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ON RAYLEIGH WAVES PROPAGATION IN LAYERED SOILS

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ABSTRACT

A numerical solution for the evaluation of the dynamic response of soils subjected to Rayleigh waves propagation is described. In the model, the soil, that could have different properties with depth, is represented by a system of lumped masses.
The exciting source due to environmental vibrations, seismic shear waves, or pulsating loads (vibrodyne), is expressed in terms of imposed vertical and horizontal accelerations. The exciting accelerogram at the ground surface is decomposed in terms of wave propagation frequencies and for each frequency it has been determined the phase velocity of the wave at the ground surface. Once determined the frequency-velocity relationship it is possible to evaluate the vertical and horizontal wave propagation from the source to any point of the layered soil. The knowledge of the wave propagation law allows to evaluate the dynamic response at every point of the layered system due to travelling waves, in terms of transfer functions. A validation of this model is carried out at the Piana di Catania (Sicily), comparing numerical results and experimental data in terms of velocity and displacements. As exciting source a vibrodyne of 2 tons was employed.

KEYWORDS

Rayleigh waves, layered soils, dispersion curves, transfer function, vibrodyne

1 - INTRODUCTION

The surface waves are subjected, along the direction of propagation, to the attenuation of horizontal and vertical displacements due to mechanical and geometrical phenomena of dissipation , and due to the effects of dispersion (Richart et al, 1970). It is known that the attenuation due to geometrical dissipation is due to the reduction of energy of vibration with the distance R from the source, because the wave involves spherical volumes of soil. The law of attenuation of the displacements with the distance can be described by the factor $1/R^{1/2}$ (Lamb, 1904). The mechanical dissipation is typical of inelastic materials and it is caused by dissipation of energy due to hysteresis phenomena (Hardin, 1965).

The dispersion is the variation of the wave velocity in relation to the frequency of vibration (Ewing et al., 1957). The surface waves, in an elastic half-space, have a velocity of propagation independent by the frequency and have only one mode shape of vibration. Instead in layered soil the velocity is dependent by the frequency and the propagation is due to the different modes of vibration. The dispersion curve is the correlation between the wave velocity and the frequency of vibration (fig.1), in which is clear that, with the increase of the frequency the velocity of propagation tends to the velocity of propagation of an elastic half-space with the same dynamic properties of the superficial layer (Taniguchi e Sawada, 1979). From fig. 1 it can be seen if a layered soil is resting on an elastic base, the Rayleigh wave velocity for the fundamental mode of vibration assumes the value of wave velocity of the elastic base, for the frequency $\omega = 0$. At higher frequencies the upper soil layer is mainly vibrating and its velocity is that one of an uniform half space of the given soil. If a layered soil is based on a rigid half-space, the dispersion curves tend to assume an infinite velocity, for the frequency $\omega = 0$. For higher frequencies, the R-wave velocity of layered system tends to assume the velocity of the homogeneous elastic half-space, because the horizontal transmission of the energy is condensed in the upper layers.

2 - DISPERSION CURVES FOR A LAYERED SOIL.

2.1 - THE LUMPED MASS SYSTEM

The layered continuous system is simulated by an equivalent lumped masses system (Lysmer, 1970), as shown in fig. 2. All the elements are chosen with the same width h and thickness b. For each element are known the phisical and mechanical properties (density ρ, shear modulus G, Lame's constant λ and Poisson's coefficient ν).
The subdivision in finite elements, on the base of Zinkiewicz's and Cheung's theory (1967), implies three assumptions:
a) concentration of mass of each rectangular element at the corners; b) variation of horizontal and vertical displacements according to the law: $\delta x = C_1 x + C_2 y + C_3 xy + C_4$, $\delta y = C_5 x + C_6 y + C_7 xy + C_8$, with C_S (s = 1,...,8) arbitrary constants and x and y horizontal and vertical coordinates; c) the transmission of forces between all elements is through the corners only, where the masses are concentrated. This approach leads to the equation of dynamic equilibrium in a typical form:

$$[K(k) - \omega^2 M]\, \underline{u} = \underline{0} \qquad [1]$$

where K is the real and symmetric stiffness matrix, related to the wave number $k = \omega / V_R$, , M is the mass matrix, $\underline{u}$ the displacements vector and V_R is the Rayleigh wave velocity.

