ :

$$\ln(L) = a_1 + a_2 b_2 \ln(D_f) + b_1 b_3 [\ln(D_f)]^2 + b_1 \epsilon_G + \epsilon_L \quad (9)$$

which has variance:

$$\sigma_{\ln(L)}^2 = b_1^2 \sigma_{\epsilon_G}^2 + \sigma_{\epsilon_L}^2 + 2 \rho b_1 \sigma_{\epsilon_G} \sigma_{\epsilon_L} \quad (10)$$

where

$\sigma_{\ln(L)}$  = standard deviation of observed loss,

$\sigma_{\epsilon_G}$  = standard deviation of ground motion (MMI,  $\ln \text{SA (10 Hz)}$ ,  $\ln \text{SA (1 Hz)}$ , etc.)

$\sigma_{\epsilon_L}$  = standard deviation of observed loss given a value of the ground motion,

$\rho$  = correlation coefficient between deviations of ground motion from their predicted values and deviations of  $\ln(L)$  from their predicted values given  $G$ .

This formulation recognizes that ground motion prediction is uncertain, that loss prediction given the ground motion is uncertain, and that there are correlations in the deviations of the two from their predicted values. All of these factors affect the precision of loss predictions. This formulation follows that in [3].

Table 2 summarizes the values for  $b_1$ ,  $\sigma_{\epsilon_L}$ ,  $\sigma_{\epsilon_G}$ , and  $\rho$  used in equation (10), and lists the calculated value of

$\sigma_{\ln \text{ loss}}$ . From Table 2 it is apparent that all 10 Hz measures of ground motion except NHE give estimates of loss that are slightly more precise than MMI. Also, the 10 Hz ground motion deviations are negatively correlated with loss. Note that  $\sigma = 1$  means that one standard deviation corresponds to a factor of 2.7 uncertainty in loss, and  $\sigma = 0.9$  corresponds to a factor of 2.46.

## CONCLUSIONS

This investigation of reported residential losses from the Northridge earthquake has allowed us to correlate observed losses with damage measures from instrumental ground motions. Using correlation of loss with observed MMI for comparison, elastic SA at 10 Hz is marginally less precise in predicting observed loss, but non-linear measures of motion (ductility, calculated damage factor, and normalized hysteretic energy) are in some cases more precise in predicting observed loss.

Prediction of future losses is always made using some ground motion measure as an intermediate parameter. To evaluate the precision of alternative ground motion measures, we must take account of correlations of deviations in predicted ground motions, and deviations in loss given an observed level of ground motion. These correlations are negative for MMI and 10 Hz ground motion measures. The total uncertainty in predicting future losses has a standard deviation corresponding to a factor of 2.4 to 2.7, and is slightly more accurate using 10 Hz ground motion measures than MMI.

The implication is that ground motions can be characterized by quantitative parameters with no loss in precision, and in some cases with a gain in precision. This is important for methods of predicting future ground motions, and indicates the characteristics of motion that are important to replicate for loss estimates.

The standard deviations of MMI and linear (and non-linear) instrumental ground motion measures calculated here are lower than what we would expect for future earthquakes characterized only by a magnitude and fault distance. Given current research and data collection trends, it is likely that scatter in instrumental measures will decrease in the future through better modeling, but scatter in MMI will not. This implies that instrumental methods to emphasize loss will gain precision in the future.

An additional perspective, of course, is that there should be more emphasis on improving loss-estimation techniques, i.e., on reducing  $\sigma_{\in L}$ . This has the potential to increase the accuracy of predictions more than marginal decreases in  $\sigma_{\in G}$ .