The equation [1], once fixed the wave number k, leads to a standard eigenvalue-eigenvector problem, where the eigenvalues are the frequencies of vibration and the eigenvectors the wave mode shapes. The equation of equilibrium [1] also may be expressed in the inverse form, in terms of the wave number k:

$$[k^2 \cdot K_2 + k \cdot K_1 + K_0 - \omega^2 M]\, \underline{u} = \underline{0} \qquad [2]$$

in this case, fixing the frequency ω, it must be solved a generalized eigenvalue problem, with the inverse iteration technique (Wilkinson, 1965).

2.2 - THE STIFFNESS MATRIX

The displacements and forces vectors for a single element of the mesh J, shown in fig.3, are: $\{\delta\}^T = \{\delta_1, \delta_2, \delta_3, ..., \delta_8\}$ and $\{P\}^T = \{P_1, P_2, P_3, ..., P_8\}$respectively. The relation between displacements and forces of the J element is:

$$\{P\} = \{K\}_J \,\{\delta\} \qquad [3]$$

where $\{K\}_J$ is the symmetric stiffness matrix 8x8 of the rectangular element (Lysmer, 1970), related to the elastic parameters G and λ, and to the geometry of the single element.

To convert the two-dimensional problem into a one-dimensional problem, as shown in fig. 4, it is necessary that the width h of the rectangular element tends to zero. For superficial harmonic waves, we can write the horizontal and vertical displacements in the form: $\delta_x = u_x \cdot \exp i(\omega t\text{-}kx)$ and $\delta_y = iu_y \cdot \exp i(\omega t\text{-}kx)$; thus the global relationship forces-displacements for the whole system (fig. 4) is:

$$\{Q\} = [K]\{U\} \qquad [4]$$

where: $\{U\}^T = \{U_1\ U_2\ U_3\ ...\ U_{2N}\}$ is the horizontal and vertical vector of N layered elements; {Q}is the vector of the applied forces in the N masses, and [K] is the global stiffness matrix, obtained assembling the simple matrixes 4 x 4 $[K]_J$ of each single layer, given by Lysmer (1970). It is clear that the stiffness matrix [K] is a simple function of wave number k, when the phisical-mechanical parameters of the layers are known.

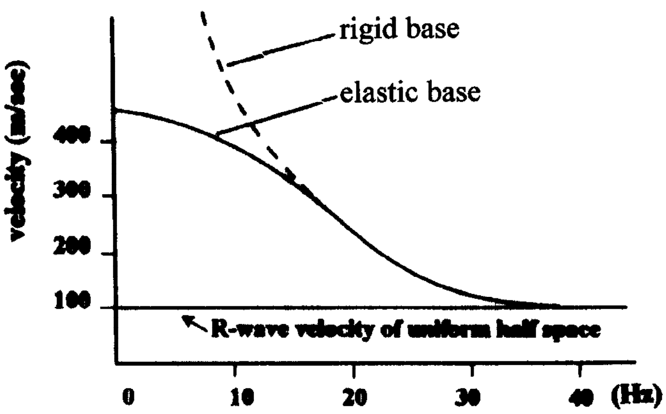


FIG. 1 Dispersion curves of the Rayleigh wave

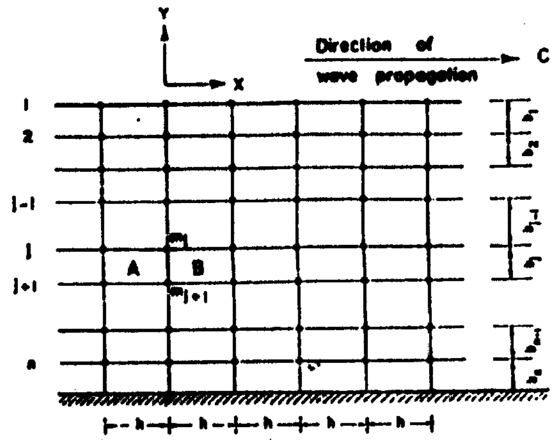


FIG.2 Lumped mass model for layered system (Lysmer, 1970)

2.3 THE MASS MATRIX

In a one-dimensional system with lumped masses, as described above, the mass matrix has non-zero terms along the main diagonal only and the generic term is:

$$m_j = (b_{j-1}\, \rho_{j-1} + b_j\, \rho_j)/2 \qquad [5]$$

with: b_j = height of the layer j and ρ_j = mass density of the layer j. Because each mass m_j yields two inertial components, one horizontal and one vertical, the mass matrix elements 2Nx2N, are given by $m_{2j-1,2j-1} = m_{2j,2j} = m_j$, with j=1,2,.....,N.

2.4 - EQUATION OF MOTION AND DISPERSION CURVES

Imposing the dynamic equilibrium between elastic and inertial forces, we can write the equation of motion for the m_j mass, subjected to the forces of overhanging and underlying layers: $-\omega^2 U_{2j-1}\, m_j = -Q_{2j-1}$ and $-i\omega^2 U_{2j}\, m_j = -iQ_{2j-1}$ respectively. Writing in a compact form the equations of motion for the N masses, we obtain:

$$\omega^2 [\,M\,] \{\,U\,\} = \{\,Q\,\} \qquad [6]$$

Substituting to the forces vector {Q} his expression given by [4], the determination of unknown displacements leads to the eigenvalue problem:

$$([\,K\,] - \omega^2 [\,M\,]) \{\,U\,\} = \{0\} \qquad [7]$$

Since the stiffness matrix is defined in relation to the wave number, once fixed a k value, we obtain a particular system among the family of system [7], its solution gives a ω_i eigenvalues, for every i-th mode of vibration, and so phase velocity $V_i = \omega_i / k$. The dispersion curve can be obtained plotting ω_i and V_i for several k values.

3 DYNAMIC RESPONSE OF LAYERED SOIL TO THE PROPAGATION OF R-WAVES

3.1 DEFINITION OF TRANSFER FUNCTION

Displacements, velocity and acceleration in a generic point of the soil, at distance x from the surface wave source, are been evaluated through the transfer function, which allows the conversion of the kinematic input of source of motion into the output vector in a generic point of the half-space. Particularly, following a procedure by Gomez-Masso et al. (1983), given a known input acceleration at the top of the first stratum, we can write the Fourier's series expressions as follows:

$$\ddot{y}(t) = \sum_{S=0}^{N-1} \ddot{Y}_S \exp(i\omega_s t) \qquad [8]$$

with: $\ddot{y}(t)$ = acceleration in time domain

$\ddot{Y}_S$ = complex amplitude, evaluated with the F.F.T. technique

$\omega_S = 2\pi s/(N \cdot t)$ = circular frequency of s-th harmonic

Expanding this Fourier'series to the whole vector of displacements {U}, for all 2N degrees of freedom of the system, each j-th U_j displacement, time depending, can be written:

$$U_j(t) = \sum_{S=0}^{N-1} U_{js} \exp(i\omega_s t) \qquad [9]$$

in which U_{jS} is the j-th complex displacement, s-th frequency depending, valuable by the F.F.T. technique (Hall, 1982).

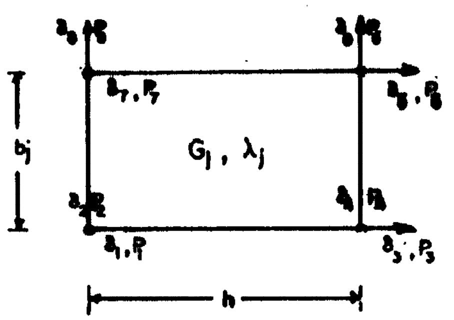


FIG.3 Forces and displacements of rectangular element (Lysmer, 1970)

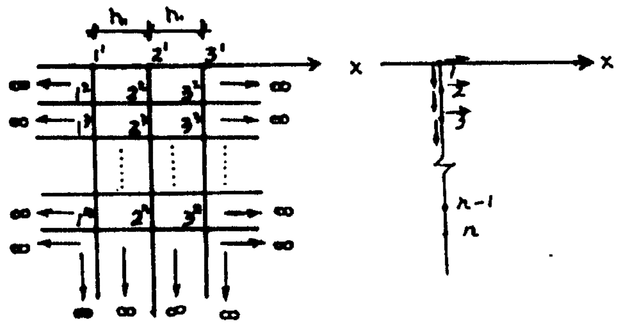


FIG.4 Element mesh of a layered half-space (Chen et al., 1991)