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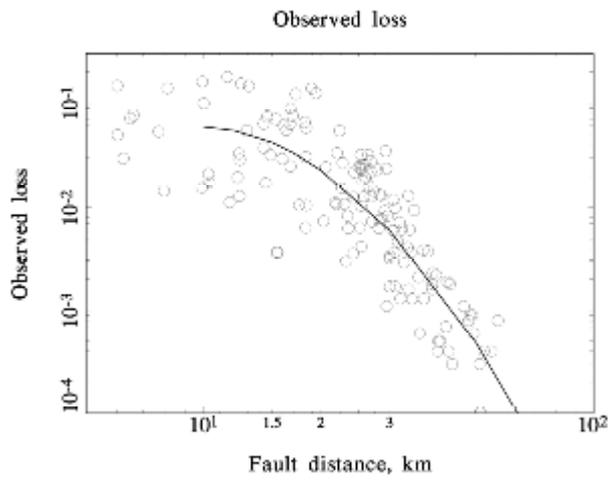
**Table 1 Regressions of ln(observed loss) on ln(ground motion measure) for 10 Hz Oscillator**

SA yield, g	at	Elastic SA	Ductility	DF	NHE*	Regression constant	Residual deviation	standard
—		2.953	—	—	—	-2.25	.932	
0.05		—	2.314	—	—	-14.74	1.163	
0.05		—	—	4.301	—	-14.30	1.466	
0.05		—	—	—	64.05, -7.407	-142.90	1.554	
0.10		—	3.475	—	—	-11.83	1.172	
0.10		—	—	3.167	—	-10.86	1.059	
0.10		—	—	—	-4.193, 0.944	-3.301	1.078	
0.20		—	2.380	—	—	-8.14	.801	
0.20		—	—	2.470	—	-7.87	.788	
0.20		—	—	—	0.293, 0.200	-7.59	.819	
0.20		-1.128	3.211	—	—	-10.28	.793	
0.20		-1.704	—	3.772	—	-10.95	.768	
0.20		1.884	—	—	0.355	-4.136	.912	
0.30		—	2.353	—	—	-1.080	.842	
0.30		—	—	2.456	—	-6.774	.894	
0.30		—	—	—	0.669, 0.0652	-6.241	.822	

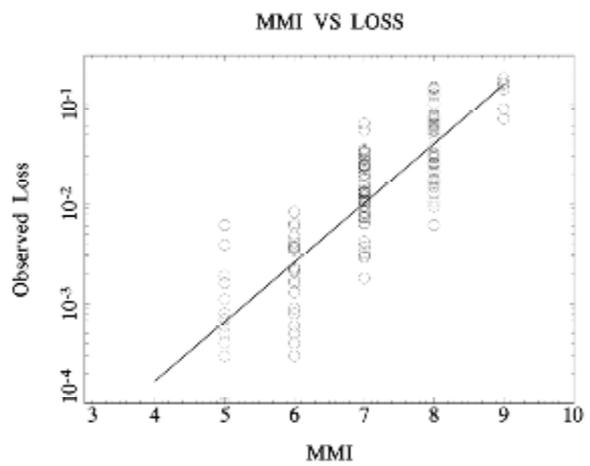
\* where two coefficients are shown, they are for ln(NHE) and [ln(NHE)]<sup>2</sup>.

**Table 2 Standard Deviation of Loss for Predictions (Equation (10))**

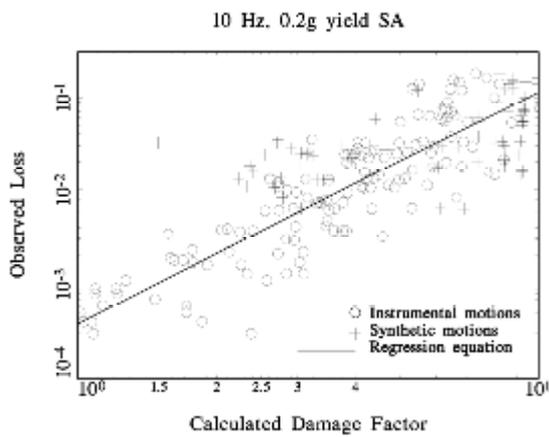
Ground Motion	b1	$\sigma \in L$	$\sigma \in G$	$\rho$	$\sigma \ln \text{loss}$
MMI	1.449	.856	.661	-.433	.970
SA (10 Hz)	2.953	.932	.217	-.358	.923
duct (10 Hz)	2.380	.801	.299	-.270	.917
DF (10 Hz)	2.470	.788	.258	-.182	.919
NHE (10 Hz)	1.895	.819	.749	-.432	1.297



**Figure 1** Observed loss vs. distance, Northridge earthquake



**Figure 2** Observed loss vs. MMI, Northridge earthquake



**Figure 3** Observed loss vs. Damage Factor DF