Writing the [9] in vectorial form, we obtain the expression good for all the displacements:

$$\{U\} = \sum_{S=0}^{N-1} \{U\}_S \exp(i\omega_S t) \qquad [10]$$

Similarly to the complex response method, each harmonic of the expression [10] can be treated in indipendent way, so it is possible to define a vector $\{A\}_S$ as a transfer function:

$$\{U\}_S = \{A\}_S \cdot \ddot{Y}_S \qquad [11]$$

this expression allows the conversion from acceleration to displacement of the complex amplitude, for a generic harmonic. The vector of transfer funtions is known, and from the [10] we have:

$$\{U\} = \sum_{S=0}^{N-1} \{A\}_S \cdot \ddot{Y}_S \exp(i\omega_S t) \qquad [12]$$

Applying the I.F.F.T. technique can be rebuilded the displacement function from frequency domain to time domain.

3.2 EXPRESSION OF TRANSFER FUNCTION

The R-wave transfer funtion can be obtained by the technique described by Lysmer (1978). It is assumes that between two contiguous layers, the displacements change in linear way and, below them, there is a rigid half-space. With these assumptions the general motion of a N-horizontal layered system, subjected to a vibration of ω_S frequency, is given by:

$$\{U(x)\}_S = \sum_{j=1}^{2N} R_j \{V\}_j \exp[i(\omega_S t - k_j x) \qquad [13]$$

with: x = direction of wave propagation
$k_j = k_j(\omega_S)$ = wave number of ω_S frequency
$\{V\}_j = \{V(\omega_S)\}_j$ = eigenvector
$R_j = R_j(\omega_S)$ = modal partecipation factor

In the expression [13] only the modal partecipation factors are unknown, which cannot be evaluated for all the N vibration modes. Assuming that the fundamental mode of vibration is predominant on the other frequencies, as confirmed by experimental data shown in fig.5 (Tokimatsu, 1995), the displacement vector, connected with a generic s-th frequency, is expressed by:

$$\{U\}_S = R_S \cdot \{V\}_S \cdot \exp\,[i(\omega_S t - k_S x)] \qquad [14]$$

The acceleration amplitude of the free-field motion is:

$$\{\ddot{U}\}_S = -\omega^2_S \cdot \{U\}_S = -\omega^2_S \cdot R_S \cdot \{V\}_S \qquad [15]$$

where the complex acceleration component for the ω_S frequency is known in the point of wave source. For the R-th degree of freedom can be derived: $Y_S = -\omega^2_S \cdot R_S \cdot V_{SR}$ and $R_S = -\ddot{Y}_S / (\omega^2_S \cdot V_{SR})$, where V_{SR} is the R component of the correspondent eigenvector of the s-th harmonic. Comparing the previous formulas we found:

$$\{A\}_S = [-1/(\omega^2_S V_{SR})]\ \{V\}_S \qquad [16]$$

It is clear that for ω_S=0 the transfer function is not defined, so it is necessary to fix Y_0=0.

In conclusion, the soil response to the Rayleigh waves can be obtained in the frequency domain, once it is known the $\ddot{y}(t)$ vector at the source, evaluating the Y_S complex amplitude, so for each s-th harmonic the wave number related and the {A} transfer function can be evaluated through the [16]. At last when it is known the complex amplitude of the displacements in the form $\{U\}_S = \{A\}_S\ Y_S$, we can evaluate the displacement vector {U} by the I.F.F.T. technique in the form [12].

4 - RESPONSE OF THE SOIL TO THE R-WAVES AT PIANA DI CATANIA.

4.1 - SOIL DYNAMIC CHARACTERIZATION

The Piana di Catania is formed by clays of various consistency with frequent intercalation of slime-sandy strata soil up to the the depth of about 40 m. Particularly a typical stratigraphy for the zone investigated is given by a higher stratum of soft clay, of poor quality up to a depth of about 10 m. This superficial layer is overhanging to a more consistent plastic clay, with increasing properties with the depth. The mechanical and dynamic properties of the clays of the Piana di Catania are well-known, by the accurate geotechnical investigation including in situ and in laboratory tests, either in static or dynamic field (Maugeri, 1983; Carrubba and Maugeri, 1985, 1988). On the basis of the results obtained from these investigations the dynamic characterization reported in tab.1 can be deduced for the test site. As far as the ν Poisson's coefficient is concerned, we have assumed its value constant for all the layers considered (tab. 1). However a parametric study of the dynamic response is made considering three distinct values 0.25, 0.30 and 0.35.

4.2 THE R-WAVES SOURCE BY VIBRODYNE

The design of an industrial building for the production of electronic components, sited on the piana di Catania, required the execution of a geotechnical investigation in dynamic field, because it was necessary to limit the amplitude of the displacements of the building less then 10^{-6} cm. The dynamic in situ vibration tests, consisted in the recording, with seismometer placed on the ground and on the near buildings (fig.6), of the environmental vibration and of the forced vibration given by vibrodyne with known amplitude and frequency.

Particularly, regarding the forced vibration tests, the soil was excited by a vertical force generated by a vibrodyne of 2 tons, installed on a monolithic concrete block of 14 tons. The vibrations on the ground due to the vibrodyne excitation were measured by 18 accelerometers.

The vibrodyne is a mechanical vibrator which generates a sinusoidal force with known amplitude and frequency, changeable into settled limits. The variations of imposed vibrations were possible by changing the position of two eccentric masses placed on two mutually opposed rotating disks on a vertical plane.

The results of seismic tests above described were fully reported by Maugeri (1983) and analyzed by Maugeri and Frenna (1988). The vibrations were impressed for a number of cycles such that they can be considered stationary and representable by a force with a sinusoidal law: $f_v(t) = F_v \,\text{sen}(\omega t+\phi_v)$, where: F_v = amplitude of excitating force, ω = pulsation of excitating force and ϕ_v = phase angle of excitating force. The vibrodyne employed allows variations in frequency between 1 - 23,8 Hz; the amplitudes of the exciting forces related to the frequency chosen are reported in fig. 7.

4.3 THE DISPERSION CURVES OF THE PIANA DI CATANIA CLAYS

The adopted model is a layered soil on a rigid half-space. The soil was subdivided in 12 layers with constant height of 1.5 m., for a total sampled depth of 18 m. Because the depth of soil crossed by Rayleigh waves is equal to the wave length, so the frequency of the waves will have values of wave length less of 18 m. On the basis of mechanical and phisical characteristics reported in tab.1, the evaluation of the dispersion curve for the first mode of vibration is reported in fig.8, for several values of ν Poisson's coefficient.

From fig.8 it is clear that for the frequency of 2.345 Hz, for a 18 meters wave length, equal to the total thickness of the soil, and for the wave number 0.35, there is a vertical asymptote, caused by the impossibility to sample the underlying strata, because we assumed the presence of an infinitely rigid bedrock. The curve of fig.8 is typical for this kind of models, likewise Lysmer (1970) found for analogous systems.

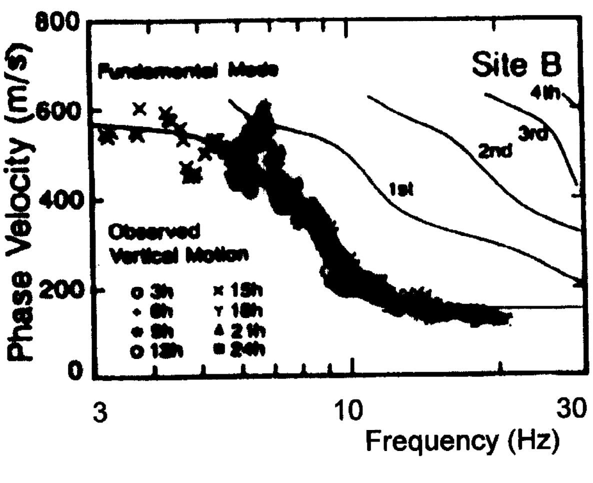


FIG.5 Dispersion curves compared with experimental data (Tokimatsu, 1995)

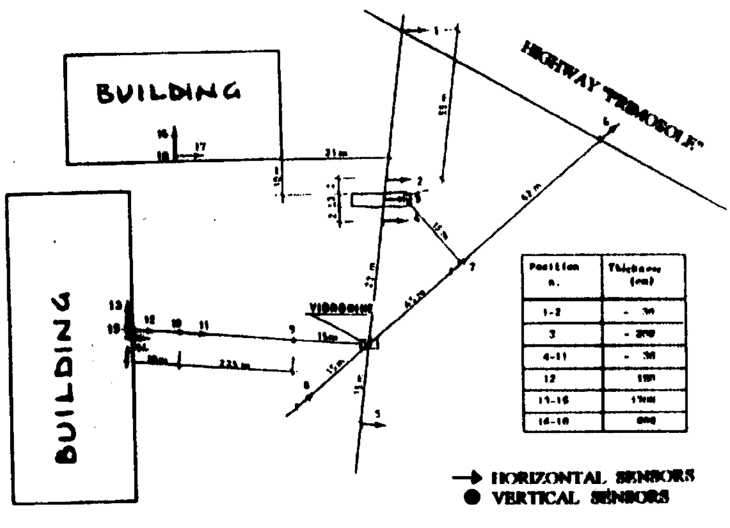


FIG.6 Position of the sensors

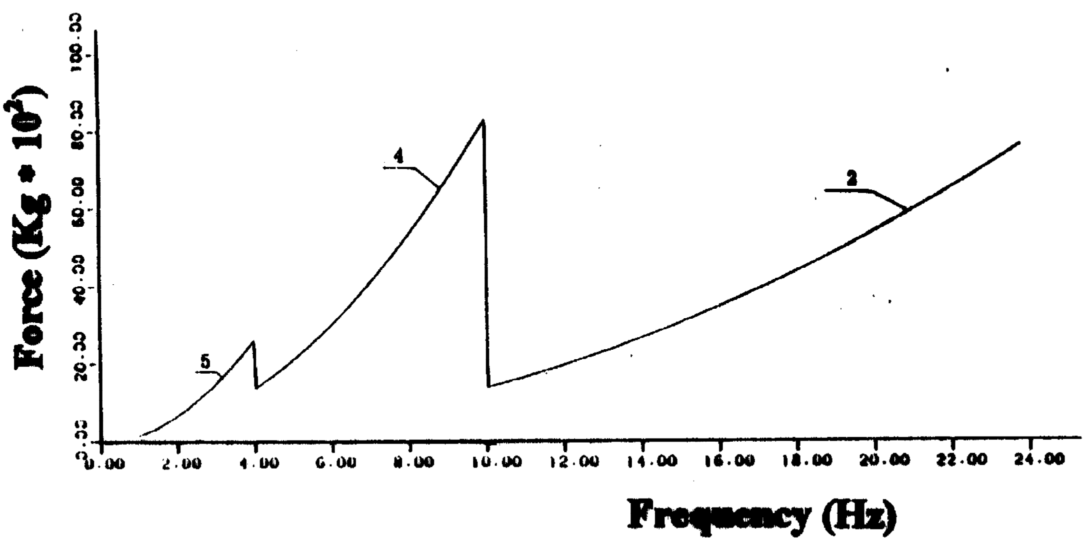


FIG. 7 Force of vibrodyne in different experiments

N	H (m)	G (KN/m²)	V_s (m/s)	ρ (KN/m³)
1	1.5	[illegible]	[illegible]	1.835
2	1.5	[illegible]	[illegible]	1.835
3	1.5	[illegible]	[illegible]	1.633
4	1.5	[illegible]	110	1.633
5	1.5	[illegible]	110	1.633
6	1.5	[illegible]	[illegible]	1.633
7	1.5	[illegible]	[illegible]	1.791
8	1.5	[illegible]	[illegible]	1.835
9	1.5	[illegible]	177	1.770
10	1.5	[illegible]	[illegible]	1.989
11	1.5	[illegible]	[illegible]	1.813
12	1.5	[illegible]	[illegible]	1.813

TABLE 1 Characteristics of layers

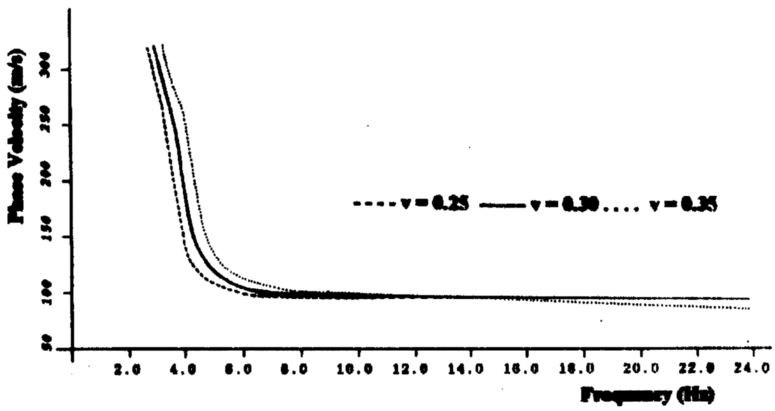


FIG.8 Dispersion curves for elastic layered soil on rigid base

4.4 THE TRANSFER FUNCTIONS

On the basis of the tests carried out, the empirical transfer functions were derived to compare it with that evaluated by numerical solution. Since the Rayleigh waves have only vertical and horizontal displacements along the direction of motion, the comparison between experimental and theoretic results was extended only to the recording stations number 6,7,8,9,10,11, for which the sensibility axis of the seismographs is parallel to the direction of vibration of R-wave.

Considering the vibrodyne as a localized source, the transfer h_{jv} function is the ratio V_j/V_v between the velocity of the generic j position and the velocity at the vibrodyne position. The transfer function can be written:

$$V_j = V_v \,(1 / r_j)^{1/2} \, \exp(-\alpha \, r_j) \qquad [17]$$

where: r_j = distance of the j position of recording from the point of application of the force
α = attenuation's coefficient $\xi 2\pi\omega/V_R$

The shear dumping ratio ξ, for a very small value of strain $\gamma \cong 10^{-4}$ mobilized by the vibrodyne (Maugeri and Frenna, 1988), can be assumed as $\xi = 0.02 \div 0.05$, and for these values several transfer functions have been evaluated.

The velocity V_R of the R-waves is related to the frequency, calculable by the dispersion curve (fig. 8).

In the figures 9 and 10 are reported, with regard to the frequency of vibration, the transfer functions of velocity, whether obtained by theorical way or deduced by experimental data.
Also it was investigated the attenuation of the vibrations between two generic position of recording, which are not source of vibrations. In this case the horizontal pair positions 6 - 7 and vertical 9 - 10 have been considered. To take into account the geometrical and mechanical attenuation, it was applied the Bornitz's formula (1931) in the form:

$$u_A = u_B(r_B / r_A)^n \exp[-\alpha(r_A-r_B)] \qquad [18]$$

where: u_A , u_B = horizontal or vertical displacements in the recording points
r_A, r_B = distance between the source and the recording points
α = attenuation coefficient
n = exponent to be assumed 0.5, for a localized source of the wave.

The results of the analysis are shown in figures 11 and 12.

5 - DISCUSSION AND CONCLUSION

The theoretical and experimental transfer functions show, for the higher frequencies, a good agreement even if only the first mode of vibration has been taken into account; a decrease of values with the increase of the frequency can be noted. For the low frequencies, a scattering between theoretical and experimental curves can be due to the assumptions made for the model. With an infinitely rigid base under the layered stratum the numerical solution so far developed is not able to take into account the interaction with the existing elastic base. Because of that the numerical dispersion curve shows an increasing of R-velocity with decreasing of frequency, while the experimental data does not show any increasing of the R-velocity when the frequency is less then 10 Hz. However the numerical procedure can be extended to consider the case of elastic base under the layered soil stratum; also the inverse problem can be solved. At last, it can be underlined that the Bornitz's attenuation law, deduced for the vertical vibrations, can be extended also to the evaluation of the horizontal vibrations; as a good agreement can be seen between theoretical and experimental results (figures 11 and 12).

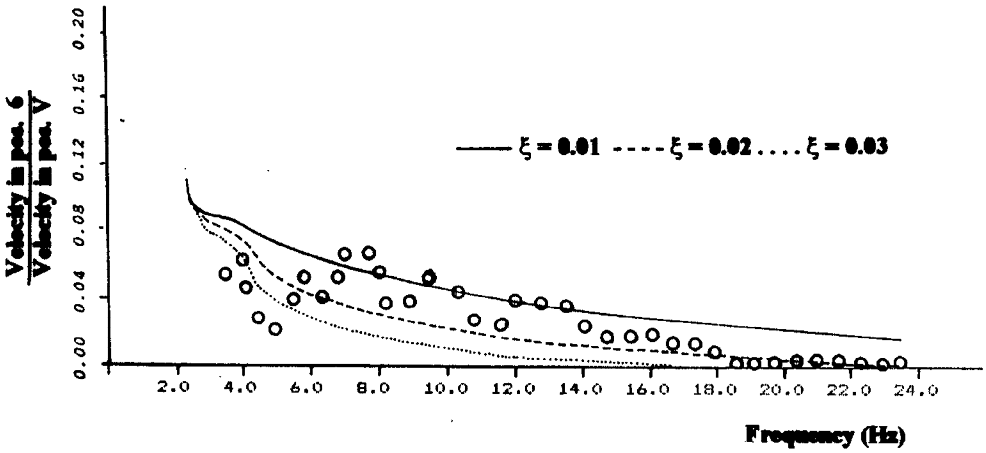


FIG.9 Velocity observed in position 6 compared with theoretical solution

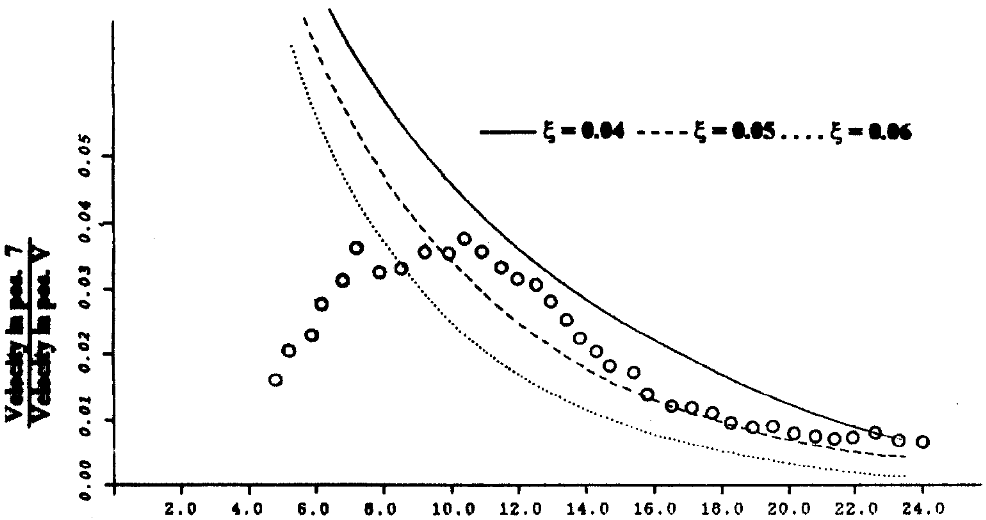
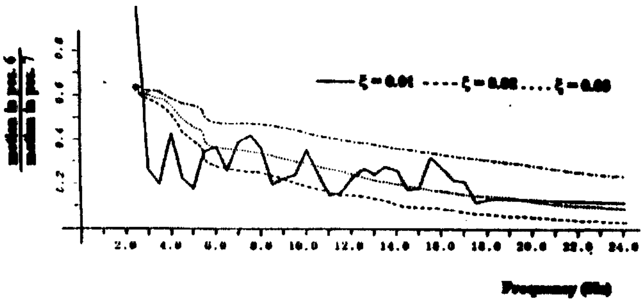


FIG. 10 Velocity observed in position 7 compared with theoretical solutions



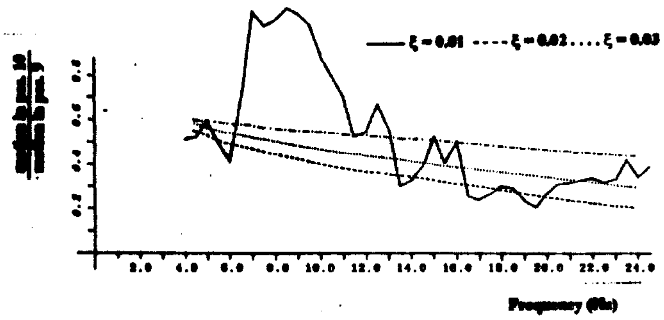


FIG. 11 Motion in pos.6 and 7 compared with theoretical solutions

FIG.12 Motion in pos.10 e 9 compared with theoretical solutions

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